Socialistic Decision Making Approach for Bipolar Fuzzy Soft H-Ideals over Hemi Rings

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Abstract: *In this paper, we provide a general algebraic frame work for handling bipolar information by combining the theory of bipolar fuzzy soft sets with hemi rings. First, we present the concepts of bipolar fuzzy soft h-ideals and normal bipolar fuzzy soft h-ideals. Second, the characterizations of bipolar fuzzy soft h-ideals are investigated by means of positive t-cut, negative s-cut and homomorphism. Third, we give a general algorithm to solve decision making problems by using bipolar fuzzy soft set.*

Keywords: fuzzy set, soft set, hemi ring ,bipolar fuzzy soft set , bipolar fuzzy soft h-ideal, normal, comparison table, extremal, endomorphism .

1. Introduction

In our real life, bipolar fuzzy theory is a core feature to be considered: positive information represents what is possible or preferred, while negative information represents what is forbidden or surely false. Bipolarity is important to distinguish between (i) positive information, which represents what is guaranteed to be possible, for example because it has already been observed or experienced, and (ii) negative information, which represents what is impossible or forbidden, or surely false [10, 11]. This domain has recently invoked many interesting research topics in database query [9], psychology [5], image processing[4], multi criteria decision making [8], argumentation [3], human reasoning [7], etc.

Fuzzy set is a type of important mathematical structure to represent a collection of objects whose boundary is vague. There are several types of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval fuzzy sets, vague sets etc. bipolar fuzzy set is another an extension of fuzzy set whose membership degree range is different from the above extensions. In 2000 , Lee [19] imitated an extension of of fuzzy set named bipolar valued fuzzy sets. He gave two kinds of representations of the notion of bipolar- valued fuzzy sets. In case of Bi-polar-Valued fuzzy sets membership degree range is enlarged from the interval [0,1] to [-1,0].

Molodtsov [24] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Molodtsov applied this theory to several directions [24, 25, 26], and then formulated the notions of soft number, soft derivative, soft integral, etc. in [27]. The soft set theory has been applied to many different fields with great success. Maji et al. [22] worked on theoretical study of soft sets in detail, and [21] presented an application of soft set in the decision making problem using the reduction of rough sets [30]. Chen et al. [6] proposed parametrization reduction of soft sets, and then Kong et al. [16] presented the normal parametrization reduction of soft sets. The algebraic structure of soft set theory dealing with uncertainties has also been studied in more detail. Aktas. and Cagman [2]

introduced a definition of soft groups, and derived their basic properties. Park et al. [29] worked on the notion of soft WS-algebras, soft sub algebras and soft deductive systems. Jun [14] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. Jun and Park [15] presented the notion of soft ideals, idealistic soft and idealistic soft BCK/BCI-algebras. Maji et al. [20] presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets [34]. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. Roy and Maji [21] gave some applications of fs-sets. Som [32] defined soft relation and fuzzy soft relation on the theory of soft sets. Mukherjee and Chakraborty [28] worked on intuitionistic fuzzy soft relations. Aktas. and Cagman [2] compared soft sets with the related concepts of fuzzy sets and rough sets. Yang et al. [33] defined the operations on fuzzy soft sets which are based on three fuzzy logic operators: negation, triangular norm and triangular conorm. Zou and Xiao [35] introduced the soft set and fuzzy soft set into the incomplete environment

Ideals of hemi rings, as a kind of special hemi ring, play a crucial role in the algebraic structure theories since many properties of hemi rings are characterized by ideals. However, in general, ideals in hemi rings do not coincide with the ideals in rings. Observing this problem, Henriksen [12] introduced *k*-ideas of semi rings, which is a class of more restricted ideals in semi rings. After that, another more restricted ideals, *h*-ideals of hemirings, was considered by Iizuka [13]. Subsequently, La Torre [17] studied thoroughly the properties of the *h*-ideals and *k*-ideals of hemi rings. Minzhou et.al [23] studied the applications of bipolar fuzzy theory to hemi rings. The rest of this paper organized as follows. Section-2 reviews some basic ideas related with this paper. In section-3, we propose main results of bipolar fuzzy soft h-ideals. Normal bipolar fuzzy soft h-ideals are studied in chapter-4. An algorithm approach is proposed in section-5 to present the application of bipolar fuzzy soft set in decision making followed by a numerical example. Finally the key conclusions are given in section-5.

2. Preliminaries

In this section, we review some definitions, regarding hemi rings [23] and bipolar fuzzy soft sets [19]. Suppose that $(S,+)$ and $(S, \bullet)$ are two semi groups, then the algebraic system $(S,+,\bullet)$ is called a semi ring , in which the two algebraic structures are connected by the distributive laws: $a.(b+c) = a.b + a.c$ and $(b+c).a = b.a + c.a$ for all $a,b,c \in S$.

