

frame work of bipolar fuzzy soft h-ideals of hemi rings because the characterization of bipolarity.

As a consequence, we defined positive t-cut and negative s-cut.

Definition 2.9:[23] Let A is a bipolar fuzzy soft set of S and $(s,t) \in [-1,0] \times [0,1]$. we define $A_t^+ = \{x \in S / \mu_A^+(x) \geq t\}$ and $A_s^- = \{x \in S / \mu_A^-(x) \leq s\}$ and call them positive t-cut and negative s-cut of A respectively. For any $k \in [0,1]$, the set $A_k^+ \cap A_k^-$ is called the k-cut of A. From the definition 2.9, we can easily obtained the relation of bipolar fuzzy soft h-ideals of hemi rings.

3. Main Results

In this section we discuss the properties of the cut sets, image and pre-image of bipolar fuzzy soft h-ideals by homomorphism of hemi rings.

Theorem-3.1: Let A be a bipolar fuzzy soft set S. Then A is a bipolar fuzzy soft left (resp., right) h-ideal of S if and only if the following hold;

- (i) For all $t \in [0,1]$, $A_t^+ \neq \Phi$ implies A_t^+ is a left (resp., right) h-ideal of S.
- (ii) For all $s \in [-1,0]$, $A_s^- \neq \Phi$ implies A_s^- is left (resp., right) h-ideal of S.

Proof: Let A be bipolar fuzzy soft h-ideal of S and $t \in [0,1]$ with $A_t^+ \neq \Phi$.

Then $\mu_A^+(x) \geq t, \mu_A^+(y) \geq t$ for all $x,y \in A_t^+, s \in S$. It implies that $\mu_A^+(x+y) \geq \min \{ \mu_A^+(x), \mu_A^+(y) \} \geq t$ and $\mu_A^+(xy) \geq \max \{ \mu_A^+(x), \mu_A^+(y) \} \geq t$, that is $x+y, xy \in A_t^+$.

Moreover $x,z \in S, a,b \in A_t^+$ with $x+a+z = b+z$. Then $\mu_A^+(x) \geq \min \{ \mu_A^+(a), \mu_A^+(b) \} \geq t$. This means that $x \in A_t^+$. Hence A_t^+ is a left h-ideal of S.

Analogously, we can prove (ii).

Conversly, assume (i), (ii) are all valid.

For any $x \in S$, if $\mu_A^+(x) = t, \mu_A^-(x) = s$, then $x \in A_t^+ \cap A_s^-$. Thus A_t^+ and A_s^- are non empty. Suppose that A is not a bipolar fuzzy soft h-ideal of S, then there exists $x,z,a,b \in S$, such that $x+a+z = b+z, \mu_A^+(x) < t < \min \{ \mu_A^+(a), \mu_A^+(b) \}$ and $\mu_A^-(x) > s > \max \{ \mu_A^-(a), \mu_A^-(b) \}$. Therefore $a,b \in A_t^+$ but $x \notin A_t^+$ and $a,b \in A_s^-$ but x does not belong to A_s^- . This is a contradiction. Therefore A is a bipolar fuzzy soft h-ideal of S.

As immediate consequence of theorem 3.1, we have the following.

Corollary 3.1: If A is a bipolar fuzzy soft h-ideal of S, then the k-cut of A is a bipolar soft h-ideal of S for all $k \in [0,1]$. For the sake of simplicity, we denote $S^{(t,s)}$ for the set $\{x \in S / \mu_A^+(x) \geq t \cap \{x \in S / \mu_A^-(x) \leq s\}$ where $A = (\mu_A^+(x), \mu_A^-(x))$.

Corollary 3.2 : If A is a bipolar fuzzy soft left (resp., right) h-ideal of S, then $S^{(t,s)}$ is a left (resp., right) h-ideal of S for

all $(t,s) \in [0,1] \times [-1,0]$. In particular, the non empty k-cut of A is an h-ideal of S for all $k \in [0,1]$.

Theorem 3.2: Assume that A BfShI(S) and $\mu_A^+(x) + \mu_A^-(x) \geq 0$ for all $x \in S$, then $A_k^+ \cup A_k^-$ is a left (resp., right) h-ideal of S for all $k \in [0,1]$.

