

Improving Post-Secondary Students' Algebraic Skills in the Learning of Complex Numbers

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Abstract: *Complex numbers is one of the mathematics topics newly introduced at the post-secondary level within the vocational and technical education context. With the implementation of the new national education reform to vocational and technical education in June 2012 in Brunei Darussalam, a few of the topics for Diploma of Computer Engineering and Diploma in Electrical and Communications Engineering, such as complex numbers and waveforms, that were usually taught in Year 2 have now been introduced into the Year 1 syllabus. The teachers involved in this study aspired in using an action research approach to improve the students' algebraic skills on complex numbers. A set of pre- and post-tests were given to two groups of students in one of the vocational and technical institutions in the country to determine whether they have shown improvement in their understanding before and after the intervention stage. The marks obtained were compared and the students have shown improvement overall. The teachers were also interviewed to investigate their opinions regarding the usefulness of this research approach in order to improve their teaching and the students' learning. Their involvement in this study has made them realized the importance of having deep content knowledge in complex numbers and simultaneously being aware of their students' difficulties not only in understanding complex numbers but also other related mathematics topics as well.*

Keywords: Complex numbers, vocational and technical education, action research.

1. Introduction

A complex number is a number with a real part and an imaginary part usually represented in algebraic form: $Z = a \pm jb$ where 'a' is the real part, 'b' is the imaginary part and 'j' represents $\sqrt{-1}$ (where in mathematics, 'i' is in mathematical notation, and in engineering context, 'j' is engineering notation). The word complex numbers may have sounded difficult and alien to some students hearing it for the first time because complex means complicated or hard to understand [13]. More unknown terms such as imaginary, conjugate, phasors and Argand diagram (a coordinate plane where horizontal axis is denoted as real axis and vertical axis is denoted as imaginary axis, also known as a complex plane) introduced were too much to the students' dismay. The j-notation was introduced when the squared root of a negative number has no solution.

In the context of Brunei Darussalam's post-secondary vocational and technical education level, for the Diploma of Computer Engineering and Diploma in Electronic and Communications Engineering courses, complex numbers is applied in calculating the alternating current signal in an electric circuit and waveforms. With the implementation of the National Education System for the 21st Century (translated from *Sistem Pendidikan Negara Abad Ke-21* in the Malay Language, or better known as SPN21) to vocational and technical education in June 2012 in Brunei [4], [16], [37], a few topics in the Diploma of Computer Engineering and Diploma in Electrical and Communications Engineering courses such as complex numbers and waveforms that were usually taught in Year 2 have now been introduced into Year 1 syllabus. There are other forms of complex numbers, which are the polar form ($Z = r\angle\theta$) and

exponential form ($Z = re^{j\theta}$); however the focus of the study is on complex numbers in algebraic form. The aim of this study is to improve the students' understanding in complex numbers of algebraic form.

2. Literature Survey

Egan (2008) [15] wrote an interesting online article regarding the teaching of complex numbers, understanding the impression and difficulties that the students will face when they are learning complex numbers for the first time. Egan stated that the arithmetic of complex numbers was probably the first topic the students will come across where the power of their mathematical training will exceed the power of their imagination. The experience of learning about complex numbers reinforces tremendous power of abstract thinking and the mathematical tools that facilitate it. He added that by introducing complex numbers, it is not simply adding on an extra piece to students' prior understanding of arithmetic, it is to show that the prior understanding only gave a one-dimensional picture of a two-dimensional world. Using the Argand diagram, to represent the complex numbers graphically, the one-dimensional number line of the real numbers is expanded into the two-dimensional plane of complex numbers.

Tan and Toh (2013) [38] stated below that even students encountered learning difficulties when learning the representation of complex numbers in the Argand diagram.

Students are confused between the algebraic representation of complex numbers and the geometric representation of complex numbers. An understanding of one representation does not necessarily imply an

understanding of the other representation... Even when students are able to view the algebraic and geometric representations as just two different ways of representing complex numbers, they are still much less comfortable carrying out operations on complex numbers in the Argand diagram than by direct calculation. ([38], p. 75).