The zero element of a semi ring $(S,+,\bullet)$ is an element $0 \in S$ satisfying $0 \bullet x = x \bullet 0 = 0$ and $x+0 = 0+x = x$ for all $x \in S$. A semi ring with zero and a commutative semi group $(S,+)$ is called a hemi ring.

A non-empty subset I of a hemi ring S is called a left (resp., right) ideal of S if I is closed with respect to addition and SI is subset of I (resp., IS is subset of I) I is called an ideal of S if it is both a left and a right ideal of S.

A left (resp., right) ideal of a hemi ring S is called a left (resp., right) h-ideal if any $x, z \in S$, any $a,b \in A$ and $x+a+z = b+z$ implies $x \in A$.

A mapping f from a hemi ring S to A semi ring T is said to be a homomorphism if for all $x,y \in S$. $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x). f(y)$.

Through out this paper, we only give the proof of results about left cases because the proof of results about right classes can be conducted by similar methods. In order to facilitate discussion, S and T are hemi rings unless otherwise specified.

Definition2.1:[23] A bipolar fuzzy set A is a universe U is an object having the form $A = \{x, \mu_A^+(x), \mu_A^-(x) : x \in U \}$ where $\mu_A^+ : U \to [0,1]$, $\mu_A^- : U \to [-1,0]$. So μ_A^+ denote the positive information and μ_A^- denote for negative information.

Definition2.2:[2] Let U be an initial universe, E be the set of parameters, A is subset of E and P(U) is the power set of U. Then (F, A) is called a soft set, where $F : A \to P(U)$.

Definition2.3:[23] Let U be an initial universe, E be the set of parameters, A is subset of E . Define F: $A \to BFU$, where BFU is the collection of all bipolar fuzzy subsets of U. Then (F,A) is said to be a bipolar fuzzy soft set over a universe U. It is defined by $(F,A) = \{ (x, \mu_e^+(x), \mu_e^-(x) : \text{for all } x \in U \text{ and } e \in A\}$.

Example: Let $U = \{ c_1,c_2,c_3,c_4 \}$ be the set of four cars under consideration and $E = \{ e_1 =\text{costly}, e_2=\text{beautiful}, e_3 = \text{fuel efficient}, e_4 = \text{modern technology} \}$ be the set of parameters and $A = \{e_1,e_2,e_3\}$ is subset of E. Then

$$\begin{cases} (F,A) = F(e_1) = \{ (c_1, 0.3, -0.4), (c_2, 0.3, -0.5), (c_3,0.1,-0.2), \\ \qquad\qquad (c_4, 0.7, -0.6)\} \\ F(e_2) = \{(c_1, 0.2, -0.6), (c_2, 0.1, -0.7), (c_3,0.3,-0.7), (c_4, 0.5, -0.6)\} \\ F(e_3) = \{(c_1, 0.1, -0.3), (c_2, 0.3, -0.5), (c_3,0.7,-0.2), (c_4, 0.3, -0.7)\} \end{cases}$$

Definition 2.4:[23] Let U be a universe and E a set of attributes. Then,(U,E) is the collection of all bipolar fuzzy soft sets on U with attributes from E and is said to be bipolar fuzzy soft class.

Definition 2.5:[23] A bipolar fuzzy soft set (F,A) is said to be a null bipolar fuzzy soft set denoted by empty set Φ, if for all $e \in A$, $F(e) = \Phi$.

Definition 2.6:[23] A bipolar fuzzy soft set (F,A) is said to be an absolute bipolar fuzzy soft set, if for all $e \in A$, $F(e) = BFU$.

Definition 2.7[23] The complement of a bipolar fuzzy soft set (F,A) is denoted $(F,A)^c$ and is denoted by $(F,A)^c = \{ (x, 1- \mu_A^+(x), 1- \mu_A^-(x) ; x \in U\}$.

Definition 2.8: A bipolar fuzzy soft set A (μ_A^+, μ_A^-) of S is called a bipolar fuzzy soft left (resp., right) h-ideal of S provided that for all $x,y,z,a,b \in S$;
(BFShI1) $\mu_A^+(x+y) \geq \min \{ \mu_A^+(x), \mu_A^+(y) \}$, $\mu_A^-(x+y) \leq \max \{ \mu_A^-(x), \mu_A^-(y) \}$,
(BFShI2)) $\mu_A^+(xy) \geq \max \{ \mu_A^+(x), \mu_A^+(y)\}$, $\mu_A^-(xy) \leq \min \{ \mu_A^-(x), \mu_A^-(y) \}$
(BFShI3) $x+a+z = b+z$ implies $\mu_A^+(x) \geq \min \{ \mu_A^+(a), \mu_A^+(b) \}$, $\mu_A^-(x) \leq \max \{ \mu_A^-(a), \mu_A^-(b) \}$,

A bipolar fuzzy soft set which is a bipolar fuzzy left and right h-ideal of S is called a bipolar fuzzy soft h- ideal of S. In this paper, the collection of all bipolar fuzzy soft h-ideals of S is denoted by BFShI(S) in short.

