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Some Examples on Generalized Two-Dimensional Fractional Cosine Transform In the Range $-\infty\ to\ \infty$

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Abstract: ***Fractional Cosine Transform (FRCT) is a generalization of the ordinary cosine transform and it has similar relationship with Fractional Fourier Transform (FRFT) as the ordinary cosine and sine transforms have with the Fourier Transform (FT). Fractional domain is useful for solving some problems, which cannot be solved in the original domain. In this paper Operation transform formulae on two dimensional fractional Cosine transform are discussed in the range $-\infty$ to ∞***

Keywords: fractional Fourier transform, fractional Cosine transform, fractional Sine transform

1. Introduction

Integral transformations have been successfully used for almost two centuries in solving many problems in applied mathematics, mathematical physics, and engineering science and one of best transform is Fourier transform. Fourier transform (FT) is named in the honor of Joseph Fourier (1768-1830). In the theory of Integral transform, Fourier analysis is one of the most frequently used tools in signal processing and many other scientific fields. The FrFT is a generalization of the ordinary Fourier transform [1] with an order parameter α and is identical to the ordinary Fourier transform when this order α is equal to p/2. Since the ordinary Fourier transform and related techniques are of importance in various different areas like communications, signal processing and control systems, it is natural to expect the FrFT to find many applications in these fields as well. In fact, the FrFT has already found many applications in the areas of signal processing and communications [2-3].

We know that the Cosine and Sine transforms and their discrete versions are useful tools in signal and image processing, such as signal coding [4], watermarking [5] and restoration of de-focused images [6]. In the Ref. [7] Pei and Yeh extended the Cosine transform to the discrete fractional cosine transform (DFrCT) and the discrete fractional sine transform (DFrST). Both of them possess well the angle additivity property of the DFrFT. Moreover, the DFrCT and DFrST are used in the digital computation of FrFT for the reducing computational load of the DFrFT.

The success of FrFT in its application has promoted the development of other kinds of fractional transforms like fractional Hartley transform, fractional Hadamard transform, fractional cosine transform and fractional sine transform (FrST). Pei Soo-Chang redefined the fractional cosine transform and fractional sine transform based on fractional Fourier transform in 2001 [8-9]. FrST is the extension of sine transform and it has been widely used in domain of digital signal and image processing [10].

The idea of fractionalization of CT and ST was proposed in [8]. There the real and imaginary parts of the fractional FT kernel were chosen as the kernels for a fractional CT and a fractional ST respectively. The fractional CT can be used for separately processing the even part of two sided function. Although, the generalization to the two –and-higher dimensional cases is straight forward. The fractional CT can be considered as Elementary fractional transforms of causal signals, which are able to treat the even part of general (non-causal) signals separately. In rest of paper we discussed some examples on generalized two dimensional fractional cosine transform in the range$-\infty\ to\ \infty$.

2. Examples

2.1 If $F_c^\alpha\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Cosine transform of f(x,y)then

$$F_C^\alpha\{1\}(u,v) = \sqrt{2\pi}\sqrt{tan^2\alpha - itan\alpha}\, e^{\frac{-i}{2}(u^2+v^2)tan\alpha}$$

Solution:

$$F_C^\alpha\{1\}(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 1\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\sqrt{\frac{1-icot\alpha}{2\pi}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dxdy$$

$$F_C^\alpha\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(u^2+v^2)cot\alpha}{2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 1\, e^{\frac{i(x^2+y^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dx\,dy$$

Let, A= $\sqrt{\frac{1-icot\alpha}{2\pi}}$ B=$e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_C^\alpha\{1\}(u,v) = AB\int_{-\infty}^{\infty} e^{\frac{i(x^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\,dx \int_{-\infty}^{\infty} e^{\frac{i(y^2)cot\alpha}{2}}\cos(cosec\alpha.vy)\,dy$$

