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Bayesian Estimation of Parameter of Inverse Maxwell Distribution via Size-Biased Sampling

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Abstract: ***In this paper, we have discussed Bayesian estimation of the parameter of an Inverse Maxwell distribution via Size-Biased sampling. Bayes estimators of the scale parameter θ of the Inverse Maxwell distribution under squared error, precautionary, entropy, and another two loss functions for using quasi-prior have been obtained. The risk functions of these estimators relative to squared error loss function have been obtained for the sake of comparison. The corresponding graphs have also been plotted.***

Keywords: Bayes theorem, squared error loss function, precautionary loss function, entropy loss function, quasi prior, Risk function

1. Introduction

In Bayesian approach it is assumed that parameter θ is itself a random variable (though unobservable) with a known distribution called prior distribution. The prior distribution (specified prior) is modified in the light of the available data to determine a posterior distribution (the conditional distribution of θ given the data), which summarizes what can be said about θ on the basis of the assumptions made and the data at hand.

Let $f(y \mid \theta); \theta \in \Theta$ be the probability density function of lifetime distribution of a component or an animate, where the parameter space Θ is known but the true value of θ is unknown. Let g (θ) be the prior density function of the random variable θ. Let $\underline{y} = (y_1, \dots, y_n)$ be a n independent observations from f(y;θ). Then using Bayes' theorem (1763) the posterior distribution $f\left(\theta \mid \underline{y}\right)$ of θ is given by

$$f\left(\theta \mid \underline{y}\right) = \frac{f\left(\underline{y} \mid \theta\right) g(\theta)}{\int_{\Theta} f\left(\underline{y} \mid \theta\right) g(\theta) d\theta} \qquad (1)$$

where $f\left(\underline{y} \mid \theta\right)$ is the joint probability density function of $\underline{y} = (y_1, \dots, y_n)$. For a given sample $\underline{y}$, the posterior p.d.f. $f\left(\theta \mid \underline{y}\right)$ is the basis for most types of Bayesian inference. In order to define Bayes estimators we must specify a loss function

$L(\hat{\theta}, \theta) \geq 0$, for all $\hat{\theta}$ and θ ;

The corresponding Bayes risk is defined as the expected value of the risk $R(\hat{\theta}, \theta)$ with respect to the prior distribution g(θ) on Θ and is given as,

$$r(\hat{\theta}, \theta) = E[R(\hat{\theta}, \theta)] = \int_{\Theta} R(\hat{\theta}, \theta)\, g(\theta) dy$$

where the risks function $R(\hat{\theta}, \theta)$ is defined as

$$R(\hat{\theta}, \theta) = \int_{\chi} L(\hat{\theta}, \theta)\, f\left(\underline{y} \mid \theta\right) d\underline{y}$$

where χ stands for the sample space of $\underline{y}$. The fundamental problems in Bayesian analysis is that of the choice of prior distribution g(θ) and loss function $L(\hat{\theta}, \theta)$ which may be appropriate for the situation at hand.

2. Prior Distributions

A prior distribution of a parameter is the probability distribution that represents your uncertainty about the parameter before the current data are examined. Multiplying the prior distribution and the likelihood function together leads to the posterior distribution of the parameter. We use the posterior distribution to carry out all inferences. We cannot carry out any Bayesian inference or perform any modeling without using a prior distribution.

In Bayesian analysis the fundamental problem is that of the choice of prior distribution $g(\theta)$ and a loss function L(.,.). Let us consider a suitable prior (quasi–prior) for θ to obtain the bayes estimators in this case assuming independence among the parameters is:

$$\mathbf{g(\boldsymbol{\theta}) = \frac{1}{\boldsymbol{\theta}^d};\ \theta > 0,\ d > 0} \qquad (2)$$

Loss Functions

The Bayes estimation $\hat{\theta}$ of θ is the course optimal relative to the loss function chosen. A commonly used loss function is the squared error loss function (SELF)