Meanwhile, Karakok, Soto-Johnson and Dyben (2013) [12] outlined the importance for teachers to have deep content knowledge and knowledge about teaching the field of complex numbers. Deep content knowledge of the field of complex numbers involves knowing the multiple representations (algebraic and geometric) and forms (algebraic, exponential and polar), understanding the connections among them, translating the forms flexibly and reorganising which representations and forms would be more suitable to use in a given task. They investigated the content knowledge of complex numbers from three secondary mathematics teachers with different numbers of years of teaching.

The teachers attended a three-day professional development program conducted by Karakok and colleagues [12] intended to strengthen their content knowledge of complex numbers. After a 90-minute interview, it was found that the teachers did not have dual conception of complex numbers and their ability to develop dual conception of complex numbers was bound by their conceptualization of various forms, which in turn was deterred by their representations of each form.

In another study by Conner, Rasmussen, Smith and Zandieh (2007) [3], they examined prospective high school mathematics teachers' understanding of complex numbers. They noticed that the teachers' conception of complex numbers often failed to extend much past $i = \sqrt{-1}$. This will become a problem especially when the teachers will teach complex numbers to the next generation of students. There were ten teachers who agreed to participate in pre and post interviews, in-class video recording, in-class assessments, homework assignments and their responses to two items on the final exam. The results from the study revealed that in the pre-interviews, all ten teachers were able to perform addition and multiplication of complex numbers; three teachers expressed prior knowledge in addition and multiplication of the complex plane and eight teachers were able to perform addition of complex numbers using a complex plane. In addition, Conner and colleagues also found that two teachers thought a complex number as a single thing while eight teachers viewed it as a pair of things [3].

The above-mentioned previous studies on complex numbers were conducted in the context of secondary school levels but none in the post-secondary level, specifically the vocational and technical education. Therefore, this study will provide an interesting opportunity to fill in the gaps otherwise still unknown in this context in Brunei.

A team of mathematics teachers in one of the vocational and technical institutions in Brunei was formed to conduct this research. The team that consisted of three teachers, one male and two female, teaching the same topic was formed so that

the members were able to share their different knowledge and teaching difficulties on complex numbers, to improve their teaching and learn the applications of complex numbers in engineering context. As mentioned earlier, in Brunei, since complex numbers and waveforms had been introduced to the Year 1 Diploma of Computer Engineering and Diploma in Electronic and Communications Engineering syllabus, the topics may be unfamiliar to the teachers who will be teaching the topics for the first time. As there were no specific textbooks for engineering mathematics provided in the institution [37], the mathematics teachers have to depend on educational websites, O and A Levels textbooks as well as the 'Learning Package' provided by mathematics department as the resources and guidelines for teaching.

After two weeks of conducting the lessons in studying complex numbers in algebraic form, the teachers noticed, from the assignments and class activities, that the students still had difficulties in learning the complex numbers especially in arithmetic (addition, subtraction, multiplication and division) and solving quadratic equations. They were not able to grasp the concept of j -notation as a simplified form of $\sqrt{-1}$. The students need to have good algebraic and arithmetic skills in order to familiarize themselves with the complex numbers.

3. Objectives of the Study

There are two objectives to this study, firstly, to investigate the extent of students' mathematical knowledge on complex numbers and secondly, to improve students' understanding in arithmetic of complex numbers by applying basic numeracy skills in the lesson. In the first objective, the students should have understood and know the features of complex numbers such as imaginary part, its conjugate and graphical representation of complex numbers in an Argand diagram. While, the second objective is to help students have the impression that learning complex numbers is not hard and applying the arithmetic will be interesting.

The research questions guiding this study are, will the diploma students be able to improve their algebraic skills in complex numbers and, what are the teachers' views in regards to the action research study of complex numbers in the vocational and technical education context?

4. Methodology

4.1 Research Framework

A team of three mathematics teachers who worked in the same institution conducted this study. The team was formed after two weeks of teaching the complex numbers to the diploma students. All members graduated with a bachelor degree in mathematics from one of the universities in Brunei. They also pursued postgraduate studies from the same university. Teacher A has a graduate diploma in technical education, Teacher B graduated with a Master of Teaching, and Teacher C was currently pursuing a Master program and also the first author of this study. Teacher A had more than six years of teaching experience while Teachers B and C had

less than five years of teaching experience. All members have never taught complex numbers before, and hence they would be able to experience and relate their difficulties of teaching complex numbers for the first time. The team also had brief discussions about complex numbers from a senior mathematics teacher who was experienced in teaching mathematics in the vocational and technical institutions.

