Solitary Waves in a Five Component Dusty Plasma with Kappa described Electrons and Ions

Sijo Sebastian¹, Sreekala G. ², Manesh Michael¹, Noble P. Abraham⁴, S. Antony⁵, G. Renuka⁶, Chandu Venugopal⁷

¹, ², ³, ⁴, ⁵, ⁶School of Pure & Applied Physics, Mahatma Gandhi University, Priyadarshini Hills, Kottayam, 686 560, Kerala, India
⁷Kerala State Council for Science, Technology & Environment, Sasthra Bhavan, Pattom, Thiruvananthapuram, 695 004, Kerala, India

Abstract: We investigate the existence of solitary waves in a five component dusty plasma. Positively and negatively charged dust, kappa function described photo-electrons, hot electrons and ions form the five components. The Kd-V equation is derived and the solutions plotted for different physical parameters. It is seen that different physical parameters affect the amplitude of the solitary structures differently. As the temperature of negative dust increases, the amplitude of the solitary structure increases. With the increase of positive dust number densities, the amplitude of the solitary structure decreases whereas its amplitude increases with an increase of hydrogen ion densities.

Keywords: Solitary waves, Five component Dusty Plasma, Kappa distributed ions and electrons, Kd-V equation, Soliton

1. Introduction

Dusty plasmas, which play a significant role in space, astrophysical and laboratory environments, is an interesting current field of research because dust significantly alters the charged particles’ equilibrium density leading to different phenomena. Dust is widespread in the plasmas of cometary tails, asteroid zones, rings of Saturn, interstellar clouds and the Earth’s magnetosphere [1-4]. This omnipresent dust makes nonlinear studies in plasmas very complex and extremely important, and phenomena like solitons, shocks and vortices have been investigated in great detail [5]. Also the discovery of the existence of new eigen modes like Dust-Acoustic Waves (DAWs) [6], Dust Ion-Acoustic Waves (DIAWs) [7] and Dust Lattice Waves (DLWs) [8] have given a new dimension to the study of dusty plasmas. Most of the dusty plasma studies, until recently, considered only negatively charged dust as a constituent [9-12].

However, many authors have found different environments where both positively and negatively charged dust could exist. In particular, it was shown by Ellis and Neff [10] that the sign of the charge on the dust grains in the tail of comet Halley is a function of distance. Similarly, it was shown that the grain potential could be either positive or negative in the region between the outer shock and the cometary ionopause [1]. Also, as shown by Chow et. al. [11], typical cosmic plasmas have grains of different sizes of opposite charge: the smaller ones were positively charged while larger sized grains were negatively charged. Nearer Earth, Havnes et. al. [13], analyzing data from two rockets launched from the Audoya Rocket Range, found that the negative charge density locked in the dust grains was so large that an electron bite out was the result. And the increase in the local electron density by photoionization resulted in positively charged dust grains.

Thus there are a number of space, astrophysical and cosmic plasmas that carry both positively and negatively charged dust grains.

Recently, Vöelzke and Izaguirre analysed 886 images of the comet Halley and found 41 solitary waves (solitons) in addition to other wavy structures [14].

Dust – acoustic solitary waves in unmagnetized three component dusty plasmas consisting of ions, electrons and negatively charged dust have been studied by many authors [9, 15, 16]. However, with the realization of the importance of the presence of positively charged dust, researchers turned their attention to four-component plasmas consisting of ions, electrons and positively and negatively charged dust. Thus Sayed and Maman [17] investigated solitary waves in a four component dusty plasma using the reductive perturbation technique [RPT]; a complementary study using the Sagdeev potential technique was carried by Chatterjee and Roy [18]. This wave has also been studied in a four component adiabatic, magnetized plasma [19].

Plasmas observed in different space environments deviate significantly from the well known Maxwellian distribution due to the presence of the high energy particles in the tail of the distribution. Using solar wind data Vasyliunas first predicted a non-Maxwellian distribution [20]; this distribution, which subsequently came to be known as the “kappa distribution”, has been found in many magnetospheric and astrophysical environments. Acoustic-like solitary waves, with the electrons described by kappa distributions, have been studied by many authors recently [21-26]. Linear and nonlinear waves in dusty plasmas, where both ions and electrons are described by kappa distributions, have also been studied [27-28].

