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Modulation and Parseval's Relation of Two Dimensional Fractional Sine Transform

V. D. Sharma[1], **S. A. Khapre**[2]

[1]Department of Mathematics, Arts, Commerce and Science College, Amravati- 444606(M.S), India

[2]Department of Mathematics, P. R. Patil College of Engineering and Technology, Amravati 444604 (M.S.), India

Abstract: ***As the sine transform, cosine transform and Hartley transform are widely use in signal processing, the application of their fractional version in signal/image processing is very promising. In this paper distributional generalized two dimensional fractional sine transform is studied. Properties as derivative Parsvel' s identity and shifting property for generalized two dimensional fractional sine transform is proved.***

Keywords: fractional Fourier transform, fractional Cosine transform, fractional Sine transform.

1. Introduction

Fractional calculus refers to integration or differentiation of non-integer order. Interestingly, the field has a history as old as calculus itself.

Today, fractional integration appears in diverse fields, some in the form of not-so-subtle variation and generalizations. These include solving dual integral equations [1] (and the works cited therein), seismic travel times, and stereo logy of spherical particles, spectroscopy of gas discharges, and the refractive index of optical fibers.

Nowadays, fractional transform play an important role in information processing, image reconstruction, pattern recognition, signal processing and the obvious question is: why do we need fractional transformation if we successfully apply the ordinary ones? First, because they naturally arise under the consideration of different problems for example, in optics and quantum mechanics and secondly, because fractionalization gives us a new degree of freedom (The fractional order), which can be used for more complete characterization of an object (A signal in general) or as an additional encoding parameter.

Fractional Transforms are used to compute the mixed time and frequency components of signals. Fractional operators particularly, Fractional Fourier Transform (FrFT), have been investigated in some depth in recent years. The FrFT is an extension of the ordinary Fourier Transform (FT) and successfully applied in the areas of optics, quantum mechanics and signal processing. It gives more complete representation of the signal in phase space and enlarges the number of applications of the ordinary FT [2].

Fractional cosine and sine transform are closely related to fractional Fourier transform which is most essential tool in the theory of optics and signal processing. Hence these transform are also used suitably in optics and signal processing as it reduces complexities of computation. Since these transform have additive property they are suitable to deal with the function which are specially even or odd. In particularly, when the function denoting the signal is impulse type, the generalized two-dimensional fractional cosine and sine transform are useful.

This paper is organized as follows: In section 2 we are describe modulation therom-1 and therom-2 of two dimensional fractional Sine transform. Some properties of two dimensional fractional sine transform are proved like Prasvel's identity in section 3 and shifting property in section 4.

2. Modulation

2.1. Modulation thermo-1

If $F_S^{\alpha}\big(f(x,y)\big)(u,v)$ is generalized two dimensional fractional sine transform of $f(x,y)$ then

$$F_S^{\alpha}\{f(x,y)sinax.sinby\}(u,v) = \frac{e^{\frac{i}{4}((csc^2\theta\,.PR+a^2)+(csc^2\theta QS+b^2))sin2\alpha}}{4}$$

$$\begin{bmatrix} e^{\frac{-i}{2}(P^2+Q^2)cot\theta}F_C^{\theta}\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,Q) \\ -e^{\frac{-i}{2}(P^2+S^2)cot\theta}F_C^{\theta}\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,S) \\ -\,e^{\frac{-i}{2}(Q^2+R^2)cot\theta}F_C^{\theta}\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,Q) \\ +e^{\frac{-i}{2}(R^2+S^2)cot\theta}F_C^{\theta}\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,S) \end{bmatrix}$$

Solution:

$$F_S^{\alpha}\{f(x,y)sinax.sinby\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(u^2+v^2)cot\alpha}{2}}e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^{\infty}\int_0^{\infty} sinax.sinby\, e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(coseca.ux)\,.\,dy\ \sin(coseca.vy)\,f(x,y)dx$$

Let $A = \sqrt{\frac{1-icot\alpha}{2\pi}}$ $B = e^{\frac{i(u^2+v^2)cot\alpha}{2}}e^{i(\alpha-\frac{\pi}{2})}$

$$F_S^{\alpha}\{f(x,y)sinax.sinby\}(u,v) = AB$$

$$\int_0^{\infty}\int_0^{\infty} e^{\frac{ix^2cot\alpha}{2}}sinax.\sin(coseca.ux)\ e^{\frac{iy^2cot\alpha}{2}}.\,sinby\,\sin(coseca.vy)\,f(x,y)dx\,dy$$