The team decided to conduct an action research study because the members needed to investigate the extent of the students' understanding of complex numbers after two weeks of teaching the topic. Moreover, this approach will help the team to identify the students' area of weaknesses through pre- and post-tests so that they would be able to improve their teachings. The team decided on having two action research study cycles to two groups of students namely, Cycle 1 and Cycle 2. The Cycle 1 group was the students of the Diploma of Computer Engineering course, while Cycle 2 group was the students from the Diploma of Electronic and Communication Engineering course.

In the first cycle, teacher C was chosen to conduct the lesson while Teachers A and B observed and recorded the lessons. For the second cycle, teacher B was chosen to teach the lessons and teachers A and C observed and the lesson was recorded as well. The Cycle 1 students had been given a preliminary test to investigate the extent of the mathematical knowledge on complex numbers but Cycle 2 students did not take the test, as they had not learned complex numbers yet.

The reasons for having a preliminary test were to investigate the level of the students' understanding on complex numbers after two weeks of lessons and to investigate the areas in which the students were weak on. For the preliminary test, they were tested on a few areas namely; solving a quadratic equation, stating a conjugate of a complex number, stating the complex numbers from an Argand diagram, expressing the squared root of a negative number in terms of j and addition, multiplication and division of complex numbers. The duration of the test was 45 minutes.

Once the preliminary test responses had been analyzed and the areas of weaknesses were identified, a pre-test was then prepared in accordance to the specific areas. For the first cycle, after the pre-test, a series of lesson activities were modified to rectify the areas of weaknesses. The lessons conducted were video-recorded and observed by the other team members. Once the lessons were completed, a post-test was given to the students. The duration for both the pre- and post-tests was also 45 minutes. The questions in the post-test were the same as the ones in the pre-test. Using the same questions for the pre- and post-tests will determine whether there was any change in the students' misunderstanding.

In the second cycle, the same pre- and post-tests were given to the students without conducting the preliminary test but the process was the same as the first cycle. After the completion of the second cycle, analyses of the pre- and post-tests for both cycles was carried out and the teachers were interviewed to find out their views about the action research study approach.

4.2 Scope and Limitation of the Study

The study involved a total of 41 engineering diploma students in one of the vocational and technical institutions in Brunei. The students consisted of 15 female and 12 male with ages from 18 to 22 years old for Cycle 1 and; 4 female and 10 male with ages from 18 to 33 years old for Cycle 2. There were nine students who were in their in-service training for Cycle 2. A few of them had studied complex numbers in their A Level mathematics while the others encountered this topic for the first time. For the Cycle 1 students, their O level mathematics results ranged from B to E, while for the Cycle 2 students, the results ranged from B to D and one student was ungraded. Since this study was conducted in one particular institution, the findings cannot be generalized to the entire population of diploma students in Brunei.

It should also be noted that the time for pursuing this study was limited and constrained due to school term holidays, school functions, completion of the syllabus and examination. Therefore, the preliminary test, pre- and post-tests for the first cycle were conducted before the examination week of the institution, and the pre- and post-tests for the second cycle and interviews were performed after the Cycle 2 students completed their examination. Time constraint also caused the team to focus on one topic only.

5. Results and Discussion

5.1 The Pre- and Post-Tests

The pre- and post-tests' marks were collected and each item was analyzed to investigate the students' responses. From Table 1, the mean marks of the pre-test obtained were 62.1 and 58.3 from Cycle 1 and Cycle 2 students respectively. Cycle 1 students performed slightly better than Cycle 2 students. As mentioned previously, the pre- and post-tests were performed before the examination for Cycle 1 students, and after examination for Cycle 2 students. Therefore, their prior knowledge of complex numbers was sufficient but improvement was definitely needed. Meanwhile, the mean marks of the post-test obtained from Cycle 1 and Cycle 2 students were 69.6 and 83.1 respectively. This was a big improvement for Cycle 2 students in comparison to Cycle 1 students.

