International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Impact Factor (2012): 3.358

Stability of the Ion-Acoustic Wave in Permeating Plasmas: Application to Comets

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Abstract: ***The ion-acoustic (IA) wave is a mode that is easily excited when the electron streaming velocity exceeds the ion-acoustic phase speed. We study the stability of this wave when streaming quasi-neutral electron-ion plasma passes through another quasi-neutral pair-ion electron target plasma. Such a situation occurs when the solar wind (the streaming component consisting of hydrogen (H^+) and solar wind electrons (`se')) flows past a comet. The cometary pair-ion electron plasma is composed of positively charged oxygen ions ($O^+$), negatively charged oxygen ions ($O^-$) and cometary electrons (`ce'). All five constituents have been modeled by kappa distribution functions. We find that the growth rate of the IA wave, which occurs under zero-current conditions, decreases with increasing spectral indices of the kappa distributions. The growth rate which increases with increasing O^+ and O^- densities and electron flow speeds, decreases with increasing hydrogen densities.***

Keywords: ion-acoustic wave, permeating, kappa distribution**,** growth rate**.**

1. Introduction

A cometary environment is one where a number of positive ions such as H^+, H_2^+, C^+, O^+, OH^+, H_2O^+, etc. have been observed [1]. However, the discovery of negatively charged ions in three broad mass peaks of 7-19, 22-65 and 85-110 amu and the unambiguous identification of negatively charged oxygen ions (O^-) makes a cometary plasma a genuine multi-ion plasma [2].

The solar wind which permeates this cometary plasma adds to the complexity: the streaming solar wind species can be a source of free-energy and the excitation of current driven instabilities is inevitable. And an instability that can easily be excited is the ion-acoustic (IA) instability. Indeed IA waves with a frequency of the order of a few kHz have been observed in missions sent to observe comets such as Halley and Giacobini-Zinner [3].

The flow of the solar wind past a comet, allows one to view the situation as two permeating plasmas [4]. Such permeating plasmas are also common in other astrophysical contexts such as colliding astrophysical clouds, solar and stellar winds, etc. We thus have one fast moving quasi-neutral plasma (in our case, the solar wind composed of hydrogen and electrons) flowing past a slow moving quasi-neutral target plasma (the cometary plasma which we model as containing positively and negatively charged oxygen ions and electrons).

A model that has been used to describe the fast moving plasma is the streaming Maxwellian while the target plasma was modelled by another Maxwellian with zero streaming velocity [4, 5]. However, Maxwellian distributions do not accurately model the high energy tail of the particle distributions, necessitating the use of mode generalised distribution functions such as the kappa or Lorentzian distribution functions.

We, therefore, study the stability of IA waves in a plasma composed of contributions from both the solar wind and cometary environments: the comet contributes positively and negatively charged oxygen ions (denoted, respectively, by O^+ and O^-) and electrons (denoted by `ce') while the solar wind contributes the streaming hydrogen ions (denoted by $H^+$) and electrons (denoted by `se'). All the constituents, for reasons given above, have been modelled by either kappa distributions (the cometary constituents) or streaming kappa distributions (the hydrogen ions and electrons of solar wind origin).

We find that the growth rate of the IA waves increases with increasing streaming velocities and oxygen ions densities; it however decreases with increasing hydrogen ion densities and increasing spectral indices of the kappa distributions.

2. The Dispersion Formula

We are interested, in this paper, on the stability of IA waves in a five component plasma of O^+, O^-, H^+, solar wind electrons (`se') and cometary electrons (`ce'). The particle distribution is described by

$$f_{\kappa}^{j} = \frac{n_{0j}}{\pi^{3/2}\theta_{\perp j}^{2}\theta_{\parallel j}} \frac{\Gamma(\kappa_j+1)}{\kappa_j^{3/2}\Gamma\left(\kappa_j-\frac{1}{2}\right)}\left[1+\frac{v_\perp^2}{\kappa_j\theta_{\perp j}^2}+\frac{\left(v_\parallel-V_{dj}\right)^2}{\kappa_j\theta_{\parallel j}^2}\right]^{-(\kappa_j+1)} \quad (1)$$

In (1), the thermal speed $\theta_{\|(\perp)j}$ (j indicates the species) is defined as

$$\theta_{\|(\perp)j} = \sqrt{\frac{2\kappa_j - 3}{\kappa_j}\frac{k_B T_{\|(\perp)j}}{m_j}} \qquad (2)$$

with κ_j being the spectral index for species `j' while T and m denote, respectively, their temperatures and masses. $k_B$ is the Boltzmann's constant while $\|(\perp)$ denote, respectively, the directions parallel and perpendicular to the ambient magnetic field $B_{0z}$ in the $\hat{z}$ direction. $n_{0j}$ is the equilibrium density of species `j', while Γ is the gamma function. Also the streaming velocities V_{dH^+} and V_{dse} are not equal to zero, while $V_{dce} = V_{dO^+} = V_{dO^-} = 0$