$F_s^\alpha\{f(x,y)sinax.sinby\}(u,v) = AB$

$$\int_0^\infty\int_0^\infty e^{\frac{ix^2cot\alpha}{2}}\frac{(e^{icsc\alpha ux}-e^{-icsc\alpha ux})}{2i}\frac{(e^{iax}-e^{-iax})}{2i}$$
$$e^{\frac{iy^2cot\alpha}{2}}\frac{(e^{icsc\alpha vy}-e^{-icsc\alpha vy})}{2i}\frac{(e^{iby}-e^{-iby})}{2i}\ f(x,y)dx\,dy$$

$F_s^\alpha\{f(x,y)sinax.sinby\}(u,v) =$

$$\frac{AB}{16}\int_0^\infty\int_0^\infty e^{\frac{ix^2cot\alpha}{2}}(2\cos((csc\alpha u+a)x - 2cos(csc\alpha u-a)x)$$
$$e^{\frac{iy^2cot\alpha}{2}}(2\cos((csc\alpha v+b)y - 2cos(csc\alpha v-b)y)\ f(x,y)dx\,dy$$

$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v) =$

$$\frac{AB}{4}\int_0^\infty\int_0^\infty e^{\frac{i(x^2+y^2)cot\alpha}{2}}\begin{bmatrix}\cos((csc\alpha u+a)x.\cos((csc\alpha v+b)y\\ -\cos((csc\alpha u+a)x.cos(csc\alpha v-b)y\\ -cos(csc\alpha u-a)x.\cos((csc\alpha v+b)y\\ +cos(csc\alpha u-a)x.cos(csc\alpha v-b)y\end{bmatrix}f(x,y)dxdy$$

Let $(csc\alpha u+a) = csc\theta.P$, $(csc\alpha u-a) = csc\theta.R$, $(csc\alpha v+b) = csc\theta.Q$, $(csc\alpha v-b) = csc\theta.S$

And $(csc\alpha u+a).(csc\alpha u-a) = csc\theta.P\ csc\theta.R$

$csc^2\alpha.u^2 - a^2 = csc^2\theta\,.PR\ csc^2\alpha.u^2 = csc^2\theta\,.PR + a^2$

$u^2 = sin^2\alpha(csc^2\theta\,.PR + a^2)$

$(csc\alpha v+b).(csc\alpha v-b) = csc\theta.Q\ csc\theta.S$

$csc^2\alpha.v^2 - b^2 = csc^2\theta\,.QS$

$csc^2\alpha.v^2 = csc^2\theta\,.QS + b^2$

$F_s^\alpha\{f(x,y)sinax.sinby\}(u,v) =$

$$\frac{AB}{4}\int_0^\infty\int_0^\infty e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\ e^{\frac{i}{2}(x^2+y^2)cot\theta}\begin{bmatrix}\cos(csc\theta.Px).\cos(csc\theta.Q)\,y\\ -\cos(csc\theta.P)\,x.cos(csc\theta.S)y\\ -cos(csc\theta.R)x.\cos(csc\theta.Q)\,y\\ +cos(csc\theta.R)x.cos(csc\theta.S)y\end{bmatrix}f(x,y)dxdy$$

$F_s^\alpha\{f(x,y)sinax.sinby\}(u,v) = \frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}$

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$\int_0^\infty\int_0^\infty e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\ e^{\frac{i}{2}(x^2+y^2+P^2+Q^2)cot\theta}\,e^{\frac{-i}{2}(P^2+Q^2)cot\theta}$
$\cos(csc\theta.Px).\cos(csc\theta.Qy)f(x,y)\ dxdy$

$-\int_0^\infty\int_0^\infty e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\ e^{\frac{i}{2}(x^2+y^2+P^2+S^2)cot\theta}$
$e^{\frac{-i}{2}(P^2+S^2)cot\theta}\cos(csc\theta.Px).\cos(csc\theta.Sy)f(x,y)\ dxdy$