Example 2.1: Let $S= \{ 0,1,2,3\}$ be a set with the addition operation (+) and the multiplication ($\bullet$) as follows;

+	0	1	2	3		$\bullet$	0	1	2	3
0	0	1	2	3		0	0	0	0	0
1	1	1	2	3		1	0	1	1	1
2	2	2	2	3		2	0	1	1	1
3	3	3	3	3		3	0	1	1	1

Then S is a hemi ring. Define a bipolar fuzzy soft set A as follows

	0	1	2	3
μ_A^+	0.3	0.7	0.5	0.2
μ_A^-	-0.5	-0.6	-0.7	-0.1

By routine calculations, we know that A is a bipolar fuzzy soft h-ideals of S.

An interesting consequence of bipolar fuzzy soft h-ideals of hemi rings is the following.

Proposition 2.1 : Let A be a non-empty subset of S. A bipolar fuzzy soft set A = (μ_A^+, μ_A^-) is defined by

$$\mu_A^+(x)= \begin{cases} m_1, \text{if } x \in A \\ m_2, \text{otherwise} \end{cases} \quad \text{and} \quad \mu_A^-(x)= \begin{cases} n_1, \text{if } x \in A \\ n_2, \text{otherwise} \end{cases}$$

where $0 \leq m_2 \leq m_1 \leq 1$, $-1 \leq n_1 \leq n_2 \leq 0$ is a bipolar fuzzy soft h-ideal of S if and only if A is a left (resp., right) h-ideal of S.

The research about the relationships of fuzzy sub algebras and crisp sub algebras by cut sets is usual. But important, as it is a tie which can connect abstract algebraic structures and fuzzy ones. However, now we encounter a significant challenge that the traditional cut sets are not suitable for the

frame work of bipolar fuzzy soft h-ideals of hemi rings because the characterization of bipolarity.

As a consequence, we defined positive t-cut and negative s-cut.

Definition2.9:[23] Let A is a bipolar fuzzy soft set of S and $(s,t) \varepsilon [-1,0] \times [0,1]$ .we define
$A_t^+ = \{ x \varepsilon S / \mu_A^+(x) \geq t \}$ and $A_s^- = \{ x \varepsilon S / \mu_A^-(x) \leq s \}$ and call them positive t-cut and negative s-cut of A respectively. For any $k \varepsilon [0,1]$, the set $A_k^+ \cap A_k^-$ is called the k-cut of A. From the definition 2.9 , we can easily obtained the relation of bipolar fuzzy soft h-ideals of hemi rings.

3. Main Results

In this section we discuss the properties of the cut sets, image and pre-image of bipolar fuzzy soft h-ideals by homomorphism of hemi rings.

Theorem-3.1: Let A be a bipolar fuzzy soft set S. Then A is a bipolar fuzzy soft left (resp., right) h-ideal of S if and only if the following hold;

  (i) For all $t \varepsilon [0,1]$, $A_t^+ \neq \Phi$ implies A_t^+ is a left (resp., right) h-ideal of S.
  (ii) For all $s \varepsilon [-1,0]$, $A_s^- \neq \Phi$ implies A_s^- is left (resp., right) h-ideal of S.

Proof: Let A be bipolar fuzzy soft h-ideal of S and $t \varepsilon [0,1]$ with $A_t^+ \neq \Phi$.

Then $\mu_A^+(x) \geq t$, $\mu_A^+(y) \geq t$ for all $x, y \varepsilon A_t^+$, $s \varepsilon S$. It implies that
$\mu_A^+(x+y) \geq \min \{ \mu_A^+(x) , \mu_A^+(y)\} \geq t$ and $\mu_A^+(xy) \geq \max \{ \mu_A^+(x) , \mu_A^+(y) \} \geq t$, that is $x+y$, $xy \varepsilon \mu_A^+$.

Moreover $x, z \varepsilon S$, $a, b \varepsilon A_t^+$ with $x+a+z = b+z$. Then $\mu_A^+(x) \geq \min \{ \mu_A^+(a) , \mu_A^+(b)\} \geq t$. This means that $x \varepsilon A_t^+$. Hence μ_A^+ is a left h-ideal of S.

Analogously, we can prove (ii).

Conversly, assume (i), (ii) are all valid.