Proof: Let $k \in [0,1]$, evidently, $A_k^+ \neq \Phi, A_k^- \neq \Phi$ and they are all left h-ideals of S from theorem 3.1. Let $x_1, x_2 \in A_k^+ \cup A_k^-, x, z \in S$ with $x+x_1+z = x_2+z$. To complete the proof, we just need to consider the following four cases;

- (i) $x_1 \in A_k^+, x_2 \in A_k^+$
- (ii) $x_1 \in A_k^+, x_2 \in A_k^-$
- (iii) $x_1 \in A_k^-, x_2 \in A_k^+$
- (iv) $x_1 \in A_k^-, x_2 \in A_k^-$

case(i) implies $\mu_A^+(x_1) \geq k, \mu_A^+(x_2) \geq k$. since $A \in BfShI(S)$, we can obtain

$$\mu_A^+(x_1+x_2) \geq \min \{ \mu_A^+(x_1), \mu_A^+(x_2) \} \geq k, \mu_A^+(x_1x_2) \geq \max \{ \mu_A^+(x_1), \mu_A^+(x_2) \} \geq k$$

and $\mu_A^+(x) \geq \min \{ \mu_A^+(x_1), \mu_A^+(x_2) \} \geq k$. Then $x_1+x_2, x_1x_2, x \in A_k^+ \cup A_k^-$. The proof of case (iv) is similar to case (i). For case (ii), we can easily acquire $\mu_A^+(x_1) \geq k, \mu_A^-(x_2) \leq -k$. since $\mu_A^+(x_2) + \mu_A^-(x_2) \geq 0, \mu_A^+(x_2) \geq -\mu_A^-(x_2) \geq k$, we have $\mu_A^+(x_1+x_2) \geq \min \{ \mu_A^+(x_1), \mu_A^+(x_2) \} \geq \min \{ \mu_A^+(x_1), -\mu_A^-(x_2) \} \geq k$. $\mu_A^+(x_1x_2) \geq \max \{ \mu_A^+(x_1), \mu_A^+(x_2) \} \geq k$ and $\mu_A^+(x) \geq \min \{ \mu_A^+(x_1), \mu_A^+(x_2) \} \geq \min \{ \mu_A^+(x_1), -\mu_A^-(x_2) \} \geq k$. Then $x_1+x_2, x_1x_2 \in A_t^+$ is subset of $A_k^+ \cup A_k^-$. The proof of case (iii) is similar to (ii). Hence $A_k^+ \cup A_k^-$ is left h-ideal of S.

Definition 3.1:[23] Let $\Phi : S \rightarrow T$ be a homomorphism of hemi rings, and B be a bipolar fuzzy soft set of T. Then the inverse

image of B $\Phi^{-1}(B)$ is the bipolar fuzzy soft set of S given by $\Phi^{-1}(\mu_B^+)(x) = \mu_B^+(\Phi(x)), \Phi^{-1}(\mu_B^-)(x) = \mu_B^-(\Phi(x))$, for all $x \in S$. Conversely, let A be a bipolar fuzzy soft set of S. The image of A, $\Phi(A)$ is bipolar fuzzy soft set of T defined by

$$\Phi(\mu_A^+)(x) = \begin{cases} \bigvee_{z \in \Phi^{-1}(y)} \mu_A^+(z), & \text{if } \Phi^{-1}(y) \neq \Phi \\ 0 & \text{otherwise} \end{cases}$$

$$\Phi(\mu_A^-)(x) = \begin{cases} \bigwedge_{z \in \Phi^{-1}(y)} \mu_A^-(z), & \text{if } \Phi^{-1}(y) \neq \Phi \\ 0 & \text{otherwise} \end{cases}$$

otherwise, for all $y \in T$, where $\Phi^{-1}(y) = \{x \in S / \Phi(x) = y\}$.

Theorem 3.3: Let $\Phi : S \rightarrow T$ be a homomorphism of hemi rings and B be a bipolar fuzzy soft left (resp.,right) h-ideal of T, then the inverse image $\Phi^{-1}(B)$ is a bipolar fuzzy soft left (resp., right) h-ideal of S.

Proof: Suppose that $B = (\mu_B^+, \mu_B^-)$ is a bipolar fuzzy soft left h-ideal of T and Φ is a homomorphism of hemi rings from S to T.