$L(\hat{\theta}, \theta) = \left(\hat{\theta} - \theta\right)^2$

(a) Squared error loss function (SELF)

$$\mathbf{L(\hat{\boldsymbol{\theta}}, \boldsymbol{\theta}) = (\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^2} \qquad (3)$$

The Bayes estimator under the above loss function, say is $\hat{\theta}$ the posterior mean, i.e.

$$\hat{\boldsymbol{\theta}}_B = E_{\pi}(\boldsymbol{\theta}) \qquad (4)$$

The risk function is given by:

$$R_B(\hat{\boldsymbol{\theta}}) = E_{\theta}(\hat{\boldsymbol{\theta}})^2 - 2\boldsymbol{\theta} E_{\theta}(\hat{\boldsymbol{\theta}}) + \boldsymbol{\theta}^2 \qquad (5)$$

(b) Precautionary Loss Function

Norstrom (1996) introduced an alternative asymmetric precautionary loss function and also presented a general class of precautionary loss functions with quadratic loss function as a special case (Srivastava,R.S.,et al. (2004)). A very useful and simple asymmetric precautionary loss function is given as

$$L(\hat{\theta}, \theta) = \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}} \qquad (6)$$

The posterior expectation of loss function in (6) is

$$E_{\pi}[L(\hat{\theta} - \theta)] = E_{\pi}\left(\frac{\theta^2}{\hat{\theta}}\right) + E_{\pi}(\hat{\theta}) - E_{\pi}(\theta) \qquad (7)$$

The value of $\hat{\theta}$ that minimize (7), denoted by $\hat{\theta}_p$ is obtained by solving the following equation

$\frac{d}{d\theta} E_\pi[L(\hat{\theta} - \theta))] = 0$

$$\hat{\theta}_P = [E_\pi(\theta^2)]^{\frac{1}{2}} \tag{8}$$

(c) Entropy Loss Function

In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\frac{\hat{\theta}}{\theta}$. In this case, Calabria and Pulcini (1994) points out that a useful asymmetric loss function is the entropy loss given by

$$L(\delta) = [\delta^p - p\log_e(\delta) - 1] \tag{9}$$

where

$\delta = \frac{\hat{\theta}}{\theta}$,

The posterior expectation of loss function in (9) is

$$E_\pi[L(\delta)] = b\left[E_\pi\left(\frac{\hat{\theta}}{\theta}\right) - E_\pi\left(\log_e\left(\frac{\hat{\theta}}{\theta}\right)\right) - 1\right] \tag{10}$$

The value of $\hat{\theta}$ that minimum (10), denoted by $\hat{\theta}_e$ is obtained by solving the following equation

$\frac{d}{d\theta} E_\pi[L(\Delta)] = 0$

$$\hat{\theta}_e = \left[E_\pi\left(\frac{1}{\theta}\right)\right]^{-1} \tag{11}$$

(d) Loss function-L₁ :

Consider the loss function given by

$$L_1(\hat{\theta}, \theta) = \left(\frac{\hat{\theta}}{\theta} - 1\right)^2 \tag{12}$$

The Bayes estimator under loss function- $\mathbf{L_1}$, say $\hat{\theta}_1$ using the value of $f(\theta|\underline{y})$,

$$\hat{\theta}_1 = \frac{E_\pi\left(\frac{1}{\theta}\right)}{E_\pi\left(\frac{1}{\theta^2}\right)} \tag{13}$$

(e) Loss function-L₂ :

Consider the loss function given by

$$L_2(\hat{\theta}, \theta) = \left(\frac{\theta}{\hat{\theta}} - 1\right)^2 \tag{14}$$

The Bayes estimator under loss function- $\mathbf{L_2}$, say $\hat{\theta}_2$ using the value of $f(\theta|\underline{y})$,

$$\hat{\theta}_2 = \frac{E_\pi(\theta^2)}{E_\pi(\theta)} \tag{15}$$

In this paper, we have considered the Bayesian estimation problem of the scale parameter of a Inverse Maxwell Distribution via Size- Biased sampling using the squared error loss function, precautionary, entropy, and other two loss functions under quasi- prior.