Table 1: Comparisons of the results from cycles 1 and 2 pre- and post-tests

Results	Cycle 1 (n=27)		Cycle 2 (n=14)	
	Pre-test	Post-test	Pre-test	Post-test
Mean	62.1	69.6	58.6	83.1
Standard Deviation	30.2	25.3	21.9	13.7

For Cycle 1 students, there were three students who scored full marks for the pre-test but for the post-test only one student maintained her 100% mark. However, there were seven students who scored less than 40% in the pre-test and the number reduced to three students in the post-test. From the overall post-test performance, 20 students had improved and six students had scored lower than their pre-test marks.

The range of scores for the pre-test was from 0% to 100%, while the post-test was from 16% to 100%.

All of the Cycle 2 students managed to answer the second item correctly that consequently improved their mean mark for the post-test. There were three students who scored 100% in the pre- and post-tests. Out of the 14 students, only three students scored less than 40% in the pre-test, but they subsequently improved their marks in the post-test. The scores for the pre- and post-tests ranged from 32% to 100% and 60% to 100% respectively.

The teachers analyzed the common mistakes made by Cycle 1 and Cycle 2 students in answering the four items in the pre-test, and they were: stating j^2 as zero instead of -1; not replacing $\sqrt{-1}$ as j ; performing algebraic addition and subtraction without any regards of the powers of j ; ignoring the sign of a negative number if the negative number was in front of the expression while performing addition; and j was not significant in performing addition, subtraction and multiplication. Shown in Figures 1 to 4 below are samples of the common errors made by the students.

Handwritten work for Pre-test Item 1 showing errors in simplifying square roots and complex numbers. The student incorrectly simplifies $\sqrt{4-76}$ and $\sqrt{72}$, and incorrectly handles the complex number $2 + j4.243$.

Figure 1: Pre-test Item 1

Handwritten work for Pre-test Item 2 showing errors in squaring a complex number. The student incorrectly calculates $(-3+j5)^2$ as $9 - j15 + j15 + j^2 25$, leading to $9 - 25 - j30 = -16 - j30$.

Figure 2: Pre-test Item 2

Handwritten work for Pre-test Item 3 showing errors in multiplying complex numbers. The student incorrectly calculates $j(-4+j)$ as -1 and $j(-4+j)$ as $-j4 + j^2$.

Figure 3: Pre-test Item 3

Handwritten work for Pre-test Item 4 showing errors in dividing complex numbers. The student incorrectly calculates $\frac{12-j2+j6-j^2}{36-j6+j6-j^2}$ as $\frac{12-j8-(-1)}{36-(-1)}$.

Figure 4: Pre-test Item 4

The teachers realized that the students were not used to having the variable j in front of a number as compared to any other algebraic expression because they did not understand the basic concepts of complex numbers. Having to learn the concept of j and j^2 in a short time was not easy to grasp especially to less academically inclined students. The

students also did not have strong algebraic and arithmetic skills to simplify the addition, subtraction and multiplication of complex numbers.

In the post-test however, there were some noted improvements, but the students still made some of the mistakes that were mentioned in the pre-test analysis. The mistakes made in the post-test were: not replacing $\sqrt{-1}$ as j and j^2 as -1; error in squaring negative numbers; ignoring the sign of a negative number and the powers of j while performing addition and subtraction; and not able to multiply properly. Some of the students' responses from the post-test are shown in Figures 5-8 below.

Handwritten work for Post-test Item 1 showing errors in simplifying square roots. The student incorrectly simplifies $\sqrt{b^2-4ac}$ as $2 \pm \sqrt{-30}$.

Figure 5: Post-test Item 1

Handwritten work for Post-test Item 2 showing errors in squaring a complex number. The student incorrectly calculates $(3+j5)^2$ as -24 .

Figure 6: Post-test Item 2

Handwritten work for Post-test Item 3 showing errors in multiplying complex numbers. The student incorrectly calculates $j(-4+j)$ as $-4j + j^2 - 1 = -4 - 1 - j = -5 - j$.

Figure 7: Post-test Item 3

Handwritten work for Post-test Item 4 showing errors in dividing complex numbers. The student incorrectly calculates $\frac{2+j}{6+j} \times \frac{6-j}{6-j} = \frac{12-2j+j6-j^2}{36-6j+6j-j^2}$ as $\frac{12-2j+j10}{36+1}$.