Let, a= $\frac{cot\alpha}{2}$, b=$cosec\alpha.u$, c=$cosec\alpha.v$

$$F_C^\alpha\{1\}(u,v) = AB\int_{-\infty}^{\infty} e^{ix^2a}\cos(bx)\,dx \int_{-\infty}^{\infty} e^{iy^2a}\cos(cy)\,dy$$

$$F_C^\alpha\{1\}(u,v) = AB$$

$$\left[\left(\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}e^{\frac{-ib^2}{4a}}\right)\begin{pmatrix} erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-b)\right) \\ +(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+b)\right)\end{pmatrix}\right]_{-\infty}^{\infty}$$

$$\left[\left(\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}e^{\frac{-ic^2}{4a}}\right)\begin{pmatrix} erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-c)\right) \\ +(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+c)\right)\end{pmatrix}\right]_{-\infty}^{\infty}$$

$$F_C^{\alpha}\{1\}(u,v) = AB\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}$$

$$\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)\left[\begin{pmatrix} erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-b)\right) \\ +(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+b)\right)\end{pmatrix}\right]_{-\infty}^{\infty}$$

$$\left[\begin{pmatrix} erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-c)\right) \\ +(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+c)\right)\end{pmatrix}\right]_{-\infty}^{\infty}$$

$$F_C^{\alpha}\{1\}(u,v) = AB\frac{1}{16a}(-1)^{\frac{3}{2}}\pi\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)$$

$$\left[\begin{matrix}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(\infty)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(\infty)\right)\right)- \\ \left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(-\infty)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(-\infty)\right)\right)\end{matrix}\right]$$

$$\left[\begin{matrix}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(\infty)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(\infty)\right)\right)- \\ \left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(-\infty)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(-\infty)\right)\right)\end{matrix}\right]$$

Here $erfi(\sqrt{i}\infty) = \sqrt{i}$

$$F_C^{\alpha}\{1\}(u,v) = AB\frac{1}{16a}(-1)^{\frac{3}{2}}\pi\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)[16i]$$

$$F_C^{\alpha}\{1\}(u,v) = AB\frac{1}{a}(-1)^{\frac{3}{2}}\pi\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)i$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}\frac{i(-1)^{\frac{3}{2}}\pi}{a}\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}$$

$$\frac{i(-1)^{\frac{3}{2}}\pi}{a}\left(e^{\frac{-i(csc^2\alpha.u^2+csc^2\alpha.v^2)}{4\frac{cot\alpha}{2}}}\right)$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{-i}{2}(u^2+v^2)\frac{1}{cot\alpha}}\frac{i(-1)^{\frac{3}{2}}\pi}{a}$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{-i}{2}(u^2+v^2)tan\alpha}\frac{i^4\pi}{\frac{cot\alpha}{2}}$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{-i}{2}(u^2+v^2)tan\alpha}\frac{2\pi}{cot\alpha}$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{2\pi}\sqrt{tan^2\alpha - itan\alpha}e^{\frac{-i}{2}(u^2+v^2)tan\alpha}$$

2.2. If $F_c^{\alpha}\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Cosine transform of f(x, y) then

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = K_c^{\alpha}(a,b,u,v)$$

Solution:

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\delta(x-a).\delta(y-b))$$

$$\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}$$

$$\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)$$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\delta(x-a).\delta(y-b))$$

$$\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)$$

$$dxdy$$

Let A= $\sqrt{\frac{1-icot\alpha}{2\pi}}$ $\boldsymbol{B} = e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = AB$$

$$\int_{-\infty}^{\infty}\delta(x-a)\,e^{\frac{i(x^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\,dx$$

$$\int_{-\infty}^{\infty}\delta(y-b)\,e^{\frac{i(y^2)cot\alpha}{2}}\cos(cosec\alpha.vy)\,dy$$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = ABe^{\frac{i(a^2)cot\alpha}{2}}$$

$$\cos(cosec\alpha.ua).e^{\frac{i(b^2)cot\alpha}{2}}\cos(cosec\alpha.vb)$$

We know that $\int_{-\infty}^{\infty}\delta(t-a)\,\varphi(t)dt = \varphi(a)$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = ABe^{\frac{i(a^2+b^2)cot\alpha}{2}}$$

$$\cos(cosec\alpha.ua).\cos(cosec\alpha.vb)$$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = K_c^{\alpha}(a,b,u,v)$$