Then for all $x,y \in S$, we have

$$(BfShI_1) \Phi^{-1}(\mu_B^+)(x+y) = \mu_B^+(\Phi(x+y)) = \mu_B^+(\Phi(x) + \Phi(y)) \geq \min \{ \mu_B^+(\Phi(x)), \mu_B^+(\Phi(y)) \} = \min \{ \Phi^{-1}(\mu_B^+)(x), \Phi^{-1}(\mu_B^+)(y) \}$$

$$\mu_B^+(y) \text{ and } \Phi^{-1}(\mu_B^-(x+y)) = (\mu_B^-(\Phi(x+y))) = (\mu_B^-(\Phi(x)+\Phi(y)))$$

$\leq \max \{(\mu_B^-(\Phi(x))), (\mu_B^-(\Phi(y)))\} = \max \{ \Phi^{-1}((\mu_B^-(x)), \Phi^{-1}((\mu_B^-(y))) \}$. Thus, (i) is valid of definition 2.8. By the same way, we can show that (ii) is hold. Moreover, let $x, z, a, b \in S$ with $x+a+z = b+z$. we can acquire $\Phi(x)+\Phi(a)+\Phi(z) = \Phi(b)+\Phi(z)$ and $\Phi^{-1}(\mu_B^-(x)) = \mu_B^+(\Phi(x_1)) \geq \min \{ \mu_B^+(\Phi(a)), \mu_B^+(\Phi(b)) \} = \min \{ \Phi^{-1}(\mu_B^-(a)), \Phi^{-1}(\mu_B^-(b)) \}$. Analogously, we have $\Phi^{-1}((\mu_B^-(x)) \leq \max \{ \Phi^{-1}(\mu_B^-(a)), \Phi^{-1}(\mu_B^-(b)) \}$. Hence $\Phi^{-1}(B)$ is a bipolar fuzzy soft h-ideal of S.

Theorem 3.4: Assume that $\Phi : S \rightarrow T$ be an epimorphism of hemi rings. If A is a bipolar fuzzy soft left (reso., right) h-ideal of S, then the image $\Phi(A)$ is a bipolar fuzzy soft left (resp., right) h-ideal of T.

Proof: Since Φ is an epimorphism, by theorem 3.1, it is sufficient to show that $\Phi(A)_t^+$ and $\Phi(A)_s^+$ are h-ideals of T for all $(t,s) \in [0,1] \times [-1,0]$ satisfying $\Phi(A)_t^+ \neq \Phi, \Phi(A)_s^+ \neq \Phi$.

Let $t \in [0,1]$ and $\Phi(A)_t^+ \neq \Phi$. Then for all $y_1, y_2 \in \Phi(A)_t^+$, we can obtain

$$\Phi(\mu_A^+(y_1)) = \bigvee_{x \in \Phi^{-1}(y_1)} \mu_A^+(x) \geq t \text{ and } \Phi(\mu_A^+(y_2)) = \bigvee_{x \in \Phi^{-1}(y_2)} \mu_A^+(x) \geq t.$$

This means that there exist $x_1 \in \Phi^{-1}(y_1), x_2 \in \Phi^{-1}(y_2)$ such that $\mu_A^+(x_1) \geq t, \mu_A^+(x_2) \geq t$. Then $\Phi(\mu_A^+(y_1+y_2)) = \bigvee_{x \in \Phi^{-1}(y_1+y_2)} \mu_A^+(x) \geq \mu_A^+(x_1+x_2) \geq \min \{ \mu_A^+(x_1), \mu_A^+(x_2) \} \geq t$.

Therefore $y_1+y_2 \in \Phi(A)_t^+$.

For all $y_0 \in \Phi(A)_t^+$, we have $\Phi(\mu_A^+(y_0)) = \bigvee_{x \in \Phi^{-1}(y_0)} \mu_A^+(x) \geq t$, which implies that there exists

$$x_0 \in \Phi^{-1}(y_0) \text{ such that } \mu_A^+(x_0) \geq t.$$

For each $y \in T$, since Φ is an epimorphism and A is a bipolar fuzzy soft left h-ideal of S, there exists $x \in S$ such that $\Phi(x) = y, \Phi_A^+(xx_0) \leq \max \{ \Phi_A^+(x), \Phi_A^+(x_0) \} \leq t$. Then

$$\Phi(\mu_A^+(yy_0)) = \bigvee_{x \in \Phi^{-1}(yy_0)} \max \{ \mu_A^+(x), \mu_A^+(x_0) \} = t. \text{ Thus } yy_0 \in \Phi(A)_t^+. \text{ More over, let any } y, z \in T$$

and any $m, n \in \Phi(A)_t^+$ such that $y+m+z = n+z$. Then we can acquire

$$\Phi(\mu_A^+(m)) = \bigvee_{x \in \Phi^{-1}(m)} \mu_A^+(x) \geq t \text{ and } \Phi(\mu_A^+(n)) = \bigvee_{x \in \Phi^{-1}(n)} \mu_A^+(x) \geq t.$$

$$\text{Thus } y \in \Phi(A)_t^+.$$

This means that $\Phi(A)_t^+$ is a left h-ideal of T. Analogously, we can prove that $\Phi(A)_s^+$ is a left h-ideal of T. This completes the proof.