A realistic model of a cometary plasma requires at least five components: in addition to solar wind protons and electrons, the dissociation of water group molecules would give rise to
positively charged oxygen ions and the associated photo-electrons (the second colder, electron component) [29]. And the unexpected discovery of negatively charged oxygen ions in a cometary plasma is now well known [30].

We thus investigate the existence of solitary waves in a five constituent plasma: positively charged oxygen ions (dust), negatively charged oxygen ions (dust), hydrogen ions, hotter solar wind electrons and colder photo-electrons. And for reasons we model the hydrogen ions and electrons by kappa distributions.

2. Basic equations

We are interested in solitary waves in a five constituent plasma, for reasons given above. The dust components are treated as cold, while the other three constituents are described by a Boltzmann-like distribution given by:

\[ n_s = n_{s0} \left[ 1 + \frac{e^{\psi/\kappa_s}}{k_B T_s} \right]^{n_s+1} ; \quad s = H, se, ce \quad (1) \]

In (1) \( s \) indicates the species (\( s = H \) for hydrogen, \( s = se \) for solar electrons and \( s = ce \) for cometary photo-electrons). \( n \) denotes the density (with the subscript ‘0’ denoting the equilibrium value), \( e \) the charge, \( T_s \) the temperature and \( \kappa_s \) the spectral index for the species ’s’. \( k_B \) is the Boltzmann’s constant and \( \psi \), is the potential.

The normalized form of the two continuity equations and the equations of motion for the dust particles and the Poisson’s equation are thus given by

\[ \frac{\partial n_1}{\partial t} + \frac{\partial (n_1 u_1)}{\partial x} = 0 \quad (2) \]
\[ \frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = \frac{\partial \psi}{\partial x} \quad (3) \]
\[ \frac{\partial n_2}{\partial t} + \frac{\partial (n_2 u_2)}{\partial x} = 0 \quad (4) \]
\[ \frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} = -\alpha_0 \frac{\partial \psi}{\partial x} \quad (5) \]
\[ \frac{\partial^2 \psi}{\partial x^2} = n_1 - (1 - \mu_1 + \mu_2 + \mu_3) n_2 \]
\[ + \mu_2 \left( 1 - \frac{\psi}{\sigma_s (\kappa_s - \frac{1}{2})} \right)^{\psi_s + 1} \]
\[ - \mu_1 \left( 1 - \frac{\psi}{\sigma_s (\kappa_s - \frac{1}{2})} \right)^{\psi_s + 1} \quad (6) \]

In (2) to (6) \( n_1 \) and \( n_2 \) are the negative and positive dust number densities, normalized by their equilibrium values \( n_{10} \) and \( n_{20} \) respectively. \( u_1 \) and \( u_2 \) are, respectively, the negatively and positively charged dust fluid speeds normalized by \( \frac{z_i k_B T_s}{m_1} \). \( \psi \), which is now the normalized electric potential, is normalized by \( \frac{k_B T_1}{e} \) and normalized by the Debye length \( \lambda_D = \left( \frac{z_i k_B T_s}{4 \pi e^2 n_{10}} \right)^{1/2} \) and the inverse of the plasma frequency \( \omega_p^{-1} = \left( \frac{m_1}{4 \pi e^2 n_{10}} \right)^{1/2} \) respectively. Also \( \alpha = \frac{z_2}{z_1} \), \( \beta = \frac{m_1}{m_2} \), \( \mu_1 = \frac{n_{10}}{n_{10}^{1/2}} \), \( \sigma_s = \frac{T_s}{T_1} \) where \( n_{10} \) is the equilibrium density for species s. \( T_s \) and \( T_1 \) are the temperatures of species s and negative dust respectively. \( m_1 \) and \( m_2 \) are, respectively, the masses of negatively and positively charged dust particles while \( z_1 \) and \( z_2 \) are the corresponding charge numbers.