$-\int_0^\infty\int_0^\infty e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\ e^{\frac{i}{2}(x^2+y^2+Q^2+R^2)cot\theta}$
$e^{\frac{-i}{2}(Q^2+R^2)cot\theta}\cos(csc\theta.Rx).\cos(csc\theta.Qy)f(x,y)\ dxdy$

$+\int_0^\infty\int_0^\infty e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\ e^{\frac{i}{2}(x^2+y^2+R^2+S^2)cot\theta}$
$e^{\frac{-i}{2}(R^2+S^2)cot\theta}\cos(csc\theta.Rx).\cos(csc\theta.Sy)f(x,y)\ dxdy$

$F_s^\alpha\{f(x,y)sinax.sinby\}(u,v) = \frac{e^{\frac{i}{4}((csc^2\theta\,.PR+a^2)+(csc^2\theta QS+b^2))sin2\alpha}}{4}$

$$\begin{bmatrix}e^{\frac{-i}{2}(P^2+Q^2)cot\theta}F_c^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,Q)\\ -e^{\frac{-i}{2}(P^2+S^2)cot\theta}F_c^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,S)\\ -e^{\frac{-i}{2}(Q^2+R^2)cot\theta}F_c^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,Q)\\ +e^{\frac{-i}{2}(R^2+S^2)cot\theta}F_c^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,S)\end{bmatrix}$$

2.1 Modulation thermo-2

If $F_s^\alpha(f(x,y))(u,v)$ is generalized two dimensional fractional sine transform of $f(x,y)$ then

$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v) = \frac{e^{\frac{i}{4}((csc^2\theta\,.PR+a^2)+(csc^2\theta QS+b^2))sin2\alpha}}{4}$

$$\begin{bmatrix}e^{\frac{-i}{2}(P^2+Q^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,Q)\\ +e^{\frac{-i}{2}(P^2+S^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,S)\\ +e^{\frac{-i}{2}(Q^2+R^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,Q)\\ +e^{\frac{-i}{2}(R^2+S^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,S)\end{bmatrix}$$

Solution:

$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v)$

$$=\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(u^2+v^2)cot\alpha}{2}}e^{i\left(\alpha-\frac{\pi}{2}\right)}$$
$$\int_0^\infty\int_0^\infty cosax.cosby\ e^{\frac{i(x^2+y^2)cot\alpha}{2}}$$
$\sin(coseca.ux).\sin(coseca.vy)\,f(x,y)dx\,dy$

Let $A = \sqrt{\frac{1-icot\alpha}{2\pi}}$ $B = e^{\frac{i(u^2+v^2)cot\alpha}{2}}e^{i\left(\alpha-\frac{\pi}{2}\right)}$

$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v) = AB$

$$\int_0^\infty\int_0^\infty e^{\frac{ix^2cot\alpha}{2}}cosax.\sin(coseca.ux)$$
$$e^{\frac{iy^2cot\alpha}{2}}.cosby\ \sin(coseca.vy)\,f(x,y)dx\,dy$$

$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v) =$

$$AB\int_0^\infty\int_0^\infty e^{\frac{ix^2cot\alpha}{2}}\frac{(e^{icsc\alpha ux}-e^{-icsc\alpha ux})}{2i}\frac{(e^{iax}+e^{-iax})}{2}$$
$$e^{\frac{iy^2cot\alpha}{2}}\frac{(e^{icsc\alpha vy}-e^{-icsc\alpha vy})}{2i}\frac{(e^{iby}+e^{-iby})}{2}\ f(x,y)dx\,dy$$

$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v) = \frac{AB}{-16}$

$$\int_0^\infty\int_0^\infty e^{\frac{ix^2cot\alpha}{2}}\left(e^{i(csc\alpha u+a)x} - e^{-i(csc\alpha u+a)x} + e^{i(csc\alpha u-a)x} - e^{-i(csc\alpha u-a)x}\right)$$
$$e^{\frac{iy^2cot\alpha}{2}}\left(e^{i(csc\alpha v+b)y} - e^{-i(csc\alpha v+b)y} + e^{i(csc\alpha v-b)y} - e^{-i(csc\alpha v-b)y}\right)f(x,y)dx\,dy$$

$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v) =$

$$\frac{AB}{-16}\int_0^\infty\int_0^\infty e^{\frac{ix^2cot\alpha}{2}}(2i\sin((csc\alpha u+a)x + 2isin(csc\alpha u-a)x)$$
$$e^{\frac{iy^2cot\alpha}{2}}(2i\sin(csc\alpha v+b)y + 2isin(csc\alpha v-b)y)\ f(x,y)dx\,dy$$