For any $x \varepsilon S$, if $\mu_A^+(x) = t$, $\mu_A^-(x) = s$, then $x \varepsilon A_t^+ \cap A_s^-$. Thus A_t^+ and A_s^- are non empty. Suppose that A is not a bipolar fuzzy soft h-ideal of S, then there exists $x, z, a, b \varepsilon S$, such that $x+a+z = b+z$, $\mu_A^+(x) < t < \min \{ \mu_A^+(a) , \mu_A^+(b)\}$ and $\mu_A^-(x) > s > \max \{ \mu_A^-(a) , \mu_A^-(b)\}$. Therefore $a, b \varepsilon A_t^+$ but $x \neq A_t^+$ and $a, b \varepsilon A_s^-$ but x does not belong to A_s^-. This is a contradiction. Therefore A is a bipolar fuzzy soft h-ideal of S.

As immediate consequence of theorem 3.1 , we have the following.

Corollary 3.1: If A is a bipolar fuzzy soft h-ideal of S, then the k-cut of A is a bipolar soft h-ideal of S for all $k \varepsilon [0,1]$. For the sake of simplicity, we denote $S^{(t,s)}$ for the set $\{ x \varepsilon S / \mu_A^+(x) \geq t \} \cap \{ x \varepsilon S / \mu_A^-(x) \leq s \}$ where $A = (\mu_A^+(x) , \mu_A^-(x))$.

Corollary 3.2 : If A is a bipolar fuzzy soft left (resp., right) h-ideal of S , then $S^{(t,s)}$ is a left (resp., right) h-ideal of S for

all $(t,s) \varepsilon [0,1] \times [-1,0]$. In particular, the non empty k-cut of A is an h-ideal of S for all $k \varepsilon [0,1]$.

Theorem 3.2: Assume that A BFShI(S) and $\mu_A^+(x) + \mu_A^-(x) \geq 0$ for all $x \varepsilon S$, then $A_k^+ \cup A_{-k}^-$ is a left (resp., right) h-ideal of S for all $k \varepsilon [0,1]$.

Proof: Let $k \varepsilon [0,1]$, evidently, $A_k^+ \neq \Phi$, $A_{-k}^- \neq \Phi$ and they are all left h-ideals of S from theorem 3.1. Let $x_1, x_2 \varepsilon A_k^+ \cup A_{-k}^-$, x, z ε S with $x+x_1+z = x_2 +z$. To complete the proof, we just need to consider the following four cases;

  (i) $x_1 \varepsilon A_k^+$, $x_2 \varepsilon A_k^+$
  (ii) $x_1 \varepsilon A_k^+$, $x_2 \varepsilon A_{-k}^-$
  (iii) $x_1 \varepsilon A_{-k}^-$, $x_2 \varepsilon A_k^+$
  (iv) $x_1 \varepsilon A_{-k}^-$, $x_2 \varepsilon A_{-k}^-$

case(i) implies $\mu_A^+(x_1) \geq k$, $\mu_A^+(x_2) \geq k$. since A ε BFShI(S), we can obtain
$\mu_A^+(x_1+x_2) \geq \min \{ \mu_A^+(x_1) , \mu_A^+(x_2)\} \geq k$, $\mu_A^+(x_1 x_2) \geq \max \{ \mu_A^+(x_1) , \mu_A^+(x_2)\} \geq k$ and
$\mu_A^+(x) \geq \min \{ \mu_A^+(x_1) , \mu_A^+(x_2)\} \geq k$. Then x_1+x_2 , $x_1 x_2$, x ε $A_k^+ \cup A_{-k}$. The proof of case (iv) is similar to case (i). For case (ii) , we can easily acquire $\mu_A^+(x_1) \geq k$, $\mu_A^-(x_2) \leq -k$. since $\mu_A^+(x_2) + \mu_A^-(x_2) \geq 0$, $\mu_A^+(x_2) \geq - \mu_A^-(x_2) \geq k$, we have $\mu_A^+(x_1+x_2) \geq \min \{ \mu_A^+(x_1) , \mu_A^+(x_2)\} \geq \min \{ \mu_A^+(x_1) , - \mu_A^+(x_2)\} \geq k$. $\mu_A^+(x_1 x_2) \geq \max \{ \mu_A^+(x_1) , \mu_A^+(x_2)\} \geq k$ and $\mu_A^+(x) \geq \min \{ \mu_A^+(x_1) , \mu_A^+(x_2)\} \geq \min \{ \mu_A^+(x_1) , - \mu_A^+(x_2)\} \geq k$. Then x_1+x_2 , $x_1 x_2 \varepsilon A_t^+$ is subset of $A_k^+ \cup A_{-k}$. The proof of case (iii) is similar to (ii). Hence $A_k^+ \cup A_{-k}$ is left h-ideal of S.