When observation is selected with probability proportional to their size, the resulting distribution is called size-biased. Statistical analysis based on size-biased samples has been studied in detail since the early 70's. The concept of length-biased sampling was mainly developed by Rao (1977) and Zelen & Feinleib (1969) etc. The size-biased distribution occurs naturally for some sampling plans in biometry, wildlife studies and survival analysis, among others. When dealing with the problem of sampling and selection from a size-biased distribution, the possible bias due to the nature of data collection process can be utilized to connect the population parameter to that of the sampling distribution (Olcay Akman,et.al, (2007)).

Now the probability density function of the Size-Biased Inverse Maxwell distribution is given by

$$f(y;\theta) = \frac{2}{\theta}\frac{1}{y^3} e^{-\frac{1}{\theta y^2}}, \; y > 0, \tag{16}$$

using the relationship

$f(y;\theta) = \frac{y\, h(y;\theta)}{\mu_1'}$,

where θ is a scale parameter of Inverse Maxwell distribution $h(y;\theta)$ where

$h(y;\theta) = \frac{4}{\sqrt{\pi}\theta^{\frac{3}{2}}} \frac{1}{y^4} . e^{-\frac{1}{\theta y^2}}$ $y > 0, \theta > 0$.

Moments

The r^th raw moments of sized-biased inverse Maxwell distribution are given by

$\mu_r' = E(Y^r)$

$$\mu_r' = \int_0^\infty y^r f(y) dy \tag{17}$$

$$= \frac{1}{\theta^{\frac{r}{2}}} \Gamma(1 - \frac{r}{2}) \tag{18}$$

Mean

$$\mu_1' = \sqrt{\frac{\pi}{\theta}} \tag{19}$$

Cumulative distribution function : Cumulative distribution function of Size-Biased Inverse Maxwell Distribution is given as

$$F(t) = e^{-\frac{1}{\theta t^2}} \tag{20}$$

Survival Function : Survival Function of Size-Biased Inverse Maxwell Distribution is defined a

$S(t) = 1 - F(t)$

$$= 1 - e^{-\frac{1}{\theta t^2}} \tag{21}$$

Hazard Function: Hazard Function of Size-Biased Inverse Maxwell Distribution is defined a

$\lambda(t) = \frac{f(t)}{S(t)}$

$$= \frac{2}{\theta}\frac{1}{t^3}\left[\frac{e^{-\frac{1}{\theta t^2}}}{1 - e^{-\frac{1}{\theta t^2}}}\right] \tag{22}$$

Let us suppose that n items are put to test, then the joint p.d.f. is given by:-

$$\mathbf{f(y;\theta)} = \frac{2^n}{\theta^n}\left(\prod_{i=1}^n \frac{1}{y_i^3}\right) e^{-\left(\frac{S}{\theta}\right)}; \; \mathbf{t, \theta > 0}; \tag{23}$$

where $\mathbf{S} = [\sum_{i=i}^n \frac{1}{y_i^2}]$;

the maximum likelihood estimator (MLE) $\hat{\theta}$ of θ may be obtained as;

$$\hat{\boldsymbol{\theta}} = \frac{S}{n}. \tag{24}$$

The pdf of $\hat{\theta}$ comes out to be

$$\mathbf{f(\hat{\theta})} = \frac{1}{n\Gamma(n-1)}\left(\frac{n}{\theta}\right)^n \hat{\theta}^{n-1} e^{\frac{-n\hat{\theta}}{\theta}}; \; \hat{\theta} > 0 \tag{25}$$

3. Bayesian Estimation Under g(θ)

In order to carry out the estimation procedure, let us suppose that very small information is available about the parameter (the suitable prior for this case). Assuming independence among the parameters is, we have a prior from (2) i.e quasi prior

$g(\theta) = \frac{1}{\theta^d}$

joint density function of SBIMD is given by

$$f(\underline{y}|\theta) = \left(\frac{2}{\theta}\right)^n \sum_{i=1}^n \frac{1}{y_i^3} e^{\left(\sum \frac{1}{y_i^2}\right)\left(\frac{1}{\theta}\right)} \tag{26}$$