To derive Kd-V equations of small amplitude waves, we introduce a new variables \( \xi \) and \( \tau \) as:

\[ \xi = \psi^{1/2} (X - Mt) \quad (11) \]
\[ \tau = \psi^{3/2} t \quad (12) \]

By the substitution \( \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} \) and

\[ \frac{\partial}{\partial t} = \psi^{3/2} \frac{\partial}{\partial \tau} = \psi^{3/2} \frac{\partial}{\partial \xi} - \psi^{1/2} \frac{\partial}{\partial \tau} \]

(2) - (5) become

\[ \frac{\partial n_1}{\partial \tau} - \frac{\partial (n_1 u_1)}{\partial \xi} = 0 \quad (7) \]
\[ \frac{\partial u_1}{\partial \tau} - \frac{\partial u_1}{\partial \xi} = 0 \quad (8) \]
\[ \frac{\partial n_2}{\partial \tau} - \frac{\partial (n_2 u_2)}{\partial \xi} = 0 \quad (9) \]
\[ \frac{\partial u_2}{\partial \tau} - \frac{\partial u_2}{\partial \xi} = -\alpha_0 \frac{\partial \psi}{\partial \xi} \quad (10) \]

The physical quantities in the above equations can be expressed asymptotically as a power in \( \psi \) about equilibrium state as:

\[ n_1 = 1 + \frac{\psi}{n_1^{(1)}} + \frac{\psi^2}{n_1^{(2)}} + \ldots \quad (11) \]
\[ n_2 = 1 + \frac{\psi}{n_2^{(1)}} + \frac{\psi^2}{n_2^{(2)}} + \ldots \quad (12) \]
\[ u_1 = 0 + \frac{\psi}{u_1^{(1)}} + \frac{\psi^2}{u_1^{(2)}} + \ldots \quad (13) \]
\[ u_2 = 0 + \frac{\psi}{u_2^{(1)}} + \frac{\psi^2}{u_2^{(2)}} + \ldots \quad (14) \]
\[ \psi = \psi^{(1)} + \frac{\psi^2}{\psi^{(2)}} + \ldots \quad (15) \]
Substituting above equations in (7)-(10) and equating powers of $\varepsilon^{1/2}$, we get

\[ n_1^{(1)} = \frac{u_1^{(1)}}{M}, \quad n_2^{(1)} = \frac{u_2^{(1)}}{M} \]  
(16)

\[ u_1^{(1)} = -\psi^{(1)}, \quad u_2^{(1)} = \frac{\alpha \beta}{M} \psi^{(1)} \]  
(17)

Similarly equating power of $\varepsilon$ from equation (6), we get as

\[ 0 = n_1^{(1)} - n_2^{(1)}(1-\mu_i + \mu_{se} + \mu_{ce}) \]
\[ + \frac{\mu_{se}}{\sigma_{se}}(\kappa_{se} - \frac{1}{2})\psi^{(1)} + \frac{\mu_{ce}}{\sigma_{ce}}(\kappa_{ce} - \frac{1}{2})\psi^{(1)} \]  
(18)

\[ + \frac{\mu_i}{\sigma_i}(\kappa_i - \frac{1}{2})\psi^{(1)} \]

Using equation (17), we can write equation (16) as

\[ n_1^{(1)} = -\psi^{(1)}, \quad n_2^{(1)} = \frac{\alpha \beta}{M} \psi^{(1)} \]  
(19)

With the use of above equation, (19) can be written as

\[ M^2 = \frac{\mu_{se}}{\sigma_{se}}(\kappa_{se} - \frac{1}{2}) + \frac{\mu_{ce}}{\sigma_{ce}}(\kappa_{ce} - \frac{1}{2}) + \frac{\mu_i}{\sigma_i}(\kappa_i - \frac{1}{2}) \]  
(20)

Similarly equating powers of $\varepsilon^{3/2}$ from equations (7)-(10) and $\varepsilon^2$ from equation (6), we get

\[ \frac{\partial^2 u_1^{(1)}}{\partial \xi^2} - M\frac{\partial^2 u_1^{(2)}}{\partial \xi^2} + \frac{\partial^2 u_1^{(1)}}{\partial \xi^2} + \frac{\partial^2 u_1^{(1)}}{\partial \xi^2} = 0 \]  
(21)

\[ \frac{\partial^2 u_2^{(1)}}{\partial \xi^2} - M\frac{\partial^2 u_2^{(2)}}{\partial \xi^2} + \frac{\partial^2 u_2^{(2)}}{\partial \xi^2} = 0 \]  
(22)

\[ \frac{\partial^2 u_1^{(1)}}{\partial \tau^2} - M\frac{\partial^2 u_1^{(2)}}{\partial \tau^2} + \frac{\partial^2 u_1^{(2)}}{\partial \tau^2} = \frac{\partial \psi^{(1)}}{\partial \xi} \]  
(23)

\[ \frac{\partial^2 u_2^{(1)}}{\partial \tau^2} - M\frac{\partial^2 u_2^{(2)}}{\partial \tau^2} + \frac{\partial^2 u_2^{(2)}}{\partial \tau^2} = -\frac{\partial \psi^{(2)}}{\partial \xi} \]  
(24)