Handwritten work for Post-test Item 4 showing errors in dividing complex numbers. The student incorrectly calculates $\frac{12-2j+j6-j^2}{36-6j+6j-j^2}$ as $\frac{-12-j6+j4-j^2 2}{16+j8-j8-j^2 4} = \frac{-12-2-j10}{20}$.

Figure 8: Post-test Item 4

In responding to the first research question, the diploma students were able to improve their algebraic skills on complex numbers. Their misconceptions on algebra needed to change, especially in squaring a negative number as they relied too much on their calculator, the addition of a negative algebraic term with a positive term, and also their tendency

in ignoring the powers of the j -variables. In addition, they will need to familiarize themselves with the concept of j -notation as they have shown the wrong values of j^2 in the tests such as j^2 became zero, $-j^25$ became $-j25$ and $-j^25$ became $-j5$.

A few still struggled but nevertheless this should not stop the teachers to find better ways to improve their students' understanding. Judging from the students' O level results, the teachers had expected that a few students might have been struggling to learn complex numbers and other topics as well regardless whether or not they have studied the topics beforehand. Importantly, the students should have a strong mathematical background and the willingness to study hard if they want to pursue an engineering program, which will include a lot of engineering mathematical concepts.

5.2 Interviews with Teachers B and C

The teachers who taught the lessons were interviewed to examine their findings and experience that they encountered during the research period. Both Teachers B and C agreed that the action research approach was a good method to improve students' learning especially in a topic where students find difficult to understand. The teachers did not encounter any difficulties in carrying out the activities. Furthermore, conducting the pre- and post-tests can be regarded as an alternative way to check students' understanding and improvement in the learning of the complex number topic. This action research study had also encouraged the teachers to make time and effort to discuss and analyse the results of the pre- and post-tests among them so as to find the best methods to improve students' learning.

The teachers also commented that the students will not automatically like complex numbers because the students need to understand a great deal of concepts. They also noticed that a small number of academically inclined students were more interested to study complex numbers as they can grasp the idea and concepts faster than the others. The teachers enjoyed the experience although they need to have more in-depth knowledge about complex numbers by learning from more experienced teachers and finding further resources. They reflected upon the need to find other methods to improve the activities that emphasize the knowledge of basic algebra and arithmetic.

The teachers' knowledge on complex numbers was limited as was their experience in conducting an action research study. Although it was their first time to conduct such research in the vocational and technical education context, the experience has taught them to be observant about the students' difficulties in studying not only for complex numbers but for other topics as well. The teachers thought by giving a pre-test after completing a topic to explore the students' conception was a good method to improve their understanding. This definitely added to their previous method used, which was to conduct extra classes to students who had the difficulty understanding the mathematics lessons. Based from the interviews, the teachers were happy with the results, but felt unsatisfied with some misconceptions that the students still had in the post-test.

6. Conclusion

It has been noted in several local studies in Brunei that mathematics, in general, has always been one of the subjects that students at all school levels find challenging [1], [2], [4], [6], [7], [11], [14], [15], [17]–[21], [23]–[25], [27], [33]–[35], [37], [39], [40]. The findings achieved in this action research study can be considered a mathematical success because the students were able to improve their algebraic skills, although some students may need more assistance to learn the algebra. The team acknowledged the use of action research as one of the methods to investigate any misconceptions that students have, and also to improve the teachers' teaching and students' learning. The teachers were able to share their views and appreciate the potential of action research study as a way to improve their own teaching.

Since the teachers have the confidence to teach mathematics, they had an interesting experience in conducting this action research study on complex numbers. They understood that the students have difficulties in recognizing the variables of j in the context of complex numbers, as was observed from the students' responses in the pre- and post-tests. The teachers also agreed that the students needed to improve their algebraic skills not only for complex numbers but also for other related topics as well. They also hoped that they would explore more teaching resources that would improve their knowledge and skills in order to improve teaching and learning. The improved lessons will benefit not only to the teachers but also to the new batch of students.