Definition3.1:[23] Let Φ ; S $\to$ T be a homomorphism of hemi rings, and B be a bipolar fuzzy soft set of T. Then the inverse
image of B $\Phi^{-1}(B)$ is the bipolar fuzzy soft set of S given by $\Phi^{-1}(\mu_B^+)(x) = \mu_B^+(\Phi(x))$, $\Phi^{-1}(\mu_B^-)(x) = \mu_B^-(\Phi(x))$, for all x ε S. Conversly, let A be a bipolar fuzzy soft set of S . The image of A, $\Phi(A)$ is bipolar fuzzy soft set of T defined by

$$\Phi(\mu_A^+)(x) = \begin{cases} \underset{z \varepsilon \Phi^{-1}(y)}{\vee} \mu_A^+(z) , & \text{if } \Phi^{-1}(y) \neq \Phi. \\ \\ 0 & \text{otherwise} \end{cases}$$

$$\Phi(\mu_A^-)(x) = \begin{cases} \underset{z \varepsilon \Phi^{-1}(y)}{\cap} \mu_A^+(z) , & \text{if } \Phi^{-1}(y) \neq \Phi \\ \\ \end{cases}$$

otherwise, for all y ε T, where $\Phi^{-1}(y) = \{ x \varepsilon S / \Phi(x) = y\}$.

Theorem 3.3: Let Φ : S $\to$ T be a homomorphism of hemi rings and B be a bipolar fuzzy soft left (resp.,right) h-ideal of T, then the inverse image $\Phi^{-1}(B)$ is a bipolar fuzzy soft left (resp., right) h-ideal of S.

Proof: Suppose that $B = (\mu_B^+, \mu_B^-)$ is a bipolar fuzzy soft left h-ideal of T and Φ is a homomorphism of hemi rings from S to T.
Then for all x,y ε S,we have

$(BFShI_1)$ $\Phi^{-1}(\mu_B^+)(x+y) = \mu_B^+ (\Phi (x+y)) = \mu_B^+ (\Phi(x) + \Phi(y)) \geq \min \{ \mu_B^+(\Phi(x)), \mu_B^+(\Phi(y))\} = \min \{ \Phi^{-1}(\mu_B^+)(x), \Phi^-$

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$^1(\mu_B^+)(y)\}$ and $\Phi^{-1}(\mu_B^-)(x+y) = (\mu_B^-)(\Phi(x+y) = (\mu_B^-)(\Phi(x)+\Phi(y))$

$\leq \max\{(\mu_B^-)(\Phi(x)), (\mu_B^-)(\Phi(y))\} = \max \{\Phi^{-1}((\mu_B^-)(x), \Phi^{-1}((\mu_B^-)(y)\}$. Thus , (i) is valid of definition 2.8. By the same way, we can show that (ii) is hold. Moreover, let x,z ,a,b ε S with x+a+z = b+z . we can acquire $\Phi(x)+\Phi(a)+\Phi(z) = \Phi(b)+\Phi(z)$ and $\Phi^{-1}(\mu_B^-)(x) = \mu_B^+(\Phi(x_1)) \geq \min \{ \mu_B^+(\Phi(a), \mu_B^+(\Phi(b)\}= \min \{ \Phi^{-1}(\mu_B^+)(a), \Phi^{-1}(\mu_B^+)(b)\}$. Analogously, we have
$\Phi^{-1}((\mu_B^-)(x) \leq \max \{ \Phi^{-1}(\mu_B^-)(a), \Phi^{-1}(\mu_B^-)(b)\}$. Hence $\Phi^{-1}(B)$ is a bipolar fuzzy soft h-ideal of S.

Theorem 3.4: Assume that $\Phi : S \rightarrow T$ be an epimorphism of hemi rings. If A is a bipolar fuzzy soft left (reso., right) h-ideal of S, then the image $\Phi(A)$ is a bipolar fuzzy soft left (resp., right) h-ideal of T.
Proof: Since Φ is an epimorphism, by theorem 3.1, it is sufficient to show that $\Phi(A)^+_t$ and $\Phi(A)^+_s$ are h-ideals of T for all (t,s) ε [0,1]× [-1,0] satisfying $\Phi(A)^+_t \neq \Phi$, $\Phi(A)^+_s \neq \Phi$.
Let t ε [0,1] and $\Phi(A)^+_t \neq \Phi$. Then for all y_1, y_2 ε $\Phi(A)^+_t$, we can obtain
$\Phi(\mu_A^+)(y_1) = V \mu_A^+(x) \geq t$ and $\Phi\mu_A^+((y_2) = V \mu_A^+(x) \geq t$.
  xε $\Phi^{-1}(y_1)$ xε $\Phi^{-1}(y_2)$