2.3. If $F_C^{\alpha}\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Cosine transform of f(x,y)then

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = \sqrt{\frac{-\pi 2(1-icot\alpha)}{cot^2\alpha}}$$

$$e^{\frac{i}{2}((u^2+v^2)cot\alpha - tan\alpha(csc^2\alpha u^2+csc^2\alpha v^2+2)}$$

$$\cos(sec\alpha.u).\cos(sec\alpha.v)$$

Solution:

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}cosx.cosy$$

$$\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\quad dy$$

$$\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dx$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}cosx.cosy$$

$$e^{\frac{i(x^2+y^2)cot\alpha}{2}} \cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\ dx\ dy$$

Let $A=\sqrt{\frac{1-icot\alpha}{2\pi}}$, $\boldsymbol{B}=e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i(x^2+y^2)cot\alpha}{2}}\frac{1}{2}(\cos(csc.u+1)x+\cos(csc.u-1)x)$$
$$\frac{1}{2}(\cos(csc.v+1)y+\cos(csc.v-1)\,y).dx\,dy$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\frac{1}{4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i(x^2+y^2)cot\alpha}{2}}(\cos(csc.u+1)x+\cos(csc.u-1)x)$$
$$(\cos(csc.v+1)y+\cos(csc.v-1)\,y).dx\,dy$$

Let $b_1=(csc.u+1), b_2=(csc.u-1),$
$$c_1=(csc.v+1), c_2=(csc.v-1)\ \frac{cot\alpha}{2}=a$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\frac{1}{4}\int_{-\infty}^{\infty}e^{iax^2}(\cos b_1x+\cos b_2x)dx\int_{-\infty}^{\infty}e^{iay^2}(\cos c_1y+\cos c_2\,y)\,dy$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\frac{1}{64a}(-1)^{\frac{3}{2}}\pi$$
$$\left[\begin{pmatrix}e^{\frac{-ib_1^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax-b_1)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax+b_1)\right)\right)\\+e^{\frac{-ib_2^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax-b_2)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax+b_2)\right)\right)\end{pmatrix}\begin{pmatrix}e^{\frac{-ic_1^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay-c_1)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay+c_1)\right)\right)\\+e^{\frac{-ic_2^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay-c_2)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay+c_2)\right)\right)\end{pmatrix}\right]_{-\infty}^{\infty}$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\frac{i^3\pi}{64a}\left[\left(e^{\frac{-ib_1^2}{4a}}(4\sqrt{i})+e^{\frac{-ib_2^2}{4a}}(4\sqrt{i})\right)\left(e^{\frac{-ic_1^2}{4a}}(4\sqrt{i})+e^{\frac{-ic_2^2}{4a}}(4\sqrt{i})\right)\right]$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\frac{-i\pi}{4a}\left[\left(e^{\frac{-ib_1^2}{4a}}+e^{\frac{-ib_2^2}{4a}}\right)\left(e^{\frac{-ic_1^2}{4a}}+e^{\frac{-ic_2^2}{4a}}\right)\right]$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\frac{-i\pi}{2cot\alpha}\left[\left(e^{\frac{-i(csc.u+1)^2}{2cot\alpha}}+e^{\frac{-i(csc.u-1)^2}{2cot\alpha}}\right)\left(e^{\frac{-i(csc.v+1)^2}{2cot\alpha}}+e^{\frac{-i(csc.v-1)^2}{2cot\alpha}}\right)\right]$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\frac{-i\pi}{2cot\alpha}e^{\frac{-itan\alpha}{2}(csc^2\alpha u^2+1)}e^{\frac{-itan\alpha}{2}(csc^2\alpha v^2+1)}\left[\begin{matrix}(e^{isec\alpha u}+e^{-isec\alpha u})\\(e^{isec\alpha v}+e^{-isec\alpha v})\end{matrix}\right]$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=AB\frac{-i\pi}{2cot\alpha}e^{\frac{-itan\alpha}{2}(csc^2\alpha u^2+csc^2\alpha v^2+2)}\,2\cos(sec\alpha.u).2\cos(sec\alpha.v)$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}\frac{-2i\pi}{cot\alpha}e^{\frac{-itan\alpha}{2}(csc^2\alpha u^2+csc^2\alpha v^2+2)}\cos(sec\alpha.u).\cos(sec\alpha.v)$$