$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v) =$

$$\frac{AB}{4}\int_0^\infty\int_0^\infty e^{\frac{i(x^2+y^2)cot\alpha}{2}}\begin{bmatrix}\sin((csc\alpha u+a)x.\sin((csc\alpha v+b)y\\+\sin(csc\alpha u+a)x.sin(csc\alpha v-b)y\\+sin(csc\alpha u-a)x.\sin((csc\alpha v+b)y\\+sin(csc\alpha u-a)x.sin(csc\alpha v-b)y\end{bmatrix}f(x,y)dxy$$

Let $(csc\alpha u+a)=csc\theta.P\,,(csc\alpha u-a)=csc\theta.R\,,$
$(csc\alpha v+b)=csc\theta.Q\,,(csc\alpha v-b)=csc\theta.S$

And $(csc\alpha u+a).(csc\alpha u-a)=\ csc\theta.P\ csc\theta.R$
$csc^2\alpha.u^2-a^2=csc^2\theta\,.PR\ csc^2\alpha.u^2=csc^2\theta\,.PR+a^2$

$u^2=sin^2\alpha(csc^2\theta\,.PR+a^2)$

$(csc\alpha v+b).(csc\alpha v-b)=\ csc\theta.Q\ csc\theta.S$
$,csc^2\alpha.v^2-b^2=csc^2\theta\,.QS,csc^2\alpha.v^2$
$=csc^2\theta\,.QS+b^2$
$v^2=sin^2\alpha(csc^2\theta\,.QS+b^2)$

$$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v)=\frac{AB}{4}\int_0^\infty\int_0^\infty e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\,e^{\frac{i}{2}(x^2+y^2)cot\theta}\begin{bmatrix}\sin(csc\theta.Px).\sin(csc\theta.Q)\,y\\+\sin(csc\theta.P)\,x.sin(csc\theta.S)y\\+sin(csc\theta.R)x.\sin(csc\theta.Q)\,y\\+sin(csc\theta.R)x.sin(csc\theta.S)y\end{bmatrix}f(x,y)dxdy$$

$$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v)=\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}e^{i\left(\alpha-\frac{\pi}{2}\right)}}{4}$$
$$[\int_0^\infty\int_0^\infty Ae^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\,e^{i\left(\theta-\frac{\pi}{2}\right)}\,e^{\frac{i}{2}(x^2+y^2+P^2+Q^2)cot\theta}\,e^{\frac{-i}{2}(P^2+Q^2)cot\theta}\sin(csc\theta.Px).\sin(csc\theta.Qy)f(x,y)\,dxdy$$
$$+\int_0^\infty\int_0^\infty Ae^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\,e^{i\left(\theta-\frac{\pi}{2}\right)}e^{\frac{i}{2}(x^2+y^2+P^2+S^2)cot\theta}\,e^{\frac{-i}{2}(P^2+S^2)cot\theta}\sin(csc\theta.Px).\sin(csc\theta.Sy)f(x,y)\,dxdy$$
$$+\int_0^\infty\int_0^\infty Ae^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}e^{i\left(\theta-\frac{\pi}{2}\right)}\,e^{\frac{i}{2}(x^2+y^2+Q^2+R^2)cot\theta}\,e^{\frac{-i}{2}(Q^2+R^2)cot\theta}\sin(csc\theta.Rx).\sin(csc\theta.Qy)f(x,y)\,dxdy$$
$$+\int_0^\infty\int_0^\infty Ae^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}\,e^{i\left(\theta-\frac{\pi}{2}\right)}\,e^{\frac{i}{2}(x^2+y^2+R^2+S^2)cot\theta}\,e^{\frac{-i}{2}(R^2+S^2)cot\theta}\sin(csc\theta.Rx).\sin(csc\theta.Sy)f(x,y)\,dxdy$$

$$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v)=\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}\begin{bmatrix}e^{\frac{-i}{2}(P^2+Q^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,Q)\\+e^{\frac{-i}{2}(P^2+S^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,S)\\+e^{\frac{-i}{2}(Q^2+R^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,Q)\\+e^{\frac{-i}{2}(R^2+S^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,S)\end{bmatrix}$$