This means that there exist x_1 ε $\Phi^{-1} (y_1)$, x_2 ε $\Phi^{-1}(y_2)$ such that $\mu_A^+(x_1)\geq t$, $\mu_A^+(x_2)\geq t$, Then $\Phi\mu_A^+((y_1+y_2) = V \mu_A^+(x) \geq \mu_A^+(x_1+x_2) \geq \min \{ \mu_A^+(x_1) , \mu_A^+(x_2) \}\geq t$.
  xε $\Phi^{-1}(y)$
Therefore y_1+y_2 ε $\Phi(A)^+_t$.
For all y_0 ε $\Phi(A)^+_t$, we have $\Phi(\mu_A^+)(y_0) = V \mu_A^+(x) \geq t$, which implies that there exists
  xε $\Phi^{-1}(y)$
x_0 ε $\Phi^{-1}(y_0)$ such that $\mu_A^+(x_0) \geq t$.
For each y ε T , since Φ is an epimorphism and A is a bipolar fuzzy soft left h-ideal of S , there exists x ε S such that $\Phi(x) = y_1$, $\Phi_A^+(xx_0) \leq \max \{ \Phi_A^+(x), \Phi_A^+(x_0)\} \leq t$. Then
$\Phi(\mu_A^+)(yy_0) = V \max \{ \mu_A^+(x) , \mu_A^+(x_0)\} = t$. Thus yy_0 ε $\Phi(A)^+_t$. More over, let any y,z ε T
  xε $\Phi^{-1}(yy_0)$
and any m,n ε $\Phi(A)^+_t$ such that y+m+z = n+z. Then we can acquire
$\Phi(\mu_A^+)(m) = V \mu_A^+(x) \geq t$ and $\Phi(\mu_A^+)(n) = V \mu_A^+(x) \geq t$.
Thus y ε $\Phi(A)^+_t$.
  xε $\Phi^{-1}(m)$ xε $\Phi^{-1}(n)$
This means that $\Phi(A)^+_t$ is a left h-ideal of T . Analogously, we can prove that $\Phi(A)^-_s$ is a left h-ideal of T. This completes the proof.

4. Normal Bipolar fuzzy soft h-ideals

In this section, we introduce and characterize normal bipolar fuzzy soft h-ideals of hemi rings.

By definition 2.8, it is clear that a bipolar fuzzy set A is an bipolar fuzzy soft h-ideals of S providing that $\mu_A^+(x) = 1$ and $\mu_A^-(x) = -1$ for x ε S. However, as a general rule, $\mu_A^+(x) = 1$ and $\mu_A^+(x) = -1$ may not always hold. Therefore, it is necessary for us to define the following definition.

Definition4.1: A bipolar fuzzy soft h-ideal A of S is said to be normal if there exits an element x ε S such that A(x) = (1,-1) that means $\mu_A^+(x)= 1$ and $\mu_A^-(x) = -1$

Example 4.1: Consider S = {0,1,2,3} which is described in example 2.1. Let A be a bipolar fuzzy soft set S defined by

	0	1	2	3
μ_A^+	1	1	1	0.6
μ_A^-	-1	-1	-1	-0.5

Clearly, A is a normal bipolar fuzzy soft h-ideal of S.

Definition 4.2: A element x_0ε S is called extremal for a bipolar fuzzy soft set A if $\mu_A^+(x_0) \geq \mu_A^+(x)$ and $\mu_A^-(x_0) \leq \mu_A^-(x)$, for all x ε S.
From the above definitions, we can easily derived the following properties.

Proposition 4.1: A bipolar fuzzy soft set A of S is a normal bipolar fuzzy soft h-ideal if and only if A(x) = (-1,1) for its all extremal elements.

Theorem 4.1: If x_0 is an element of a bipolar fuzzy soft left (resp., rtght) h-ideal, then a bipolar fuzzy soft set A defined by $\mu_A^+(x) = \mu_A^+(x) +1 - \mu_A^+(x_0)$ and $\mu_A^-(x) = \mu_A^-(x) -1 - \mu_A^-(x_0)$ for all x ε S is a normal bipolar fuzzy soft left (resp., right) h-ideal of S containing A.