$$F_C^\alpha\{cosx.cosy\}(u,v)=\sqrt{\frac{-\pi2(1-icot\alpha)}{cot^2\alpha}}e^{\frac{i}{2}((u^2+v^2)cot\alpha-tan\alpha(csc^2\alpha u^2+csc^2\alpha v^2+2)}\cos(sec\alpha.u).\cos(sec\alpha.v)$$

2.4. If $F_C^\alpha\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Cosine transform of f(x,y)then

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v)=\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}e^{\frac{-i}{sin2\alpha}\left(\frac{u^2}{(1+2atan\alpha)}+\frac{v^2}{(1+2btan\alpha)}\right)}\frac{2\pi}{\sqrt{(1+2atan\alpha)cot\alpha(1+2btan\alpha)cot\alpha}}$$

Solution:

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v)=\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{(ax^2+by^2)}e^{\frac{i(x^2+y^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dx\,dy$$

Let, $A=\sqrt{\frac{1-icot\alpha}{2\pi}}$ $B=e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v)=AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{(ax^2+by^2)}e^{\frac{i(x^2+y^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dx\,dy$$

$$F_C^\alpha\{e^{(ax^2+by^2)}\}(u,v)=AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}(x^2+y^2+\frac{2ax^2}{cot\alpha}+\frac{2by^2}{cot\alpha})cot\alpha}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dx\,dy$$

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v)=AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}((1+2atan\alpha)x^2+(1+2btan\alpha)y^2)cot\alpha}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dx\,dy$$

$$F_S^\alpha\{e^{(ax^2+by^2)}\}(u,v)=AB\int_{-\infty}^{\infty}e^{i(px^2)}\cos(cx)\,dx\int_{-\infty}^{\infty}e^{i(qx^2)}.\cos(dy)\,dy$$

Let $p=\frac{(1+2atan\alpha)cot\alpha}{2}$ $q=\frac{(1+2btan\alpha)cot\alpha}{2}$

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v)=AB\left[\begin{matrix}\left(\frac{-1}{4\sqrt{p}}(-1)^{\frac{3}{4}}\sqrt{\pi}e^{\frac{-ic^2}{4p}}\right)\\\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{p}}(2px-c)\right)+(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{p}}(2px+c)\right)\right)\end{matrix}\right]_{-\infty}^{\infty}$$

$$\left[\begin{matrix}\left(\frac{-1}{4\sqrt{q}}(-1)^{\frac{3}{4}}\sqrt{\pi}e^{\frac{-id^2}{4a}}\right)\\ \begin{pmatrix} erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{q}}(2qx-d)\right) \\ +(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{q}}(2qx+d)\right)\end{pmatrix}\end{matrix}\right]_{-\infty}^{\infty}$$

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v) =$$

$$\frac{-iAB\pi}{16\sqrt{pq}}\left[\begin{matrix}\left(e^{\frac{-ic^2}{4p}}\right)\\ \begin{pmatrix} erfi\left(\frac{\sqrt{i}}{2\sqrt{p}}(2px-c)\right) \\ +(erfi\left(\frac{\sqrt{i}}{2\sqrt{p}}(2px+c)\right)\end{pmatrix}\end{matrix}\right]_{-\infty}^{\infty}$$

$$\left[\left(e^{\frac{-id^2}{4a}}\right)\begin{pmatrix} erfi\left(\frac{\sqrt{i}}{2\sqrt{q}}(2qx-d)\right) \\ +(erfi\left(\frac{\sqrt{i}}{2\sqrt{q}}(2qx+d)\right)\end{pmatrix}\right]_{-\infty}^{\infty}$$

