Operation Transform Formulae for Two Dimensional Fractional Mellin Transform

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Abstract: The Mellin transform is a basic tool for analyzing the behavior of many important functions in mathematics and mathematical physics. Mellin transform has many applications such as quantum calculus, radar classification of ships, electromagnetic, stress distribution, agriculture, medical stream, statistics, probability, signal processing, optics, pattern recognition, algorithms, correlators, navigation, vowel recognition and cryptographic scheme. The aim of this paper is to provide a generalization of two dimensional fractional Mellin transform. Also derived some operation transform formulae as Derivative, Parsval’s Identity, Modulation and Scaling property for two dimensional fractional Mellin transform.

Keywords: Fractional Mellin Transform, Generalize function, Mellin Transform, Testing Function space.

1. Introduction

Here, we have generalized the integral transform and the integral transforms provide a powerful technique for solving initial and boundary value problems arising in applied mathematics, mathematical physics and engineering. There are many integral transforms including Fourier, Laplace, Mellin, Hankel, Hilbert, Stieltjes etc. Still new integral transforms are being introduced in mathematical literature [7].

Mellin transform is a natural analytical tool to study the distribution of products and quotients of independent random variables. So by using Mellin transform agricultural land classification is possible [4]. Mellin transforms has application to derive different properties in statistics and classification is possible [4]. Mellin transforms has been used in deriving densities for algebraic combination of random variables [6]. Mellin transform method is applied to fractional differential equations with a right-sided derivative and variable potential [2]. Mellin transform also use to establish the means, variances, skewness, and kurtosis of fuzzy numbers and applied them to the random coefficient autoregressive (RCA) time series models [5]. Dirichlet Boundary Value Problem is solved by Mellin Transform [6].

The two dimensional fractional Mellin transform with parameter θ of \( f(x,y) \) denoted by \( \text{FRMT} \{ f(x,y) \} \) performs a linear operation, given by the integral transform

\[
\text{FRMT}_\theta \{ f(x,y) \} = F_\theta (u,v) = \int_{0}^{\infty} \int_{0}^{\infty} f(x,y) K_\theta (x,y,u,v) \, dx \, dy
\]

(1)

where the kernel

\[
K_\theta (x,y,u,v) = x^{\sin \theta - 1} y^{\sin \theta - 1} e^{\frac{\pi i}{\sin \theta} \left[ u^2 + \frac{v^2}{\sin^2 \theta} \log x^2 + \frac{y^2}{\cos^2 \theta} \log y^2 \right]} \]

(2)

and \( \theta \) is a fractional Mellin transformable if it is a member of \( E^* \), the dual of \( E \).

2. Preliminaries

2.1 The Test Function Space \( E \)

An infinitely differentiable complex valued function \( \phi(x,y) \) on \( R^2 \) belongs to \( E(R^2) \), if for each compact set \( I \subset S_a \), \( K \subset S_b \) where

\[
S_a = \{ x : x \in R, |x| \leq a, a > 0 \}
\]

\[
S_b = \{ y : y \in R, |y| \leq b, b > 0 \}, \quad I, K \in R^2
\]

\[
\gamma_{E,q,i} (\phi(x,y)) = \sup_{x,y \in K} \left| D^{p,q}_{i,j} (\phi(x,y)) \right| < \infty
\]

Thus \( E(R^n) \) will denote the space of all \( \phi(x,y) \in E(R^n) \) with compact support contained in \( S_a \& S_b \).

Note that the space \( E \) is complete and therefore a Frechet space. Moreover, we say that \( f(x,y) \) is a fractional Mellin transformable if it is a member of \( E^* \), the dual of \( E \).

2.2 Distributional two dimensional fractional Mellin transform (FRMT)

The two dimensional fractional Mellin transform of \( f(x,y) \in E^* (R^n) \) can be defined by

\[
\text{FRMT}_\theta \{ f(x,y) \} = F_\theta (u,v) = \left( f(x,y), K_\theta (x,y,u,v) \right)
\]

where \( K_\theta (x,y,u,v) = x^{\sin \theta - 1} y^{\sin \theta - 1} e^{\frac{\pi i}{\sin \theta} \left[ u^2 + \frac{v^2}{\sin^2 \theta} \log x^2 + \frac{y^2}{\cos^2 \theta} \log y^2 \right]} \)

Right hand side of equation has a meaning as the application of \( f(x,y) \in E^* \) to \( K_\theta (x,y,u,v) \in E \).

In this paper we proposed an operation transform formulate such as Derivative, Parsval’s Indentity, Modulation and Scaling property for two dimensional fractional Mellin transform.

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3. Results

3.1 Differential Property

Prove that \( FRMT\{f'(x,y)\}((u,v)) = \frac{-2\pi i}{\sin\theta} \left[ \frac{\log x \cos \theta}{x} \right] \left[ \frac{\sin \theta}{2\pi i} \right] \cdot \frac{f'(x,y)}{x} \left[ \frac{f(x,y)}{(u,v)} \right] \). 