4. Normal Bipolar fuzzy soft h-ideals

In this section, we introduce and characterize normal bipolar fuzzy soft h-ideals of hemi rings.

By definition 2.8, it is clear that a bipolar fuzzy set A is an bipolar fuzzy soft h-ideals of S providing that $\mu_A^+(x) = 1$ and $\mu_A^-(x) = -1$ for $x \in S$. However, as a general rule, $\mu_A^+(x) = 1$ and $\mu_A^-(x) = -1$ may not always hold. Therefore, it is necessary for us to define the following definition.

Definition 4.1: A bipolar fuzzy soft h-ideal A of S is said to be normal if there exists an element $x \in S$ such that $A(x) = (1,-1)$ that means $\mu_A^+(x) = 1$ and $\mu_A^-(x) = -1$

Example 4.1: Consider $S = \{0,1,2,3\}$ which is described in example 2.1. Let A be a bipolar fuzzy soft set S defined by

	0	1	2	3
μ_A^+	1	1	1	0.6
μ_A^-	-1	-1	-1	-0.5

Clearly, A is a normal bipolar fuzzy soft h-ideal of S.

Definition 4.2: A element $x_0 \in S$ is called extremal for a bipolar fuzzy soft set A if $\mu_A^+(x_0) \geq \mu_A^+(x)$ and $\mu_A^-(x_0) \leq \mu_A^-(x)$, for all $x \in S$.

From the above definitions, we can easily derived the following properties.

Proposition 4.1: A bipolar fuzzy soft set A of S is a normal bipolar fuzzy soft h-ideal if and only if $A(x) = (-1,1)$ for its all extremal elements.

Theorem 4.1: If x_0 is an element of a bipolar fuzzy soft left (resp., rtght) h-ideal, then a bipolar fuzzy soft set A defined by $\mu_A^+(x) = \mu_A^+(x) + 1 - \mu_A^+(x_0)$ and $\mu_A^-(x) = \mu_A^-(x) - 1 - \mu_A^-(x_0)$ for all $x \in S$ is a normal bipolar fuzzy soft left (resp., right) h-ideal of S containing A.

Proof: First, we claim that \tilde{A} is normal. In fact, since $\tilde{A}^+(x) = \mu_A^+(x) + 1 - \mu_A^+(x_0), \tilde{A}^-(x) = \mu_A^-(x) - 1 - \mu_A^-(x_0)$ and x_0 is an extremal element of A. we have $\mu_A^+(x_0) = 1, \mu_A^-(x_0) = -1, \mu_A^+(x) \in [0,1]$ and $\mu_A^-(x) \in [-1,0]$ for all $x \in S$, Thus \tilde{A} is normal.

Next we show that \tilde{A} is bipolar fuzzy soft h-ideal of S. For all $x, y \in S$, we have

$$(BFShI1) \tilde{A}^+(x+y) = \mu_A^+(x+y) + 1 - \mu_A^+(x_0) \geq \min \{ \mu_A^+(x), \mu_A^+(y) \} + 1 - \mu_A^+(x_0) = \min \{ \mu_A^+(x) + 1 - \mu_A^+(x_0), \mu_A^+(y) + 1 - \mu_A^+(x_0) \} = \min \{ \tilde{A}^+(x), \tilde{A}^+(y) \} \text{ and}$$

$$\tilde{A}^-(x+y) = \mu_A^-(x+y) - 1 - \mu_A^-(x_0) \leq \max \{ \mu_A^-(x), \mu_A^-(y) \} - 1 - \mu_A^-(x_0) = \max \{ \mu_A^-(x) - 1 - \mu_A^-(x_0), \mu_A^-(y) - 1 - \mu_A^-(x_0) \} = \max \{ \tilde{A}^-(x), \tilde{A}^-(y) \}.$$