Proof: First, we claim that Ã is normal . In fact , since $\tilde{A}^+(x) = \mu_A^+(x) +1 - \mu_A^+(x_0)$, $\tilde{A}^-(x) = \mu_A^-(x) -1 - \mu_A^-(x_0)$ and x_0 is an extremal element of A. we have $\mu_A^+(x_0) = 1$, $\mu_A^-(x_0) = -1$. $\mu_A^+(x)$ ε [0,1] and $\mu_A^-(x) = [-1,0]$ for all x ε S, Thus Ã is normal.
Next we show that Ã is bipolar fuzzy soft h-ideal of S. For all x,y ε S, we have
(BFShI1) $\tilde{A}^+(x+y) = \mu_A^+(x+y) +1 - \mu_A^+(x_0) \geq \min \{ \mu_A^+(x) ,\mu_A^+(y)\} +1 - \mu_A^+(x_0) = \min \{ \mu_A^+(x) +1 - \mu_A^+(x_0), \mu_A^+(y) +1 - \mu_A^+(x_0)\} = \min \{ \tilde{A}^+(x), \tilde{A}^+(y)\}$ and
$\tilde{A}^-(x+y) = \mu_A^-(x+y) +1 - \mu_A^-(x_0) \leq \max \{ \mu_A^-(x) ,\mu_A^-(y)\} +1 - \mu_A^-(x_0) = \max \{ \mu_A^-(x) +1 - \mu_A^-(x_0), \mu_A^-(y) +1 - \mu_A^-(x_0)\} = \max \{ \tilde{A}^-(x), \tilde{A}^-(y)\}$. Thus (BFShI1) is valid. Similarly, we can prove that (BFShI2) holds. More over, let any x,z,a,b ε S such that x+a+z = b+z, we have
$\tilde{A}^+(x) = \mu_A^+(x) +1 - \mu_A^+(x_0) \geq \min \{ \mu_A^+(a) ,\mu_A^+(b)\} +1 - \mu_A^+(x_0) = \min \{ \mu_A^+(a) +1 - \mu_A^+(x_0), \mu_A^+(b) +1 - \mu_A^+(x_0)\} = \min \{ \tilde{A}^+(a) , \tilde{A}^+(b) \}$.
Analogously, we have $\tilde{A}^-(x) \leq \max \{ \tilde{A}^-(a) , \tilde{A}^-(b) \}$. Thus Ã is normal bipolar fuzzy soft h-ideal of S . Clearly A is contained in Ã.

Corollary 4.1: From the definition of Ã in theorem 4.1, we get $\tilde{\tilde{A}} = \tilde{A}$ for all A ε BFShI(S). In particular, if A is normal, then $\tilde{A} = A$.

Definition 4.2: A non empty bipolar fuzzy soft h-ideal of S is called completely normal if there exists x ε S such that A(x) = (0,0).
Let all the completely normal bipolar fuzzy soft h-ideals of S be denoted by C(S).

Theorem 4.2: Let f : [0,1] → [0,1] and g : [-1,0] → [-1,0] be two increasing functions and A be a bipolar fuzzy soft set of S. Then $A_{(f,g)} =(\mu_{Af}^+, \mu_{Ag}^-)$ where $\mu_{Af}^+(x) = f (\mu_A^+(x)$ and μ_{Ag}^-

(x) = g(μ_A(x)) for all x ε S is a bipolar fuzzy soft h-ideal of S if and only if g(μ_A^+(0)) =-1, then $A_{(f,g)}$ is normal.

Proof: Let $A_{(f,g)}$ ε BFShI(S) , then for all x,y ε S. we have

$f(\mu_A^+(x+y)) = \mu_{Af}^+(x+y) \geq$ min { μ_{Af}^+(x), μ_{Af}^+(y) }= min { $f(\mu_A^+(x))$, $f(\mu_A^+(y))$} = f (min{μ_{Af}^+(x), μ_{Af}^+(y)}).

Since f is increasing, it follows that μ_A^+(x+y) $\geq$ min { μ_A^+(x), μ_A^+(y) }. Conversely, if A ε BFShI(S), then for all x,y ε S, we have μ_{Af}^+(x+y) = f (μ_A^+(x+y)) $\geq$ f (min(μ_{Af}^+(x), μ_{Af}^+(y))= min { f(μ_{Af}^+(x), μ_{Af}^+(y))} = min {. μ_{Af}^+(x) , μ_{Af}^+(x)}. Similarly, we have μ_{As}^-(x+y) $\leq$ max {μ_{Af}^-(x) , μ_{Af}^-(x)}. Thus $A_{(f,g)}$ satisfies $(BFShI_1)$ if and only if A satisfies $(BFShI_1)$.The analogous connection between $A_{(f,g)}$ and A can be obtained in the case of axioms $(BFShI_2)$ and $(BFShI_3)$. This completes the proof.

5. Socialistic decision making approach for Bipolar fuzzy soft set

Bipolar fuzzy soft set has several application to deal with uncertainties from our different kinds of daily life problems. Here we discuss such an application for solving a socialistic decision making problem.

5.1 Comparison Table

It is a square table in which number of rows and number of colums are equal and both are labeled by the object name of the universe such as $c_1, c_2, \ldots, c_n$ and the entries d_{ij} where d_{ij} = the number of parameters for which the value of d_i exceeds or equal to the value of d_j.