Using equations (17), (18), (19), (21), (22), (23) and (24), we can write above equation as

\[ \frac{\partial \psi^{(1)}}{\partial \tau} + A\psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + B \frac{\partial^3 \psi^{(1)}}{\partial \xi^3} = 0 \]

Where

\[ A = \frac{1}{2M(1+\alpha \beta(1-\mu_i + \mu_{se} + \mu_{ce}))} \]
\[ -\frac{M\mu_{se}}{\sigma_{se}^2}(\kappa_{se} - \frac{1}{2})(\kappa_{se} + \frac{1}{2}) + \frac{M\mu_{ce}}{\sigma_{ce}^2}(\kappa_{ce} - \frac{1}{2})(\kappa_{ce} + \frac{1}{2}) \]
\[ -\frac{\mu_i}{\sigma_i^2}(\kappa_i - \frac{1}{2})(\kappa_i + \frac{1}{2}) \]

\[ B = \frac{M^2}{2(1+\alpha \beta(1-\mu_i + \mu_{se} + \mu_{ce}))} \]

3. Solution of Kd-V Equation

For the soution of the Kd-V equation, we follow the method of Kolebage and Oyewande [31]. For this we use the transformation $\psi^{(1)} = \psi_{se} \text{sech}^2 \left( \frac{Z - Z_o}{\omega} \right)$

Where $\psi_{se} = \frac{3U}{A}$ is the amplitude and $\omega = 2\sqrt{\frac{B}{U}}$ is the width of the solitons.

3. Discussions

Though our equations are valid for arbitrary values of charge numbers $z_1$ and $z_2$ on the dust particles we are interested, in this paper, on parameters relevant to comet Halley. The density of hydrogen ions observed at comet Halley was $4.95 \times 10^3$ cm$^{-3}$ at a temperature of $8 \times 10^4$ K. The solar electron temperature was $2 \times 10^5$ K [29]. The negatively charged ions were detected at an energy of 1eV, with densities $\leq 1$ cm$^{-3}$ in the 7-19 amu peak [30]. Therefore we choose a majority ion density $n_{i0} = 4.95 \times 10^3$ cm$^{-3}$ and temperature $T_{se} = 2 \times 10^5$ K and photo-electron temperature $T_{ce} = 2 \times 10^4$ K. Negatively and positively charged oxygen ions are considered in lieu of negatively and positively charged dust with densities $n_{i0} = 0.05$ cm$^{-3}$ and $n_{i0} = 0.5$ cm$^{-3}$ respectively. The temperature of negatively and positively charged oxygen is taken as $T_{i1} = 1.16 \times 10^3$ K [30].
Figure 1: Plots of $\psi$ vs. $x$ for different $z_2$ values. Curve (a) is for $z_2 = 4$, (b) for $z_2 = 5$ and (c) for $z_2 = 6$.

Figure 1 shows the plot of $\psi$ vs. $x$ for different $z_2$ values. Curve (a) is for $z_2 = 4$, (b) for $z_2 = 5$ and (c) for $z_2 = 6$. The other parameters are $M = 2$, $U = 0.1$, $n_{10} = 0.05 \text{ cm}^{-3}$, $n_{20} = 0.5 \text{ cm}^{-3}$, $n_{10} = 4.95 \text{ cm}^{-3}$, $T_{ce} = 2 \times 10^4 \text{ K}$, $T_{se} = 2 \times 10^5 \text{ K}$, $T_i = 8 \times 10^4 \text{ K}$, $z_1 = 1$, $\kappa_{ce} = \kappa_i = 4$ and $\kappa_{se} = 12$. It is seen that as the $z_2$ increases, amplitude of the solitary wave decreases.

Figure 2: Plots of $\psi$ vs. $x$ for different $U$ values. Curve (a) is for $U = 0.03$, (b) for $U = 0.09$ and (c) for $U = 0.12$.