Thus (BFShI1) is valid. Similarly, we can prove that (BFShI2) holds. More over, let any $x, z, a, b \in S$ such that $x+a+z = b+z$, we have

$$\tilde{A}^+(x) = \mu_A^+(x) + 1 - \mu_A^+(x_0) \geq \min \{ \mu_A^+(a), \mu_A^+(b) \} + 1 - \mu_A^+(x_0) = \min \{ \mu_A^+(a) + 1 - \mu_A^+(x_0), \mu_A^+(b) + 1 - \mu_A^+(x_0) \} = \min \{ \tilde{A}^+(a), \tilde{A}^+(b) \}.$$

Analogously, we have $\tilde{A}^-(x) \leq \max \{ \tilde{A}^-(a), \tilde{A}^-(b) \}$. Thus \tilde{A} is normal bipolar fuzzy soft h-ideal of S. Clearly A is contained in \tilde{A} .

Corollary 4.1: From the definition of \tilde{A} in theorem 4.1, we get $\tilde{A} = \tilde{A}$ for all $A \in \text{BFShI}(S)$. In particular, if A is normal, then $\tilde{A} = A$.

Definition 4.2: A non empty bipolar fuzzy soft h-ideal of S is called completely normal if there exists $x \in S$ such that $A(x) = (0,0)$.

Let all the completely normal bipolar fuzzy soft h-ideals of S be denoted by C(S).

Theorem 4.2: Let $f : [0,1] \rightarrow [0,1]$ and $g : [-1,0] \rightarrow [-1,0]$ be two increasing functions and A be a bipolar fuzzy soft set of S. Then $A_{(f,g)} = (\mu_{Af}^+, \mu_{Ag}^-)$ where $\mu_{Af}^+(x) = f(\mu_A^+(x))$ and μ_{Ag}^-

$(x) = g(\mu_A(x))$ for all $x \in S$ is a bipolar fuzzy soft h -ideal of S if and only if $g(\mu_A^+(0)) = -1$, then $A_{(f,g)}$ is normal.

Proof: Let $A_{(f,g)} \in \text{BFShI}(S)$, then for all $x, y \in S$. we have

$$f(\mu_A^+(x+y)) = \mu_{Af}^+(x+y) \geq \min \{ \mu_{Af}^+(x), \mu_{Af}^+(y) \} = \min \{ f(\mu_A^+(x)), f(\mu_A^+(y)) \} = f(\min \{ \mu_{Af}^+(x), \mu_{Af}^+(y) \})$$

Since f is increasing, it follows that $\mu_A^+(x+y) \geq \min \{ \mu_A^+(x), \mu_A^+(y) \}$. Conversely, if $A \in \text{BFShI}(S)$, then for all $x, y \in S$, we have $\mu_{Af}^+(x+y) = f(\mu_A^+(x+y)) \geq f(\min(\mu_A^+(x), \mu_A^+(y))) = \min \{ f(\mu_A^+(x)), f(\mu_A^+(y)) \} = \min \{ \mu_{Af}^+(x), \mu_{Af}^+(y) \}$. Similarly, we have $\mu_{As}^-(x+y) \leq \max \{ \mu_{As}^-(x), \mu_{As}^-(y) \}$. Thus $A_{(f,g)}$ satisfies (BFShI_1) if and only if A satisfies (BFShI_1) . The analogous connection between $A_{(f,g)}$ and A can be obtained in the case of axioms (BFShI_2) and (BFShI_3) . This completes the proof.

5. Socialistic decision making approach for Bipolar fuzzy soft set

Bipolar fuzzy soft set has several application to deal with uncertainties from our different kinds of daily life problems. Here we discuss such an application for solving a socialistic decision making problem.

5.1 Comparison Table

It is a square table in which number of rows and number of columns are equal and both are labeled by the object name of the universe such as c_1, c_2, \dots, c_n and the entries d_{ij} where d_{ij} = the number of parameters for which the value of d_i exceeds or equal to the value of d_j .

5.2 Algorithm

- (i) Input the ACE of choice of parameters of the X .
- (ii) Consider the bipolar fuzzy soft set in tabular form.
- (iii) Compute the comparison table of positive values function and negative values function.
- (iv) Compute the positive values and negative values score.
- (v) Compute the final score by averaging positive values score and negative values score.