Now using the Bayes theorem, the joint density function (26) along with the prior pdf given in (2), we obtain the following joint posterior density function of SBIMD is

$$f(\theta \mid \underline{y}) = \frac{f(y|\theta)g(\theta)}{\int_0^\infty f(y|\theta)g(\theta)d\theta} \tag{27}$$

substituting the value of $g(\theta)$ and $f(y|\theta)$ in the equation (27), we get

$$f(\theta \mid \underline{y}) = \frac{[S]^{n+d-1} e^{-\left(\frac{S}{\theta}\right)}}{\theta^{n+d}\Gamma(n+d-1)}; \tag{28}$$

(a) **Squared error loss function** : the Bayes estimator under squared error loss function is the posterior mean given by

$$\hat{\theta}_S = \int_0^\infty \theta\, f\left(\theta \middle| \underline{y}\right) d\theta \tag{29}$$

Substituting the value of $f(\theta \mid \underline{y})$ from equation (28) in equation (29) and solving it, we get

$$\hat{\theta}_S = \int_0^\infty \theta \frac{[S]^{n+d-1} e^{-\left(\frac{S}{\theta}\right)}}{\theta^{n+d}\Gamma(n+d-1)} \tag{30}$$

Solving equation (30), we get

$$\hat{\theta}_S = \frac{S}{n+d-2} \tag{31}$$

(b) **Precautionary loss function** :The Bayes estimator under precautionary loss function

$$\hat{\theta}_P = [E_\pi(\theta^2)]^{\frac{1}{2}}$$

$$= \left[\int_0^\infty \theta^2 f(\theta \mid \underline{y})\, d\theta\right]^{\frac{1}{2}} \tag{32}$$

Which on simplification leads to

$$\hat{\theta}_P = \frac{S}{[(n+d-2)(n+d-3)]^{\frac{1}{2}}} \tag{33}$$

(c) **Entropy loss function**: The Bayes estimator under entropy loss function

$$\hat{\theta}_e = \left[E_\pi\left(\frac{1}{\theta}\right)\right]^{-1}$$

$$= \left[\int_0^\infty \frac{1}{\theta} f(\theta \mid \underline{y})\, d\theta\right]^{-1} \tag{34}$$

Which on simplification comes out to be

$$\hat{\theta}_e = \frac{S}{(n+d-1)} \tag{35}$$

(d) Other loss function-L_1 :

Consider the loss function given by

$$L_1(\hat{\theta}, \theta) = \left(\frac{\hat{\theta}}{\theta} - 1\right)^2$$

The Bayes estimator under loss function- L_1 say $\hat{\theta}_1$ using the expression of $f(\theta|\underline{y})$ in equation (28) is the solution of equation given

$$\hat{\theta}_1 = \frac{E_\pi\left(\frac{1}{\theta}\right)}{E_\pi\left(\frac{1}{\theta^2}\right)}$$

$$= \frac{\int_0^\infty \frac{1}{\theta} f(\theta \mid \underline{y})\, d\theta}{\int_0^\infty \frac{1}{\theta^2} f(\theta \mid \underline{y})\, d\theta} \tag{36}$$

$$\text{Or } \hat{\theta}_1 = \frac{S}{(n+d)} \tag{37}$$

(e) Other loss function- L_2 :

Consider the loss function given by

$$L(\hat{\theta}, \theta) = \left(\frac{\theta}{\hat{\theta}} - 1\right)^2$$

The Bayes estimator under loss function-L_2, say, $\hat{\theta}_2$ using the value of $f(\theta|\underline{y})$ from equation (28) is the solution of equation given by

$$\hat{\theta}_2 = \frac{E_\pi(\theta^2)}{E_\pi(\theta)}$$

$$= \frac{\int_0^\infty \theta^2 f(\theta \mid \underline{y})\, d\theta}{\int_0^\infty \theta f(\theta \mid \underline{y})\, d\theta} \tag{38}$$