$$F_s^\alpha\{f(x,y)cosax.cosby\}(u,v)=\frac{e^{\frac{i}{4}((csc^2\theta\,.PR+a^2)+(csc^2\theta QS+b^2))sin2\alpha}}{4}$$
$$\begin{bmatrix}e^{\frac{-i}{2}(P^2+Q^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,Q)\\+e^{\frac{-i}{2}(P^2+S^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(P,S)\\+e^{\frac{-i}{2}(Q^2+R^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,Q)\\+e^{\frac{-i}{2}(R^2+S^2)cot\theta}F_s^\theta\left\{e^{\frac{i}{2}(x^2+y^2)(cot\alpha-cot\theta)}f(x,y)\right\}(R,S)\end{bmatrix}$$

3. Parsvel's Identity

Parsvel's identity for generalized two-dimensional fractional sine transforms. If $F_s^\alpha(u,v)=G_s^\alpha(u,v)\ \overline{F_s^\alpha(u,v)}=\overline{G_s^\alpha(u,v)}$

Then

i) $$\int_0^\infty\int_0^\infty f(x,y)\ \overline{g(x,y)}\,dxdy=\frac{8csc\alpha}{\pi}e^{i\left(\alpha-\frac{\pi}{2}\right)}\int_0^\infty\int_0^\infty\overline{G_s^\alpha(u,v)}\ F_s^\alpha\{f(x,y)\}(u,v)dudv$$

ii) $$\int_0^\infty\int_0^\infty|f(x,y)|^2\ dxdy=\frac{8csc\alpha}{\pi}e^{i\left(\alpha-\frac{\pi}{2}\right)}\int_0^\infty\int_0^\infty|F_s^\alpha(u,v)|^2\ dudv$$

Solution:
By definition of generalized two dimensional sine transform

$$F_s^\alpha\{g(x,y)\}(u,v)=G_s^\alpha(u,v)=\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}e^{i\left(\alpha-\frac{\pi}{2}\right)}\int_0^\infty\int_0^\infty g(x,y)\,e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(coseca.ux).\sin(coseca.vy)\,dx\,dy$$

Using inversion formula of fractional sine transform

$$g(x,y)=\frac{4}{\pi^2}e^{-i\left(\alpha-\frac{\pi}{2}\right)}\int_0^\infty\int_0^\infty G_s^\alpha(u,v)\,e^{-\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\sin(coseca.ux).\sin(coseca.vy)\ csc^2\alpha\sqrt{\frac{2\pi}{1-icot\alpha}}dudv$$

$$g(x,y)=\frac{4}{\pi^2}\sqrt{\frac{2\pi}{1-icot\alpha}}e^{-\frac{i(x^2+y^2)cot\alpha}{2}}csc^2\alpha e^{-i\left(\alpha-\frac{\pi}{2}\right)}\int_0^\infty\int_0^\infty G_s^\alpha(u,v)\,e^{-\frac{i(u^2+v^2)cot\alpha}{2}}\sin(coseca.ux).\sin(coseca.vy)\,dudv$$

$$\overline{g(x,y)}=\frac{4}{\pi^2}\sqrt{\frac{2\pi}{1+icot\alpha}}e^{\frac{i(x^2+y^2)cot\alpha}{2}}csc^2\alpha e^{i\left(\alpha-\frac{\pi}{2}\right)}\int_0^\infty\int_0^\infty\overline{G_s^\alpha(u,v)}\,e^{\frac{i(u^2+v^2)cot\alpha}{2}}\sin(coseca.ux).\sin(coseca.vy)\,dudv$$

$$\int_0^\infty\int_0^\infty f(x,y)\ \overline{g(x,y)}\,dxdy=\int_0^\infty\int_0^\infty f(x,y)dxdy\frac{4}{\pi^2}\sqrt{\frac{2\pi}{1+icot\alpha}}e^{\frac{i(x^2+y^2)cot\alpha}{2}}csc^2\alpha\ e^{i\left(\alpha-\frac{\pi}{2}\right)}\int_0^\infty\int_0^\infty\overline{G_s^\alpha(u,v)}\,e^{\frac{i(u^2+v^2)cot\alpha}{2}}\ \sin(coseca.ux)\sin(coseca.vy)\,dudv$$

$$\int_0^\infty\int_0^\infty f(x,y)\ \overline{g(x,y)}\,dxdy$$

$$=\frac{4}{\pi^2}\sqrt{\frac{2\pi}{1+icot\alpha}}csc^2\alpha\ e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^\infty\int_0^\infty f(x,y)\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}$$