5.3 Bipolar socialistic decision making problem.

Assume that a real estate agent has a set of different types of houses $U = \{ u_1, u_2, u_3, u_4, u_5 \}$ which may be characterized by a set of parameters $E = \{ x_1, x_2, x_3, x_4 \}$ for $j = 1, 2, 3, 4$ the parameters x_j stand for in "good location", "cheap", "modern", "large", respectively. Suppose that a married couple, Mr.X and Mrs. X, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners' parameters by using bipolar fuzzy soft sets as follows.

Assume that $U = \{ u_1, u_2, u_3, u_4, u_5 \}$ is a universal set and $E = \{ x_1, x_2, x_3, x_4 \}$ set of all parameters. Our aim is to find the attractive houses for Mr. X. Suppose the wishing parameters of Mr.X be A is subset of E , where $A = \{ e_1, e_2, e_5 \}$.
 $F(e_1) = \{(c_1, 0.6, -0.7), (c_2, 0.3, -0.2), (c_3, 0.7, -0.3), (c_4, 0.8, -0.4)\}$

$$F(e_2) = \{(c_1, 0.4, -0.6), (c_2, 0.7, -0.5), (c_3, 0.9, -0.4), (c_4, 0.5, -0.3)\}$$

$$F(e_5) = \{(c_1, 0.9, -0.6), (c_2, 0.3, -0.1), (c_3, 0.8, -0.9), (c_4, 0.7, -0.4)\}$$

For the maximum score, if it occurs in i -th row, then Mr.X buy to $d_i, 1 \leq i \leq 4$

Step-1 Positive values function and Negative values function of the given data

\bullet	e_1	e_2	e_5
c_1	0.6	0.4	0.9
c_2	0.3	0.7	0.3
c_3	0.7	0.9	0.8
c_4	0.8	0.5	0.7

\bullet	e_1	e_2	e_3
c_1	-0.7	-0.6	-0.6
c_2	-0.2	-0.5	-0.1
c_3	-0.3	-0.4	-0.9
c_4	-0.4	-0.3	-0.4

Step-2: Comparison tables of step-1

\bullet	c_1	c_2	c_3	c_4
c_1	3	2	1	1
c_2	1	3	1	1
c_3	2	3	3	1
c_4	2	2	2	3

\bullet	c_1	c_2	c_3	c_4
c_1	3	2	3	3
c_2	0	3	1	1
c_3	2	1	3	2
c_4	0	2	2	3

Step-3: Membership score tables

\bullet	Row sum (a)	Column sum (b)	Membership score (a-b)
c_1	7	8	-1
c_2	6	10	-4
c_3	9	7	2
c_4	9	6	3

\bullet	Row sum (A)	Column sum (B)	Non-Membership score (A-B)
c_1	11	5	6
c_2	5	8	-3
c_3	8	9	-1
c_4	7	9	-2

Step-5 Final score table

\bullet	Positive value score (P)	Negative value score (N)	Final score (P+N/2)
c_1	-1	6	2.5
c_2	-4	-3	-3.5
c_3	2	-1	0.5
c_4	3	-2	0.5

Clearly the maximum score is 2.5 scored by the house c_1 .

Decision: Mr.X will buy c_1 . If he does not want to buy due to certain reason, his second choice will be c_3 or c_4 .

6. Conclusion and Future Work

Bipolarity plays a very important role in many branches of pure and applied mathematics. The combination of bipolar fuzzy set theory and algebraic system have resulted in many interesting research topics, which have been drawing a wide spread attention of many mathematical researchers and computer scientists. In this paper, we have applied bipolar fuzzy sets theories to hemirings and have discussed some basic properties on the subject of bipolar fuzzy h -ideals of hemirings, which is, in fact, just a incomplete beginning of the study of the hemiring theory, so it is necessary to carry out more theoretical researches to establish a general

framework for the practical application. We believe that the research in this direction can invoke more new topics and can provide more applications in some fields such as mathematical morphology, logic and information science, engineering, medical diagnosis.

7. Future Work

(i) By employing bipolar fuzzy h-ideals of hemi rings, we establish bipolar fuzzy topologies of hemi rings and discuss the correspondences between bipolar fuzzy topologies and bipolar fuzzy ideals of hemi rings. (ii) The study about bipolar fuzzy h-bi-ideals, bipolar fuzzy h-quasi-ideals, bipolar fuzzy h-interior ideals and so on. (iii) The study about applications, especially in information sciences and general systems.

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