Or

$$\hat{\theta}_2 = \frac{S}{(n+d-3)} \tag{39}$$

4. The Risk Functions Under The Squared Error Loss Function

(i) The risk function of $\hat{\theta}_S$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_S)$ and accordance with (5), is given by

$$R_S(\hat{\theta}_S) = E_\theta\left(\hat{\theta}_S{}^2\right) - 2\theta E_\theta(\hat{\theta}_S) + \theta^2 \tag{40}$$

Substituting the value of $\hat{\theta}_S$ from (31) and evaluating various expectations in (40), we get

$$R_S(\hat{\theta}_S) = \theta^2\left[\frac{(n-1)(n+1)}{(n+d-2)^2} - \frac{2(n-1)}{(n+d-2)} + 1\right] \tag{41}$$

(ii) The risk function of $\hat{\theta}_p$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_p)$ is given by

$$R_S(\hat{\theta}_p) = E_\theta\left(\hat{\theta}_p{}^2\right) - 2\theta E_\theta(\hat{\theta}_p) + \theta^2 \tag{42}$$

Substituting the value of $\hat{\theta}_p$ from (33) and evaluating various expectations in (42), we get

$$R_S(\hat{\theta}_p) = \theta^2\left[\frac{(n-1)(n+1)}{[(n+d-2)(n+d-3)]} - \frac{2(n-1)}{[(n+d-2)(n+d-3)]^{\frac{1}{2}}} + 1\right]$$

(43)

(iii) the risk function of $\hat{\theta}_e$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_e)$ is given by

$$R_S(\hat{\theta}_e) = E_\theta\left(\hat{\theta}_e{}^2\right) - 2\theta E_\theta(\hat{\theta}_e) + \theta^2 \tag{44}$$

Substituting the value of $\hat{\theta}_e$ from (34) and evaluating various expectations in (44), we get

$$R_S(\hat{\theta}_e) = \theta^2\left[\frac{(n-1)(n+1)}{(n+d-1)^2} - \frac{2(n-1)}{(n+d-1)} + 1\right] \tag{45}$$

(iv) the risk function of $\hat{\theta}_1$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_S)$, is given by

$$R_S(\hat{\theta}_1) = E_\theta\left(\hat{\theta}_1{}^2\right) - 2\theta E_\theta(\hat{\theta}_1) + \theta^2 \tag{46}$$

Substituting the value of $\hat{\theta}_1$ from (36) and evaluating various expectations in (46) we get

$$R_S(\hat{\theta}_1) = \theta^2\left[\frac{(n-1)(n+1)}{(n+d)^2} - \frac{2(n-1)}{(n+d)} + 1\right] \tag{47}$$

(v) the risk function of $\hat{\theta}_2$, relative to squared error loss function is denoted by $R_S(\hat{\theta}_2)$, is given by

$$R_S(\hat{\theta}_2) = E_\theta\left(\hat{\theta}_2{}^2\right) - 2\theta E_\theta(\hat{\theta}_2) + \theta^2 \tag{48}$$

Substituting the value of $\hat{\theta}_2$ from (38) and evaluating various expectations in (48), we get

$$R_S(\hat{\theta}_2) = \theta^2\left[\frac{(n-1)(n+1)}{(n+d-3)^2} - \frac{2(n-1)}{(n+d-3)} + 1\right] \tag{49}$$

5. Conclusion

In this paper, we have discussed Bayesian estimation of parameter of one parameter of Inverse Maxwell Distribution using size-biased sampling. Which is same as the estimation of the parameter of the corresponding SBIMD. It is evident from the equations (24), (31), (33), (35), (37) and (39) that the MLE's of $\widehat{\theta}$, Bayes estimators of the scale parameter θ of the SBIMD under squared error loss function, precautionary and entropy loss functions using prior probability distribution (29) have different expressions for their definitions. Here it is clear that the Bayes estimators do depend upon the parameters of the prior distribution.