$\sin(cosec\alpha.ux).\sin(cosec\alpha.vy)\ dxdy$

$$\int_0^\infty\int_0^\infty \overline{G_s^\alpha(u,v)}\ dudv$$

$$\int_0^\infty\int_0^\infty f(x,y)\ \overline{g(x,y)}\,dxdy$$

$$=\sqrt{\frac{32*2(1-icot\alpha)}{2\pi^3(1+icot\alpha)(1-icot\alpha)}}$$

$$csc^2\alpha\ e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^\infty\int_0^\infty f(x,y)\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\ \sin(cosec\alpha.ux).\sin(cosec\alpha.vy)\ dxdy$$

$$\int_0^\infty\int_0^\infty \overline{G_s^\alpha(u,v)}\ dudv$$

$$\int_0^\infty\int_0^\infty f(x,y)\ \overline{g(x,y)}\,dxdy$$

$$=\sqrt{\frac{64}{\pi^2(1+cot^2\alpha)}}csc^2\alpha\ e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^\infty\int_0^\infty \overline{G_s^\alpha(u,v)}\ F_s^\alpha\{f(x,y)\}(u,v)dudv$$

$$\int_0^\infty\int_0^\infty f(x,y)\ \overline{g(x,y)}\,dxdy=\frac{8csc^2\alpha}{\pi csc\alpha}e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^\infty\int_0^\infty \overline{G_s^\alpha(u,v)}\ F_c^\alpha\{f(x,y)\}(u,v)dudv$$

$$\int_0^\infty\int_0^\infty f(x,y)\ \overline{g(x,y)}\,dxdy=\frac{8csc\alpha}{\pi}e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^\infty\int_0^\infty \overline{G_s^\alpha(u,v)}\ F_s^\alpha\{f(x,y)\}(u,v)dudv$$

ii) Let $f(x,y)=g(x,y)$ then

$$\int_0^\infty\int_0^\infty f(x,y)\,\overline{f(x,y)}\ dxdy=\frac{8csc\alpha}{\pi}e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^\infty\int_0^\infty F_s^\alpha(u,v)\ \overline{F_s^\alpha(u,v)}dudv$$

$$\int_0^\infty\int_0^\infty |f(x,y)|^2\ dxdy=\frac{8csc\alpha}{\pi}e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^\infty\int_0^\infty |F_s^\alpha(u,v)|^2\ dudv$$

4. Shifting Property

If $F_s^\alpha\big(f(x,y)\big)(u,v)$ is generalized two dimensional fractional sine transform of $f(x,y)$ then

$F_s^\alpha\{f(x+a,y+b)\}(u,v)=$

$\cos(csc\alpha.ua)\cos(csc\alpha.vb)$

$$F_s^\alpha\left\{e^{i(\frac{a^2+b^2}{2}-(ta+sb))cot\alpha}f(t,s)\right\}(u,v)$$

$$+\ \sin(csc\alpha ua)\sin(csc\alpha vb)\ e^{i\left(\theta-\frac{\pi}{2}\right)}$$

$$F_c^\alpha\left\{e^{i(\frac{a^2+b^2}{2}-(ta+sb))cot\alpha}f(t,s)\right\}(u,v)$$

$$-\cos(csc\alpha.ua)\sin(csc\alpha vb)e^{\frac{i}{2}(a^2+b^2)cot\alpha}$$

$$\sqrt{\frac{2\pi}{1-icot\alpha}}G_s^\alpha\{e^{-itacot\alpha}.1\}(u)G_c^\alpha\{e^{-isbcot\alpha}f(t,s)\}(v)$$

$$-\ \cos(csc\alpha.vb)\sin(csc\alpha ua)e^{\frac{i}{2}(a^2+b^2)cot\alpha}$$

$$\sqrt{\frac{2\pi}{1-icot\alpha}}G_s^\alpha\{e^{-isbcot\alpha}.1\}(v)G_c^\alpha\{e^{-itacot\alpha}f(t,s)\}(u)$$

Solution:

