

- **Edge detection** - large wavelet coefficients coincide with image edges.
- **Edge clustering** - the edge coefficients within each sub band tend to form spatially connected clusters

During a two level of decomposition of an image using a scalar wavelet, the two-dimensional data is replaced with four blocks. These blocks correspond to the sub bands that represent either low pass filtering or high pass filtering in each direction. The procedure for wavelet decomposition consists of consecutive operations on rows and columns of the two-dimensional data. The wavelet transform first performs one step of the transform on all rows. This process yields a matrix where the left side contains down sampled low pass coefficients of each row, and the right side contains the high pass coefficients. Next, one step of decomposition is applied to all columns; this results in four types of coefficients, HH, HL, LH and LL.

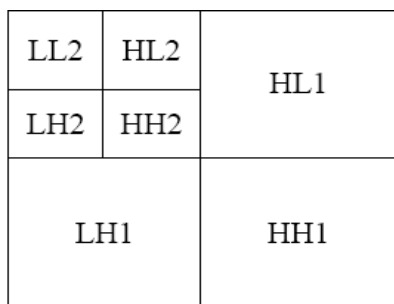


Figure 1: Two-Level Image decomposition by using DWT

3. Introduction to Denoising

De-noising plays a very important role in the field of the medical image pre-processing. It is often done before the image data is to be analyzed. Denoising is mainly used to remove the noise that is present and retains the significant information, regardless of the frequency contents of the signal. It is entirely different content and retains low frequency content. De-noising has to be performed to recover the useful information. In this process much attention is kept on, how well the edges are preserved and how much of the noise granularity has been removed [8][9]. The main purpose of an image-denoising algorithm is to eliminate the unwanted noise level while preserving the important features of an image. In wavelet domain, the noise is uniformly spread throughout the coefficients while mostly the image information is concentrated in the few largest coefficients. The most important way of distinguishing information from noise in the wavelet domain consists of thresholding the wavelet coefficients. Mainly hard and soft thresholding techniques are performed

4. Methodology

The reduction of noise present in images is an important aspect of image processing. Denoising is a procedure to recover a signal that has been corrupted by noise. After discrete wavelet decomposition the resulting coefficients can be modified to eliminate undesirable signal components. To implement wavelet thresholding a wavelet shrinkage method

for de-noising the image has been verified. The proposed algorithm to be used is summarized in Algorithm 1 and it consists of the following steps

4.1 Algorithm 1: Wavelet image de-noising

- Choice of a wavelet (e.g. Haar, symmlet, etc) and number of levels or scales for the decomposition. Computation of the forward wavelet transform of the noisy image.
- Estimation of a threshold.
- Choice of a shrinkage rule and application of the threshold to the detail coefficients.
- Application of the inverse transform (wavelet reconstruction) using the modified (threshold) coefficients.

4.2 Flow Diagram

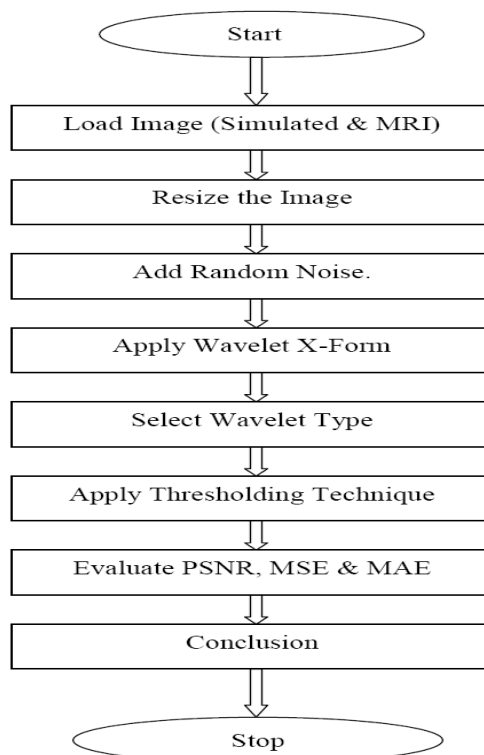


Figure 2: Flowcharts for Image Denoising Algorithm Using Wavelet Transform

For better and easy understanding, a complete flowchart of the discussed methodology has been shown above. The main algorithm, followed in order to fulfill the aim of this thesis, is as follows:

Step 1:

Read Simulated and original standard image (MRI.jpg, BRAIN.tif).