7. Implications

The teachers were inspired to conduct an action research study due to the concern with the students' algebraic misconceptions and in the learning of complex numbers. The action research approach has contributed to the teachers' professional development in improving their content knowledge, teaching methods, and also being aware of the students' learning. As far as is known, this study investigating students' learning in complex numbers was first of its kind for vocational and technical education context in Brunei. We hoped that this would serve as a reference for future studies in mathematics education and promote more educational researches in the vocational and technical institutions.

8. Recommendations

In order to improve on this study in the future, we suggest that teachers could attempt to continue using the same research approach but with different groups of students with related electrical engineering background and perhaps, to conduct the research in another vocational and technical institution. Moreover, they could also change the algebraic form of complex numbers from $Z = a \pm jb$ into $Z = a \pm bj$, a form that the students had been familiar with as the j -variable would be at the back rather than in front of a number similar to any algebraic expression. In terms of the arithmetic of complex numbers, teachers should attempt to

emphasize the commutative and associative laws to the real and imaginary parts. Other suggestions would be to conduct activities by learning through videos and animations, common assessment tasks [40] or applicable representations [21], the appropriate use of textbooks [14] or even through facilitating engagements [39] and group discussions [4] among the students by instructing them to find the background of complex numbers in order to gain better understanding and familiarize themselves with complex numbers.

Egan (2008) [5] suggested that teachers should start with the lesson with the idea of imaginary numbers, by introducing the square root of -1 as i and proceed with basic addition, subtraction, multiplication and division. Once the students have grasped the concept of imaginary numbers, the lesson can proceed with showing them the complex numbers in the form of $Z = a + ib$. By having a number of repetitive exercises, the students may gain the necessary confidence in dealing with complex numbers.

Studies investigating misconceptions [26], [28], [29], particularly in Brunei are still confined within specific mathematics topics [1], [2], [6], [25]. The approach used in this study can be seen as one of the ways for vocational and technical education teachers to investigate their students' misconceptions in the learning of algebra and complex numbers, and also their own teaching practices. Teachers of the same subject should form a research team to investigate any learning issues faced by students [7], [15], [27], [35] by implementing future action research studies. Additionally, teachers are encouraged to assist molding the correct conceptions of learning held by students from their early schooling stages, and for teachers to be reflective practitioners of their own practices in their classroom teaching, so that meaningful learning can be achieved [8]–[10], [14], [20], [22], [33], [40].

This study has made teachers become more aware that diploma students still have difficulty in understanding algebra not only in complex numbers but also other topics as well, and to investigate ways to improve the learning of less academically inclined students. Perhaps, a targeted technique in diagnosing students' inclination to learn may be found in the appropriate usage level of questioning in the mathematics lessons [24], [30], [31], [34], [36]. In addition, teachers are encouraged to pursue research for their professional development, such as leadership or peer mentoring studies of teachers [11], [32], and of students [4], as well as to keep up with the challenges in the ever changing vocational and technical education system.