In the tables 1(a),1(b),1(c) and 1(d) , we have shown the ratio to risk function with respect to θ^2,B_S, B_P, B_e, B_1 and B_2 of the bayes estimators $\widehat{\theta}_S$,$\widehat{\theta}_P$, $\widehat{\theta}_e$, $\widehat{\theta}_1$,and $\widehat{\theta}_2$ respectively .the parameter θ under squared error loss function as given in equation (41), (43), (45), (47) and (49) for n=5(5)15 and d=0.5(0.5)5.0.

It is evident that none of the estimators dominate the other uniformly The tables show that the value of d has a major role in deciding the estimator to be used for precision point of view , It may be used as a guideline as to the choice of the estimator according to the situation at hand, If there is any bias in the mind of the experimenter with regard to the loss function and it is ignorant of the value of d then the table can be used to decide that what value of d is to be chosen to be used in quasi-density

Table 1(a): Risk function (θ = 1, n= 5)

N	d	R_S	R_p	R_e	R_1	R_2
5	0.0	1	1.734014	0.5	0.36	3
5	0.5	0.673469	1.038364	0.407407	0.338843	1.64
5	1.0	0.5	0.690599	0.36	0.333333	1
5	1.5	0.407407	0.507999	0.338843	0.337278	0.673469
5	2.0	0.36	0.411146	0.333333	0.346939	0.5
5	2.5	0.338843	0.361636	0.337278	0.36	0.407407
5	3.0	0.333333	0.339407	0.346939	0.375	0.36
5	3.5	0.337278	0.333341	0.36	0.391003	0.338843
5	4.0	0.346939	0.337002	0.375	0.407407	0.333333
5	4.5	0.36	0.346524	0.391003	0.423823	0.337278
5	5.0	0.375	0.359526	0.407407	0.44	0.346939

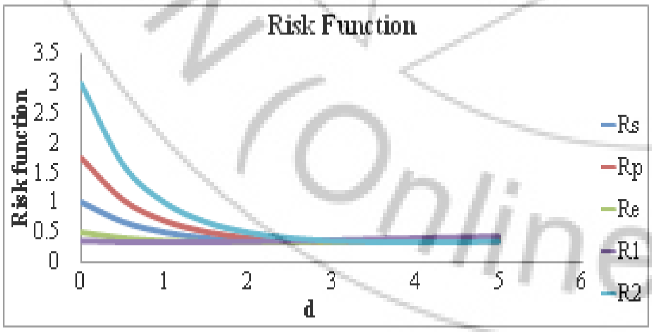


Table 1(b): Risk function (θ = 1, n= 10)

n	d	R_S	R_p	R_e	R_1	R_2
10	0.0	0.296875	0.362506	0.222222	0.19	0.44898
10	0.5	0.252595	0.298534	0.202216	0.183673	0.36
10	1.0	0.222222	0.25368	0.19	0.181818	0.296875
10	1.5	0.202216	0.222913	0.183673	0.183365	0.252595
10	2.0	0.19	0.202633	0.181818	0.1875	0.222222
10	2.5	0.183673	0.190227	0.183365	0.1936	0.202216
10	3.0	0.181818	0.183767	0.1875	0.201183	0.19
10	3.5	0.183365	0.181819	0.1936	0.209877	0.183673
10	4.0	0.1875	0.183301	0.201183	0.219388	0.181818
10	4.5	0.1936	0.187392	0.209877	0.229489	0.183365
10	5.0	0.201183	0.193462	0.219388	0.24	0.1875