5.2 Algorithm

(i) Input the ACE of choice of parameters of the X.
(ii) Consider the bipolar fuzzy soft set in tabular form.
(iii) Compute the comparision table of positive values function and negative values function.
(iv) Compute the positive values and negative values score.
(v) Compute the final score by averaging positive values score and negative values score.

5.3 Bipolar socialistic decision making problem.

Assume that a real estate agent has a set of different types of houses U = { u_1, u_2, u_3, u_4, u_5 }which may be characterized by a set of parameters E= {x_1, x_2, x_3, x_4 } for j = 1,2,3,4 the parameters x_j stand for in "good location", "cheap", "modern", "large", respectively. Suppose that a married couple , Mr.X and Mrs. X, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners' parameters by using bipolar fuzzy soft sets as follows.

Assume that U = { u_1, u_2, u_3, u_4, u_5 } is a universal set and E= {x_1, x_2, x_3, x_4 } set of all parameters. Our aim is to find the attractive houses for Mr. X. Suppose the wishing parameters of Mr.X be A is subset of E, where A = {e_1, e_2, e_5}.
$F(e_1)$ = {(c_1, 0.6, -0.7), (c_2, 0.3, -0.2), (c_3,0.7,-0.3), (c_4, 0.8, -0.4)}

$F(e_2)$ = {(c_1, 0.4, -0.6), (c_2, 0.7, -0.5), (c_3,0.9,-0.4), (c_4, 0.5, -0.3)}
$F(e_5)$ = {(c_1, 0.9, -0.6), (c_2, 0.3, -0.1), (c_3,0.8,-0.9), (c_4, 0.7, -0.4)}
For the maximum score, if it occurs in i-th row, then Mr.X buy to d_i, $1 \leq i \leq 4$

Step-1 Positive values function and Negative values function of the given data

•	e_1	e_2	e_5
c_1	0.6	0.4	0.9
c_2	0.3	0.7	0.3
c_3	0.7	0.9	0.8
c_4	0.8	0.5	0.7

•	e_1	e_2	e_3
c_1	-0.7	-0.6	-0.6
c_2	-0.2	-0.5	-0.1
c_3	-0.3	-0.4	-0.9
c_4	-0.4	-0.3	-0.4

Step-2: Comparison tables of step-1

•	c_1	c_2	c_3	c_4
c_1	3	2	1	1
c_2	1	3	1	1
c_3	2	3	3	1
c_4	2	2	2	3

•	c_1	c_2	c_3	c_4
c_1	3	2	3	3
c_2	0	3	1	1
c_3	2	1	3	2
c_4	0	2	2	3

Step-3: Membership score tables

•	Row sum (a)	Column sum (b)	Membership score (a-b)
c1	7	8	-1
c2	6	10	-4
c3	9	7	2
c4	9	6	3

•	Row sum (A)	Column sum (B)	Non-Membership score (A-B)
c_1	11	5	6
c_2	5	8	-3
c_3	8	9	-1
c_4	7	9	-2

Step-5 Final score table

•	Positive value score (P)	Negative value score (N)	Final score (P+N / 2)
c_1	-1	6	**2.5**
c_2	-4	-3	-3.5
c_3	2	-1	0.5
c_4	3	-2	0.5

Clearly the maximum score is **2.5** scored by the house c_1.

Decision: Mr.X will by c_1 . If he does not want to buy due to certain reason, his second choice will be c_3 or c_4.

6. Conclusion and Future Work

Bipolarity plays a very important role in many branches of pure and applied mathematics. The combination of bipolar fuzzy set theory and algebraic system have resulted in many interesting research topics, which have been drawing a wide spread attention of many mathematical researchers and computer scientists. In this paper, we have applied bipolar fuzzy sets theories to hemi rings and have discussed some basic properties on the subject of bipolar fuzzy h-ideals of hemi rings, which is, in fact, just a incomplete beginning of the study of the hemi ring theory, so it is necessary to carry out more theoretical researches to establish a general

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framework for the practical application. We believe that the research in this direction can invoke more new topics and can provide more applications in some fields such as mathematical morphology, logic and information science, engineering, medical diagnosis.

7. Future Work

(i) By employing bipolar fuzzy h-ideals of hemi rings, we establish bipolar fuzzy topologies of hemi rings and discuss the correspondences between bipolar fuzzy topologies and bipolar fuzzy ideals of hemi rings. (ii) The study about bipolar fuzzy h-bi-ideals, bipolar fuzzy h-quasi-ideals, bipolar fuzzy h-interior ideals and so on. (iii) The study about applications, especially in information sciences and general systems.

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