Step 2:

Resize the loaded image to a standard size of 256 × 256. The images taken for test have a lot of variation in their sizes and hence cannot be compared on the same basis. For large sized images, such as 512 × 512, the computation time for denoising is found to be more. And if the image size is taken

smaller than 256×256 , then the useful data is liable to get lost.

Step 3:

Random Noise is added to the standard test images.

Step 4:

Make the noisy image to undergo wavelet transform i.e. DWT.

Step 5:

Now select the desired Wavelet & Level. After that noisy image is decomposed into approximation and detail coefficients using wavelet transform.

Step 6:

Select the desired thresholding technique (Global, Level-Dependent & Optimal). After the decomposed image coefficients are threshold using the above mentioned three thresholding technique, the denoised image is reconstructed using inverse wavelet transforms- IDWT.

4.3 Thresholding Technique

Thresholding is the simplest method of image denoising. In this from a gray scale image, thresholding can be used to create binary image. Thresholding is used to segment an image by setting all pixels whose intensity values are above a threshold to a foreground value and all the remaining pixels to a background value. Thresholding is mainly divided into two categories:

A. Hard Thresholding: Hard threshold is a "keep or kill" procedure and is more intuitively appealing. The transfer function of the Hard thresholding is shown in the figure. Hard thresholding may seem to be natural. Sometimes pure noise coefficients may pass the hard threshold and this thresholding method is mainly used in medical image processing.[10][11].

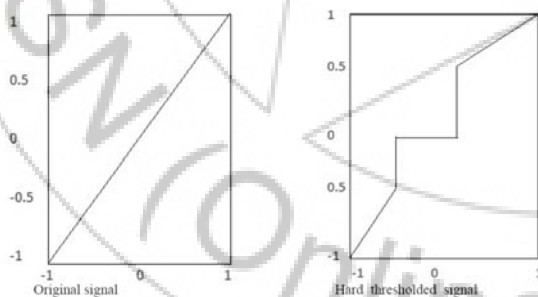


Figure 3: Original and Hard thresholded signal

B. Soft Thresholding: Soft threshold shrinks coefficients above the threshold in absolute value. The false structures in hard thresholding can be overcome by soft thresholding. Now a days, wavelet based denoising methods have received a greater attention. Important features are characterized by large wavelet coefficient across scales in most of the timer scales.[10]

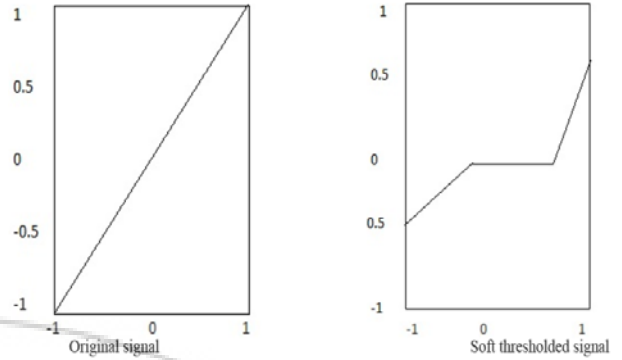


Figure 4: Original and Soft thresholded signal

4.4 Selection of Thresholding Technique

In the application of De-noising, the threshold level parameter T plays an essential role. Values too small cannot effectively get rid of noise component, while values too large will eliminate useful signal components. There are a variety of ways to determine the threshold value T as we will discuss in this section Depending on whether or not the threshold value T changes across wavelet scales and spatial locations, the thresholding can be:

1. Global Threshold: The global threshold method derived by Donoho is given by Eq. (2) has a universal threshold:

$$\lambda = \sigma \sqrt{2 \log(N)} \tag{2}$$

Where N is the size of the coefficient arrays and σ^2 is the noise variance of the signal samples.