$F_s^\alpha\{f(x+a,y+b)\}(u,v)$

$$=\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(cosec\alpha.ux).\sin(cosec\alpha.vy)$$

$$f(x+a,y+b)dxdy$$

Let, A=$\sqrt{\frac{1-icot\alpha}{2\pi}}$ B=$e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$x+a=t,y+b=s\,,dx=dt\,,dy=ds\,,t\to-\infty\ to\ \infty\,,$

$s\to-\infty\ to\ \infty$

$F_s^\alpha\{f(x+a,y+b)\}(u,v)=AB$

$$\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{i((t-a)^2+(s-b)^2)cot\alpha}{2}}\sin\big(cosec\alpha.u(t-a)\big).ds$$

$$\sin(cosec\alpha.v(s-b))f(t,s)\ dt$$

$F_s^{\square}\{f(x+a,y+b)\}(u,v)$

$$=AB\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{i}{2}(t^2+a^2-2ta)cot\alpha}\sin\big(cosec\alpha.(ut-ua)\big)$$

$$e^{\frac{i}{2}(s^2+b^2-2sb)c0t\alpha}\sin\big(cosec\alpha.(vs-vb)\big)f(t,s)dtds$$

$F_s^\alpha\{f(x+a,y+b)\}(u,v)$

$$=AB\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{i}{2}(t^2+a^2)cot\alpha}\ e^{-itacot\alpha}\sin\big(cosec\alpha.(ut-ua)\big)$$

$$e^{\frac{i}{2}(s^2+b^2)cot\alpha}\ e^{-isbcot\alpha}\sin\big(cosec\alpha.(vs-vb)\big)f(t,s)dtds$$

$F_s^\alpha\{f(x+a,y+b)\}(u,v)=$

$$AB\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{i}{2}(t^2+a^2)cot\alpha}\ e^{-itacot\alpha}$$

$[\sin(csc\alpha.ut)\cos(csc\alpha ua)-\cos(csc\alpha ut)\sin(csc\alpha ua)]$

$e^{\frac{i}{2}(s^2+b^2)cot\alpha}e^{-isbcot\alpha}$

$[\sin(csc\alpha.vs)\cos(csc\alpha vb)$

$-\ \sin\ (csc\alpha vb)\cos(csc\alpha vs)]\,f(t,s)dtds$

$F_s^\alpha\{f(x+a,y+b)\}(u,v)=$

$$AB\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{i}{2}(t^2+s^2+a^2+b^2)cot\alpha}\ e^{-i(ta+sb)cot\alpha}$$

$\cos(csc\alpha.ua)\sin(csc\alpha ut)$

$\cos(csc\alpha.vb)\sin(csc\alpha vs)\,f(t,s)dtds$

$$-AB\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{i}{2}(t^2+s^2+a^2+b^2)cot\alpha}\ e^{-i(ta+sb)cot\alpha}$$

$\cos(csc\alpha.ua)\sin(csc\alpha ut)$

$\sin(csc\alpha vb)\ \cos(csc\alpha vs)\,f(t,s)dtds$

$$-AB\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{i}{2}(t^2+s^2+a^2+b^2)cot\alpha}\ e^{-i(ta+sb)cot\alpha}$$

$\sin(\csc\alpha ua)\cos(\csc\alpha ut)$
$\cos(\csc\alpha . vb)\sin(\csc\alpha vs) f(t,s)dtds$
$+ AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(t^2+s^2+a^2+b^2)\cot\alpha}\, e^{-i(ta+sb)\cot\alpha}$
$\sin(\csc\alpha ua)\cos(\csc\alpha ut)$
$\sin(\csc\alpha vb)\cos(\csc\alpha vs) f(t,s)dtds$