Proof:

\[
\begin{align*}
    \int_0^\infty \int_0^\infty f'(x,y) x^{1-2\pi u} y^{1-2\pi v} dx dy &= \frac{-2\pi i}{\sin\theta} \left[ \frac{\log x \cos \theta}{x} \right] \left[ \frac{\sin \theta}{2\pi i} \right] \frac{f'(x,y)}{x} \left[ \frac{f(x,y)}{(u,v)} \right]. \\
    \int_0^\infty \int_0^\infty f'(x,y) x^{1-2\pi u} y^{1-2\pi v} dx dy &= \frac{-2\pi i}{\sin\theta} \left[ \frac{\log x \cos \theta}{x} \right] \left[ \frac{\sin \theta}{2\pi i} \right] \frac{f'(x,y)}{x} \left[ \frac{f(x,y)}{(u,v)} \right].
\end{align*}
\]

3.2 Parseval’s Identity for two dimensional fractional Mellin Transform:

If \( FRMT\{f(x,y)\}((u,v)) = F_0(u,v) \) & \( FRMT\{g(x,y)\}((u,v)) = G_0(u,v) \) then

i) \( \int_0^\infty \int_0^\infty f(x,y)g(x,y)dx dy = \int_0^\infty \int_0^\infty \frac{-1}{\sin^2\theta} G_0(u,v) dudv \) \( FRMT\{xyf(x,y)\}((u,v)) \)

ii) \( \int_0^\infty \int_0^\infty f(x,y)^2 dx dy = \int_0^\infty \int_0^\infty \frac{-1}{\sin^2\theta} \left[ \int f_0(u,v) \right]^2 dudv \)

Proof:

\[
\begin{align*}
    \int_0^\infty \int_0^\infty f(x,y)^2 dx dy &= \int_0^\infty \int_0^\infty \frac{-1}{\sin^2\theta} \left[ \int f_0(u,v) \right]^2 dudv. \\
    \int_0^\infty \int_0^\infty f(x,y)^2 dx dy &= \int_0^\infty \int_0^\infty \frac{-1}{\sin^2\theta} \left[ \int f_0(u,v) \right]^2 dudv.
\end{align*}
\]
\[
\int_{0}^{\infty} \int_{0}^{\infty} f(x, y) g(x, y) dx dy = \int_{0}^{\infty} \int_{0}^{\infty} 2 \pi i u \sin \theta \cos \theta (u^2 + v^2 + \log^2 x + \log^2 y) e^{i(ax + by)} f(x, y) dx dy
\]

\[
= \int_{0}^{\infty} \int_{0}^{\infty} x \frac{2 \pi i u}{\sin \theta} y \frac{2 \pi i}{\sin \theta} e^{i(ax + by)} (u^2 + v^2 + \log^2 x + \log^2 y) f(x, y) dx dy
\]

\[
= \frac{1}{2} \left( \int_{0}^{\infty} \int_{0}^{\infty} \frac{2 \pi i u}{\sin \theta} y \frac{2 \pi i}{\sin \theta} e^{i(ax + by)} e^{i(ax + by)} f(x, y) dx dy \right)
\]

Putting

\[
f(x, y) = g(x, y)
\]

\[
F_\theta (u, v) = G_\theta (u, v)
\]

\[
F_\theta (u, v) = \overline{G_\theta (u, v)}
\]

By using above result

\[
\int_{0}^{\infty} \int_{0}^{\infty} f(x, y) \overline{f(x, y)} dx dy
\]

\[
= \frac{\pi}{2} \left( \int_{0}^{\infty} \int_{0}^{\infty} \frac{2 \pi i u}{\sin \theta} y \frac{2 \pi i}{\sin \theta} e^{i(ax + by)} e^{-(ax + by)} f(x, y) dx dy \right)
\]

3.3 Modulation property:

(i) Prove that:

\[
FRMT[f(x, y) \cos (ax + by)](u, v) = \frac{1}{2} \left( FRMT[e^{i(ax+by)} f(x, y)](u, v) + FRMT[e^{-i(ax+by)} f(x, y)](u, v) \right)
\]

Proof: \[
\int_{0}^{\infty} \int_{0}^{\infty} f(x, y) \cos (ax + by) dx dy
\]

\[
= \int_{0}^{\infty} \int_{0}^{\infty} 2 \pi i u \sin \theta \cos \theta (u^2 + v^2 + \log^2 x + \log^2 y) e^{i(ax + by)} (u^2 + v^2 + \log^2 x + \log^2 y) e^{i(ax + by)} f(x, y) dx dy
\]

(ii) Prove that:

\[
FRMT[f(x, y) \cos (ax + by)](u, v) = \frac{1}{2l} \left( FRMT[e^{i(ax+by)} f(x, y)](u, v) - FRMT[e^{-i(ax+by)} f(x, y)](u, v) \right)
\]

Proof: \[
\int_{0}^{\infty} \int_{0}^{\infty} f(x, y) \cos (ax + by) dx dy
\]

\[
= \int_{0}^{\infty} \int_{0}^{\infty} 2 \pi i u \sin \theta \cos \theta (u^2 + v^2 + \log^2 x + \log^2 y) e^{i(ax + by)} (u^2 + v^2 + \log^2 x + \log^2 y) e^{i(ax + by)} f(x, y) dx dy
\]
3.4 Scaling property:

**Proof**: Let $f(x,y) = \frac{1}{ab}$ then

$$\frac{1}{ab}\int_0^\infty \int_0^\infty f(p,q) \left( \frac{p}{a} \right)^{n-1} \left( \frac{q}{b} \right)^{n-1} \alpha \beta \gamma \delta \epsilon \zeta \theta \iota \kappa \lambda \mu \nu \xi \omicron \rho \sigma \tau \upsilon \phi \chi \psi \omega \alpha \beta \gamma \delta \epsilon \zeta \theta \iota \kappa \lambda \mu \nu \xi \omicron \rho \sigma \tau \upsilon \phi \chi \psi \omega \right) dpdq$$

4. Conclusion

In this paper we have defined Distributional two dimensional fractional Mellin transform with compact support. This paper is mainly focused on some operation transform formulae such as Derivative, Parsval’s Identity, Modulation and Scaling property for two dimensional fractional Mellin transform is proved.

References