2. Level Dependent Threshold: Level dependent thresholding method is done by using Eq. (3). Estimation of the noise standard deviation σ_k is done by using the robust median estimator in the highest sub-band of the wavelet transform

$$\lambda_k = \sigma_k \sqrt{2 \log(N)} \tag{3}$$

Where the scaled MAD noise estimator is computed by:

$$\sigma_k = \frac{MAD_k}{0.6745} = \frac{(median(|w_i|))_k}{0.6745} \tag{4}$$

Where MAD is the median absolute deviation of the magnitudes of all the coefficients at the finest decomposition scale and w_i are the coefficients for each given sub-band, the factor 0.6745 in the denominator rescales the numerator so that σ_k is also a suitable estimator. The threshold estimation method is repeated for each sub-band separately, because the sub-bands exhibit significantly different characteristics.

3. Optimal Threshold Estimation: Estimate the mean square error function to that compute the error of the output to minimize the function, the minimum MSE serves as a solution to the optimal threshold. A function of the threshold value which is minimized is defined in Eq. (5).

$$G(\lambda) = MSE(\lambda) = \frac{1}{N} ||y - y_\lambda||^2 \tag{5}$$

If y_λ is the output of the threshold algorithm with a threshold value λ and y is the vector of the clean signal, the remaining noise on this result equals $e_\lambda = y_\lambda - y$. As the notation indicates, the MSE is a function of the threshold value λ .

Find the optimal value of λ that minimizes MSE (λ) and the convergence of the algorithm.

Step 7:

Then three parameters, PSNR (peak signal to noise ratio), MAE (mean absolute error) and MSE (mean square error) are calculated for all the standard images with their noisy and denoised counterparts, respectively. Hence, we get a good amount of comparison between the noisy and denoised images keeping the set standard image intact.

Step 8:

A usual way to de-noise is to find a processed image such that it minimizes mean square error MSE, MAE and increases the value of the PSNR. Hence depending upon the values of above three parameter, we conclude that which wavelet & Thresholding technique gives best denoised result.

5. Performance Evaluation

To get the measure of the wavelet performance, the experimental results are evaluated according to three error criteria namely, the mean square error (MSE), the mean absolute error (MAE) and the peak signal to noise ratio (PSNR).

1. **Peak signal to Noise Ratio (PSNR):** PSNR measures the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the quality and reliability of its representation. It defines the purity of the output signal. PSNR is calculated as follows:

$$PSNR = 10 \cdot \log_{10} \left(\frac{MAX_I^2}{MSE} \right) = 20 \cdot \log_{10} \left(\frac{MAX_I}{\sqrt{MSE}} \right) \quad (6)$$

Where, MSE = Mean Squared Error, MAX_I is the maximum possible pixel value of the image.

2. **Mean Squared Error (MSE) :** Mean Square Error (MSE) function is commonly used because it has a simple mathematical structure that is easy to compute and it is differentiable implying that a minimum can be sought. The MSE is the difference between the original image and the denoised image. Given by

$$MSE = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (y(m,n) - \hat{y}(m,n))^2 \quad (7)$$

3. **Mean of absolute error (MAE) :** Another criterion measure include: Mean of absolute error (MAE) which is given by

$$MAE = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |y(m,n) - \hat{y}(m,n)| \quad (8)$$

The goal of de-noising is to find an estimate image such that MAE is minimum.

6. Results

For our test experiments we have considered an additive noise with a uniform distribution which has been used to corrupt our simulated and real MR test image objects. Artificially adding noise to an image allows us to test and assess the performance of various wavelet functions.

We used MATLAB to implement the de-noising algorithm. MATLAB has a wavelet toolbox and functions which are very convenient to do the DWT. A usual way to de-noise is to find a processed image such that it minimizes mean square error MSE, MAE and increases the value of the PSNR.