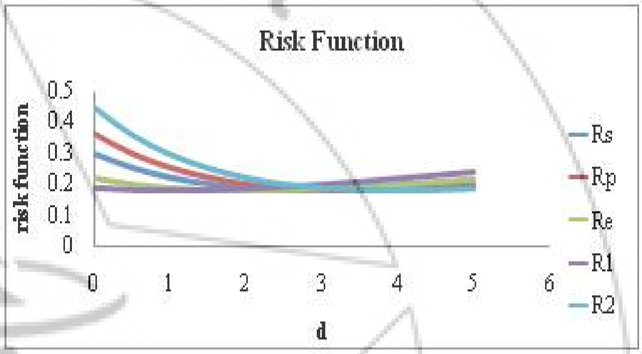
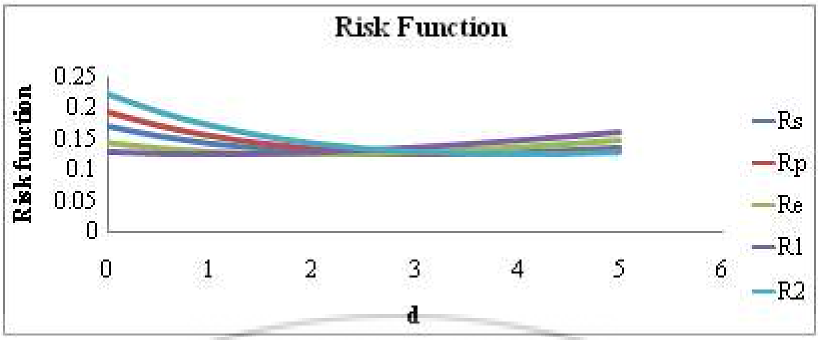


Table 1(c): Risk function (θ = 1, n= 15)

n	d	R_S	R_p	R_e	R_1	R_2
15	0.0	0.171598	0.194103	0.142857	0.128889	0.222222
15	0.5	0.155007	0.171966	0.134364	0.125911	0.1936
15	1.0	0.142857	0.155271	0.128889	0.125	0.171598
15	1.5	0.134364	0.14304	0.125911	0.125803	0.155007
15	2.0	0.128889	0.134483	0.125	0.128028	0.142857
15	2.5	0.125911	0.128958	0.125803	0.131429	0.134364
15	3.0	0.125	0.125941	0.128028	0.135802	0.128889
15	3.5	0.125803	0.125	0.131429	0.140979	0.125911
15	4.0	0.128028	0.12578	0.135802	0.146814	0.125
15	4.5	0.131429	0.127986	0.140979	0.153189	0.125803
15	5.0	0.135802	0.131373	0.146814	0.16	0.128028



References

[1] Bayes, T.: "An Essay towards solving a problem in the Doctrine of Chance," Philos. Trans. Roy. Soc., 53, pp. 370-418, 1763.

[2] Norstrom, J.G.: "The use of precautionary loss functions in risk analysis," IEEE Trans. Reliability., 45(3), pp. 400-403, 1996.

[3] Calabria, R. and Pulcini, G.s An engineering approach to Bayes estimation for the Weibull distribution, Micro-electron.Reliab.34 (5), 789–802,1994.

[4] Olcay, A.,et.al,"A simple test for detection of length-biased sampling", JP Journal of Biostatistics,Vol.1,No.2, pp 189-195,2007.

[5] Rao, C.R. :"A Natural Example of A Weighted Distribution," Amer. Stat. 31, pp. 24-26, (1977).

[6] Singh, Kusum lata & Srivastava, R.S.,"*Inverse Maxwell Distribution as a survival models, genesis and parameter estimation*". Research Journal of Mathematical and Statistical Sciences, Vol. 2(7), 1-3, July 2014.

[7] Singh,Kusum lata & Srivastava,R.S., "Estimation of the parameter in the Size-Biased Inverse Maxwell distribution", "International journal of statistika and mathematika ,ISSN;2277-2790 E-ISSN;2249-8605,vol 10.issue 3,pp 52-55,2014.

[8] Srivastava, R.S., Kumar, V., and Rao, A.K.: "Bayesian Estimation of the Shape Parameter and Reliability of Generalized Pareto Distribution using Precautionary Loss Function with Censoring" South East Asian J. Math & Math. Sc. Vol. 2, No. 2, pp. 47-56,2004.

[9] Zelen, M. and Feinleib, M. :"On the theory of Screening for Chronic Diseases," Biometrika, Vol. 56, pp. 601-614, 1969.