Figure 2 shows the plot of $\psi$ vs. $x$ for different $U$ values. Curve (a) is for $U = 0.03$, (b) for $U = 0.09$ and (c) for $U = 0.12$. The other parameters are $M = 1.5$, $n_{10} = 0.05 \text{ cm}^{-3}$, $n_{20} = 0.5 \text{ cm}^{-3}$, $n_{10} = 4.95 \text{ cm}^{-3}$, $T_{ce} = 2 \times 10^4 \text{ K}$, $T_{se} = 2 \times 10^5 \text{ K}$, $T_i = 8 \times 10^4 \text{ K}$, $z_1 = 1$, $z_2 = 3$, $\kappa_{ce} = \kappa_i = 4$ and $\kappa_{se} = 12$. It is seen that as the $U$ increases, amplitude of the solitary wave increases significantly.

Figure 3: Plots of $\psi$ vs. $x$ for different $T_i$ values. Curve (a) is for $T_i = 1 \times 10^4 \text{ K}$, (b) for $T_i = 1.4 \times 10^4 \text{ K}$ and (c) for $T_i = 1.6 \times 10^4 \text{ K}$.

Figure 3 shows the plot of $\psi$ vs. $x$ for different $T_i$ values. Curve (a) is for $T_i = 1 \times 10^4 \text{ K}$, (b) for $T_i = 1.4 \times 10^4 \text{ K}$ and (c) for $T_i = 1.6 \times 10^4 \text{ K}$. The other parameters are $M = 1.5$, $U = 0.06$, $n_{10} = 0.05 \text{ cm}^{-3}$, $n_{20} = 0.5 \text{ cm}^{-3}$, $n_{10} = 4.95 \text{ cm}^{-3}$, $T_{ce} = 2 \times 10^4 \text{ K}$, $T_{se} = 2 \times 10^5 \text{ K}$, $T_i = 8 \times 10^4 \text{ K}$, $z_1 = 1$, $z_2 = 3$, $\kappa_{ce} = \kappa_i = 4$ and $\kappa_{se} = 12$. From the graph, it is clear that as the negative dust temperature increases, the amplitude of the solitary structure increases; i.e., negative dust temperature has a significant role in the amplitude of the solitary structures.

Figure 4: Plots of $\psi$ vs. $x$ for different $\kappa_{ce} = \kappa_i$ values.

Figure 4 shows the plot of $\psi$ vs. $x$ for different $\kappa_{ce} = \kappa_i$ values. Curve (a) is for $\kappa_{ce} = \kappa_i = 3$, (b) for $\kappa_{ce} = \kappa_i = 4$ and (c) for $\kappa_{ce} = \kappa_i = 9$.

The other parameters are $M = 1.5$, $U = 0.1$, $n_{10} = 0.05 \text{ cm}^{-3}$, $n_{20} = 0.5 \text{ cm}^{-3}$, $n_{10} = 4.95 \text{ cm}^{-3}$, $T_{ce} = 2 \times 10^4 \text{ K}$, $T_{se} = 2 \times 10^5 \text{ K}$, $T_i = 8 \times 10^4 \text{ K}$, $z_1 = 1$, $z_2 = 3$ and $\kappa_{se} = 12$. As kappa value increases, amplitude of the solitary structures decreases. There is only a slight change in the amplitude for higher values of kappa.
Figure 5: Plots of $\psi$ vs. $x$ for different $n_{20}$ values. Curve (a) is for $n_{20} = 0.2$, (b) for $n_{20} = 0.3$ and (c) for $n_{20} = 0.5$.

Figure 5 shows the plot of $\psi$ vs. $x$ for different $n_{20}$ values. Curve (a) is for $n_{20} = 0.2$, (b) for $n_{20} = 0.3$ and (c) for $n_{20} = 0.5$. $T_1$ is taken as $1.16 \times 10^4 \text{ K}$. Other relevant parameters are same as figure 3. From the graph, we can see that amplitude of the solitary waves decreases with increase of positive dust number density.

Figure 6: Plots of $\psi$ vs. $x$ for different $n_{10}$ values. Curve (a) is for $n_{10} = 3$, (b) for $n_{10} = 5$ and (c) for $n_{10} = 6$.

Figure 6 shows the plot of $\psi$ vs. $x$ for different $n_{10}$ values. Curve (a) is for $n_{10} = 0.2$, (b) for $n_{10} = 0.3$ and (c) for $n_{10} = 0.5$. $n_{20}$ is taken as 0.5. Other values remain the same as in the previous case. From the graph, it is seen that amplitude of the solitary structures increases with increase of hydrogen ion densities.

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References