We have done simulations with uniform random noise added to the MR image. An example of a noisy magnetic resonance image (MRI) which consists of 256X256 pixels is shown in Fig. 5. As can be seen in the background the image has been uniformly corrupted with additive noise. The de-noising techniques discussed in the previous section are applied to the noisy MR image to test the efficiency of the different threshold methods

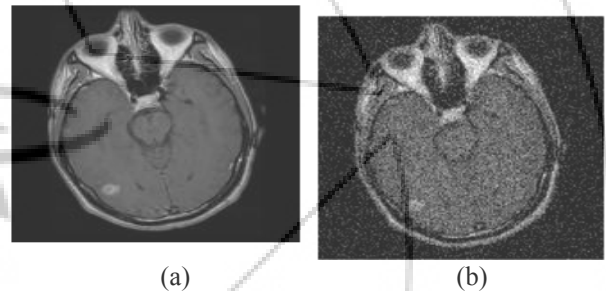


Figure 5: (a) Original image and (b) Noisy image

For comparison of the five different wavelet functions, the quantitative de-noising results of the MRI images obtained by using global, level-dependent and optimal thresholding are shown in Table I and II respectively. The MSE, MAE, PSNR error criteria are the ones which have been used to assess the performance of the wavelet functions. Their numerical results are summarized in the tables.

Table 1: Qualitative analysis (MRI image) - Level-dependent Thresholding

Type of Wavelet	LEVEL 1		LEVEL 2			
	MSE	MAE	MSE	MAE	MSE	MAE
Haar	0.0087	0.0739	20.859	0.0130	0.0887	19.377
db2	0.0085	0.0729	21.176	0.0121	0.0864	19.936
db4	0.0082	0.0718	21.551	0.0121	0.0864	19.151
sym2	0.0084	0.0721	22.102	0.0119	0.0859	19.429
sym4	0.0083	0.0721	21.481	0.0116	0.0852	19.849
bior1.1	0.0092	0.0758	20.794	0.0132	0.0898	19.409
bior1.3	0.0093	0.0766	20.299	0.0135	0.0899	19.410

• It is clear from the table I, for Level Dependent thresholding technique; sym2 gives best result for level-1 & db2 performs well for level-2.

Table 2: Qualitative analysis (MRI image) - Optimal Thresholding

Type of Wavelet	LEVEL 1			LEVEL 2		
	MSE	MAE	MSE	MAE	MSE	MAE
Haar	0.009	0.075	20.814	0.008	0.0717	21.612
db2	0.008	0.07	21.398	0.009	0.0752	21.032
db4	0.007	0.069	21.882	0.009	0.0746	21.589
sym2	0.008	0.07	21.64	0.009	0.0756	20.972
sym4	0.008	0.07	21.628	0.009	0.0751	21.225
bior1.1	0.007	0.069	22.164	0.008	0.0736	21.424
bior1.3	0.007	0.068	22.244	0.008	0.0738	21.912

- From the comparison results it can be observed, that the bior1.3 wavelet & optimal thresholding technique gives greatly improved de-noising results for both level-1 & level-2.
- Hence from the above tables, we observed that for both Simulated & MRI Image, bior Wavelet & Optimal Thresholding technique gives the best denoised results. Its gives higher PSNR & lower MSE & MAE value.

7. Conclusion

The de-noising process consists of decomposing the image, thresholding the detail coefficients, and reconstructing the image. The decomposition procedure of the de-noising example is accomplished by using the DWT. Wavelet thresholding is an effective way of de-noising as shown by the experimental results obtained with the use of different types of wavelets. Thresholding methods implemented comprised of the level (sub- band) thresholding and optimal thresholding. More levels of decomposition can be performed; the more the levels chosen to decompose an image, the more detail coefficients we get. But for de-noising the noisy MR data sets, two-level decomposition provided sufficient noise reduction

In this paper we have presented the generalization of the DWT method for the 2-D case. The resulting algorithms have been used for the processing of noisy MR image. Experimental results have shown that despite the simplicity of the proposed de-noised algorithm it yields significantly better results both in terms of visual quality and mean square error values. Considering the simplicity of the proposed method, we believe these results are very encouraging for other forms of de-noising. The Biorthogonal wavelet (bior1.1) & Biorthogonal wavelet (bior1.3) gave the best results compared to other wavelets for both Simulated & MRI image respectively. Optimal thresholding gives better denoised result among the three thresholding technique.

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