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v) = \frac{-iAB\pi}{16\sqrt{pq}}e^{\frac{-ic^2}{4p}}$$

$$\begin{bmatrix} erfi(\sqrt{i}\infty)+erfi(\sqrt{i}\infty)- \\ erfi(-\sqrt{i}\infty)-erfi(-\sqrt{i}\infty)\end{bmatrix}$$

$$e^{\frac{-id^2}{4q}}\begin{bmatrix} erfi(\sqrt{i}\infty)+erfi(\sqrt{i}\infty) \\ -erfi(-\sqrt{i}\infty)-erfi(-\sqrt{i}\infty)\end{bmatrix}$$

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v) = \frac{-iAB\pi}{16\sqrt{pq}}e^{\frac{-ic^2}{4p}}[4\sqrt{i}]e^{\frac{-id^2}{4q}}[4\sqrt{i}]$$

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v)$$

$$= \frac{AB\pi}{\sqrt{pq}}e^{\frac{-i\left(\frac{(cosec\alpha.u)^2}{\frac{(1+2atan\alpha)cot\alpha}{2}}+\frac{(cosec\alpha.v)^2}{\frac{(1+2btan\alpha)cot\alpha}{2}}\right)}{4}}$$

$$F_C^\alpha\{e^{i(ax^2+by^2)}\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(u^2+v^2)cot\alpha}{2}}$$

$$e^{\frac{-i}{sin2\alpha}\left(\frac{u^2}{(1+2atan\alpha)}+\frac{v^2}{(1+2btan\alpha)}\right)}\frac{2\pi}{\sqrt{(1+2atan\alpha)cot\alpha(1+2btan\alpha)cot\alpha}}$$

3. Conclusion

We solved some examples of generalized two-dimensional fractional Cosine transform

In the range $-\infty$ to ∞

References

[1] M. Ozaktas, Z. Zalevsky, and M. A. Kutay," The Fractional Fourier Transform. West Sussex, U. K.: Wiley, 2001.

[2] L. B. Almeida, "The fractional Fourier transform and time–frequency representations," IEEE Trans. Signal Proc. 42 (11) (1994) 3084–3093.

[3] D. Mendlovic, "Advances in Imaging and Electron Physics, vol. 106, "Academic Press, New York, 1999.

[4] F. Bellifemine and R. Picco, "Video signal coding with DCT and vector quantization," IEEE. Trans. Commun. 42, (1994) 200.

[5] H. T. Chang and C. L. Tsan," Image watermarking by use of digital holography embedded in the discrete-cosine-transform domain," Appl.Opt. 44, (2005) 6211.

[6] E. Y. Lam and J. W. Goodman, "Discrete cosine transform domain restoration of defocused images," Appl. Opt. 37, (1998) 6213.

[7] S. C. Pei and M. H. Yeh," The discrete fractional cosine and sine transform, IEEE Trans. Signal Processing," 49, (2001) 1198.

[8] A. W. Lohmann, D. Mendlovic, Z. Zalevsky, and R. G. Dorch, "Some important fractional transformation for signal processing," Opt.commun,vol.125,pp.18-20,1996

[9] Pei Soo-Chang, Min-Mung, "The Discrete Fractional Cosine and Sine Transform. IEEE Trans Signal Processing," 2001, 49(6): 1198-1207.

[10] Conf on Information Sci and Eng. 2009, 1864-1867.

[11] V. D. Sharma, S. A. Khapre; "Analyticity of the generalized two dimensional fractional Cosines transforms, "J. Math. Computer Sci. ISSN. 1927-5307.

[12] V. D. Sharma, S. A. Khapre; "Inversion formula for generalized two dimensional fractional Cosine transforms, "American journal of mathematics and mathematical sciences vol.2, No.1Jan-June 2013. Pp. 83-86 ISSN – 2278 - 0874.

[13] V. D. Sharma, S. A. Khapre; "Applications on generalized two dimensional fractional Cosine transforms, "International journal of engineering and innovative technology vol.3, issue4 October 2013. ISSN – 2277 - 3754.

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