References

- [1] L.H. Ang, M. Shahrill, "Identifying Students' Specific Misconceptions in Learning Probability," *International Journal of Probability and Statistics*, 3 (2), pp. 23-29, 2014.
- [2] J.S.Y. Chong, M. Shahrill, "The Refinements Needed to Improve the Teaching of Conditional Probability," In *Proceedings of the 2nd International Conference on Social Sciences Research (ICSSR)*, pp. 389-398, 2014.
- [3] M.E. Conner, C. Rasmussen, M. Smith, M. Zandieh, "Mathematical Knowledge for Teaching: The Case of Complex Numbers," In *Proceedings for the Tenth Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education*, 2007.
- [4] D.S.M.P. Daud, M. Shahrill, "Examining the Effectiveness of Peer Mentoring in the Learning of Differentiation," In L. Gómez Chova, A. López Martínez, & I. Candel Torres (Eds.), *Proceedings of the 6th International Conference on Education and New Learning Technologies (EDULEARN)*, pp. 3305-3315, 2014.
- [5] K. Egan, "Complex Numbers," 2008. [Online]. Available: <http://www.ierg.net/publications/teaching/units/complexnum.html>. [Accessed: Mar. 11, 2013].
- [6] B.T. Go, M. Shahrill, "Investigating Students' Learning of Function Notation," In *Proceedings of the 2nd International Conference on Social Sciences Research (ICSSR)*, pp. 399-408, 2014.
- [7] M.H.S. Hamid, M. Shahrill, R. Matzin, S. Mahalle, L. Mundia, "Barriers to Mathematics Achievement in Brunei Secondary School Students: Insights into the Roles of Mathematics Anxiety, Self-esteem, Proactive Coping, and Test Stress," *International Education Studies*, 6 (11), pp. 1-14, 2013.
- [8] J.H. Jaidin, *Conceptions of Learning Held by Upper Primary Children in Government Schools in Brunei Darussalam*, (Unpublished doctoral dissertation), Queensland University of Technology, Brisbane, Australia, 2009.
- [9] R. Jawawi, *Conceptions of Economics Pre-Service Teachers' Use of Subject Knowledge in Teaching Economics and Commerce at Secondary Schools in Brunei Darussalam*, (Unpublished doctoral dissertation), University of London, London, United Kingdom, 2009.
- [10] R. Jawawi, *Reflective Practice in Teaching Economics and Commerce: A Case Study of Pre-Service Teachers in Brunei Darussalam*, VDM Verlag Dr. Müller, Saarbrücken, Germany, 2010.
- [11] N.H.A. Kani, H.N.H.M. Nor, M. Shahrill, R.H.A. Halim, "Investigating the Leadership Practices among Mathematics Teachers: The Immersion Programme," *International Journal of Contemporary Educational Research*, 1 (2), pp. 113-121, 2014.
- [12] G. Karakok, H. Soto-Johnson, S.A. Dyben, (2013). "In-Service Secondary Teachers' Conceptualization of Complex Numbers," 2013. [Online]. Available: http://pzacad.pitzer.edu/~dbachman/RUME_XVI_Linked_Schedule/rume16_submission_31.pdf. [Accessed: Mar. 11, 2013].
- [13] C. Livingstone, *Little Oxford English Dictionary and Thesaurus* (2nd ed.), Oxford University Press, New York, 2008.
- [14] M.A.H. Mahadi, M. Shahrill, "In Pursuit of Teachers' Views on the Use of Textbooks in their Classroom Practice," *International Journal of Education*, 6 (2), pp. 149-158, 2014.
- [15] R. Matzin, M. Shahrill, S. Mahalle, M.H.S. Hamid, L. Mundia, "A Comparison of Learning Styles and Study Strategies Scores of Brunei Secondary School Students by Test Anxiety, Success Attributions, and Failure Attributions: Implications for Teaching At-Risk and Vulnerable Students," *Review of European Studies*, 5 (5), pp. 119-127, 2013.