$$F_s^\alpha\{f(x+a,y+b)\}(u,v) = \cos(\csc\alpha . ua)\cos(\csc\alpha . vb)$$
$$F_s^\alpha\left\{e^{i(\frac{a^2+b^2}{2}-(ta+sb))\cot\alpha} f(t,s)\right\}(u,v)$$
$$+\sin(\csc\alpha ua)\sin(\csc\alpha vb)\, e^{i\left(\theta-\frac{\pi}{2}\right)}$$
$$F_c^\alpha\left\{e^{i(\frac{a^2+b^2}{2}-(ta+sb))\cot\alpha} f(t,s)\right\}(u,v)$$
$$-\cos(\csc\alpha . ua)\sin(\csc\alpha vb) e^{\frac{i}{2}(a^2+b^2)\cot\alpha}\sqrt{\frac{2\pi}{1-i\cot\alpha}}$$
$$\left[\int_{-\infty}^{\infty}\sqrt{\frac{1-i\cot\alpha}{2\pi}}\, e^{\frac{i}{2}(t^2+u^2)\cot\alpha} e^{-ita\cot\alpha}\, e^{i\left(\theta-\frac{\pi}{2}\right)}\sin(\csc\alpha ut)\, dt \int_{-\infty}^{\infty}\sqrt{\frac{1-i\cot\alpha}{2\pi}}\, e^{\frac{i}{2}(s^2+v^2)\cot\alpha} e^{-isb\cot\alpha}\cos(\csc\alpha vs) f(t,s)\, ds\right]$$
$$-\cos(\csc\alpha . vb)\sin(\csc\alpha ua) e^{\frac{i}{2}(a^2+b^2)\cot\alpha}\sqrt{\frac{2\pi}{1-i\cot\alpha}}$$
$$\left[\int_{-\infty}^{\infty}\sqrt{\frac{1-i\cot\alpha}{2\pi}}\, e^{\frac{i}{2}(s^2+v^2)\cot\alpha} e^{-isb\cot\alpha}\sin(\csc\alpha vs) e^{i\left(\theta-\frac{\pi}{2}\right)}\, ds \int_{-\infty}^{\infty}\sqrt{\frac{1-i\cot\alpha}{2\pi}}\, e^{\frac{i}{2}(t^2+u^2)\cot\alpha} e^{-ita\cot\alpha}\cos(\csc\alpha ut) f(t,s)\, dt\right]$$

$$F_s^\alpha\{f(x+a,y+b)\}(u,v) =$$
$$\cos(\csc\alpha . ua)\cos(\csc\alpha . vb)$$
$$F_s^\alpha\left\{e^{i(\frac{a^2+b^2}{2}-(ta+sb))\cot\alpha} f(t,s)\right\}(u,v)$$
$$+\sin(\csc\alpha ua)\sin(\csc\alpha vb)\, e^{i\left(\theta-\frac{\pi}{2}\right)}$$
$$F_c^\alpha\left\{e^{i\left(\frac{a^2+b^2}{2}-(ta+sb)\right)\cot\alpha} f(t,s)\right\}(u,v)$$
$$-\cos(\csc\alpha . ua)\sin(\csc\alpha vb) e^{\frac{i}{2}(a^2+b^2)\cot\alpha}\sqrt{\frac{2\pi}{1-i\cot\alpha}}$$
$$G_s^\alpha\{e^{-it\square\cot\alpha}.1\}(u)G_c^\alpha\{e^{-isb\cot\alpha} f(t,s)\}(v) -$$
$$\cos(\csc\alpha . vb)\sin(\csc\alpha ua) e^{\frac{i}{2}(a^2+b^2)\cot\alpha}\sqrt{\frac{2\pi}{1-i\cot\alpha}}$$
$$G_s^\alpha\{e^{-isb\cot\alpha}.1\}(v)G_c^\alpha\{e^{-ita\cot\alpha} f(t,s)\}(u)$$

where G_C^α , G_S^α are one dimensional fractional cosine and sine are transform respectively.

5. Conclusion

In the proposed work we have proved Parsvel's identity and shifting property for generalized two fractional Sine transform. Also Modulation Property is described in the form of theorem.

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Author Profile

Dr. V. D. Sharma is currently working as an Assistant professor in the department of Mathematics, Arts, Commerce and Science College, Kiran Nagar, Amravati-444606 (M.S.) India. She has got 18 years of teaching and research experience. She has obtained her Ph.D. degree in 2007 from SGB Amravati University Amravati. Her field of interest is Integral Transforms. Six research students are working under her supervision. She has published more than 50 research articles.

S. A. Khapre is an Assistant professor in the department of Mathematics, P. R. Patil College of engineering and technology, Amravati, Dist: Amravati-444604 (M.S.) India. She has got 6 years of teaching experience. She has obtained her master degree in 2001 and M.Phil degree in 2008 from SGB Amravati University Amravati .She has 6 research articles in journals to her credit.