- [16] Ministry of Education, "The National Education System for the 21st Century: SPN21 (Revised ed.)," Ministry of Education, Brunei Darussalam, 2013.
- [17] L. Mundia, "Implementation of SPN21 Curriculum in Brunei Darussalam: A Review of Selected Implications on School Assessment Reforms," *International Education Studies*, 3 (2), pp. 119-129, 2010a.
- [18] L. Mundia, "Problems in Learning Mathematics: Comparison of Brunei Junior High School Students in Classes with and without Repeaters," *Journal of Mathematics Research*, 2 (3), pp. 150-160, 2010b.
- [19] L. Mundia, "Policy Changes in Brunei Teacher Education: Implications for the Selection of Trainee Teachers," *The Education Forum*, 76 (3), pp. 326-342, 2012a.
- [20] L. Mundia, "The Assessment of Math Learning Difficulties in a Primary Grade-4 Child with High Support Needs: Mixed Methods Approach," *International Electronic Journal of Elementary Education*, 4 (2), pp. 347-366, 2012b.
- [21] H.N.H.M. Nor, M. Shahrill, "Incorporating the Use of Poster and Oral Presentations as an Alternative Assessment in the Teaching of Secondary Mathematics," In *Proceedings of the 2nd International Conference on Social Sciences Research (ICSSR)*, pp. 369-378, 2014.
- [22] N.A. Omar, S.R. Matarsat, N.H. Azmin, V.A.W Chung, M.M.M. Nasir, U.M.K. Sahari, M. Shahrill, L. Mundia, "The Ideal Psychology Teacher: Qualitative Analysis of Views from Brunei GCE A-Level Students and Trainee Psychology Teachers," *Asian Social Science*, 10 (12), pp. 184-194, 2014.
- [23] M.H.A. Pungut, M. Shahrill, "Students' English Language Abilities in Solving Mathematics Word Problems," *Mathematics Education Trends and Research*, pp. 1-11, 2014.
- [24] N.H.A. Salam, M. Shahrill, "Examining Classroom Interactions in Secondary Mathematics Classrooms in Brunei Darussalam," *Asian Social Science*, 10 (11), pp. 92-103, 2014.
- [25] H.R.H. Sarwadi, M. Shahrill, "Understanding Students' Mathematical Errors and Misconceptions: The Case of Year 11 Repeating Students," *Mathematics Education Trends and Research*, pp. 1-10, 2014.
- [26] M. Shahrill, *A Further Investigation of Decimal Misconceptions held by Primary and Secondary Students*, (Unpublished master dissertation), University of Melbourne, Melbourne, Australia, 2005.
- [27] M. Shahrill, *From the General to the Particular: Connecting International Classroom Research to Four Classrooms in Brunei Darussalam*, (Unpublished doctoral dissertation), University of Melbourne, Melbourne, Australia, 2009.
- [28] M. Shahrill, *Investigating Decimals Misconceptions: Cross-sectional and Longitudinal Approaches*, VDM Verlag Dr. Müller, Saarbrücken, Germany, 2011.
- [29] M. Shahrill, "Clustering of Decimal Misconceptions in Primary and Secondary Classes," *International Journal of Humanities and Social Science*, 3 (11), pp. 58-65, 2013a.
- [30] M. Shahrill, "Review of Teacher Questioning in Mathematics Classrooms," *International Journal of Humanities and Social Science*, 3 (17), pp. 224-231, 2013b.
- [31] M. Shahrill, "Comparing Teacher Questioning in American and Australian Mathematics Classrooms," *Journal of Applied Research in Education*, 17, pp. 26-40, 2013c.
- [32] M. Shahrill, "Exploring Educational Administration: The Relationship between Leadership and Management," *International Journal of Academic Research in Business and Social Sciences*, 4 (1), pp. 525-538, 2014.
- [33] M. Shahrill, N.A. Abdullah, J. Yusof, A.S. Suhaili, "Informing the Practice of Teaching Mathematics in Upper Primary Classes," In I. Sahin, S. A. Kiray, & S. Alan (Eds.), *Proceedings of the International Conference on Education in Mathematics, Science & Technology (ICEMST)*, pp. 168-172, 2014.
- [34] M. Shahrill, D.J. Clarke, "Brunei Teachers' Perspectives on Questioning: Investigating the Opportunities to 'Talk' in Mathematics Lessons," *International Education Studies*, 7 (7), pp. 1-18, 2014.
- [35] M. Shahrill, S. Mahalle, R. Matzin, M.H.S. Hamid, L. Mundia, "A Comparison of Learning Styles and Study Strategies used by Low and High Math Achieving Brunei Secondary School Students: Implications for Teaching," *International Education Studies*, 6 (10), pp. 39-46, 2013.
- [36] M. Shahrill, L. Mundia, "The Use of Low-order and Higher-order Questions in Mathematics Teaching: Video Analyses Case Study," *Journal of Studies in Education*, 4 (2), pp. 15-34, 2014.
- [37] U.F.H.M. Tahir, M. Shahrill, "Mathematics in Vocational and Technical Education: Investigating the Relationship in Engineering Work Related Problems," In L. Gómez Chova, A. López Martínez, & I. Candel Torres (Eds.), *Proceedings of the 6th International Conference on Education and New Learning Technologies (EDULEARN)*, pp. 3282-3292, 2014.
- [38] S.H. Tan, T.L. Toh, "On the Teaching of the Representation of Complex Numbers in the Argand Diagram," *Learning Science and Mathematics*, 8, pp. 75-86, 2013.
- [39] N.A. Wahid, M. Shahrill, "Pre-university Students' Engagement towards the Learning of Mathematics," In *Proceedings of the 2nd International Conference on Social Sciences Research (ICSSR)*, pp. 379-388, 2014.
- [40] R.S. Yatab, M. Shahrill, "The Differing Views in using the Common Assessment Tasks in Secondary School Science," *International Journal of Science and Research*, 3 (7), pp. 685-693, 2014.

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