Substituting (1) in to the dispersion formula for electrostatic waves of frequency ω and wave vector $\vec{k}$ and carrying out the $dv_\perp$ and $dv_\|$ integrations, we get the final form of $D(\omega,\vec{k})$ as

$$D(\omega,\vec{k}) = 1 + \sum_j \frac{2\omega_{pj}^2}{k^2\theta_{\|j}^2}\left[\frac{(2\kappa_j - 1)}{2\kappa_j} + \xi_{\kappa j} Z_\kappa^*(\xi_{\kappa j})\right] = 0 \qquad (3)$$

In (3), $\omega_{pj} = \left[\frac{4\pi n_{0j}e^2}{m_j}\right]^{1/2}$ is the plasma frequency of species `j' and `e' is the electronic charge; Z_κ^*, of argument $\xi_{\kappa j}$, is the modified plasma dispersion function which arises from the $dv_\|$ integration and is defined as [6]

$$Z_\kappa^*(\xi_{\kappa j}) = \frac{1}{\sqrt{\pi}}\frac{\Gamma(\kappa_j + 1)}{\Gamma\left(\kappa_j - \frac{1}{2}\right)}\int_{-\infty}^{\infty}(x - \xi_{\kappa j})^{-1}\left(1 + \frac{x^2}{\kappa_j}\right)^{-(\kappa_j+1)} dx \qquad (4)$$

The arguments of the modified plasma dispersion function are either

$$\xi_{\kappa j} = \frac{(\omega - k_\| V_{dj})}{k_\|\theta_{\|j}} \quad j = H^+, se \text{ with } V_{dH^+} \neq 0 \text{ and } V_{dse} \neq 0. \qquad (5a)$$

$$\xi_{\kappa j} = \frac{\omega}{k_\|\theta_{\|j}} \quad j = O^+, O^- \text{ or ce.} \qquad (5b)$$

3. The Dispersion Relation

The dispersion relation for the propagation of IA waves in the plasma under consideration is derived in this section. We consider waves in the range

$$k_\|\theta_{\|O^+}, k_\|\theta_{\|O^-} << \omega << k_\|\theta_{\|ce}$$
$$|\omega - k_\| V_{dj}| << k_\|\theta_{\|H^+}, k_\|\theta_{\|se} \qquad (6)$$

which is similar to the frequency range of Vranjes et. al. [4]. In addition, we also need the quasi-neutrality conditions [4]

$$n_{0H^+} = n_{0ce}$$
$$n_{0O^+} = n_{0O^-} + n_{0ce} \qquad (7)$$

To satisfy (6) we need the asymptotic expansion of the modified plasma dispersion function for O^+ and O^- [6] which yields

$$\xi_{\kappa j} Z_\kappa^*(\xi_{\kappa j}) = -\frac{(2\kappa_j - 1)}{2\kappa_j} - \frac{1}{2}\frac{1}{\xi_{\kappa j}^2} \quad ; j = O^+, O^-. \qquad (8)$$

with $\xi_{\kappa j}$ being defined in (5b) and the small parameter expansion for $Z_\kappa^*(\xi_{\kappa j})$ [6] which again yields

$$\xi_{\kappa j} Z_\kappa^*(\xi_{\kappa j}) = \frac{i\sqrt{\pi}\kappa_j!\,\xi_{\kappa j}}{\kappa_j^{3/2}\Gamma\left(\kappa_j - \frac{1}{2}\right)} - \frac{(2\kappa_j - 1)(2\kappa_j + 1)}{2\kappa_j^2}\xi_{\kappa j}^2; \quad j = H^+, se, ce \qquad (9)$$

with $\xi_{\kappa j}$ being defined in (5a) with $V_{dce} = 0$.

Substituting (8) and (9) into (3) and after a lengthy simplification we get the expression for the real frequency as $(\omega = \omega_r + i\omega_i)$

$$\left[1 + 2\sum_{j=ce,se,H^+}\frac{\omega_{pj}^2}{k^2 V_{Tj}^2}\frac{(2\kappa_j - 1)}{(2\kappa_j - 3)} - 4\sum_{j=O^+,O^-}\frac{\omega_{pj}^2}{k^2 V_{Tj}^2}\frac{(2\kappa_j - 1)(2\kappa_j + 1)}{(2\kappa_j - 3)^2}\frac{(\omega_r - kV_{dj})^2}{k^2 V_{Tj}^2}\right]^{-1} \qquad (10)$$

with $V_{dce} = 0$ and $k = k_\|$

Using the real part of (3) to get $\frac{\partial \operatorname{Re} D(\omega,k)}{\partial \omega_r}$ and $\operatorname{Im} D(\omega,k)$, obtained from (3) using (9) together with the formula $\omega_i = \frac{-\operatorname{Im} D(\omega,k)}{\frac{\partial \operatorname{Re} D(\omega,k)}{\partial \omega_r}}$, we can get the expression for the growth/damping rate for IA waves as

$$\omega_i = -\sqrt{\pi}\left[\sum_{j=ce,se,H^+}\frac{\omega_{pj}^2}{k^2 V_{Tj}^2}\frac{\kappa_j!}{\left(\kappa_j - \frac{3}{2}\right)^{3/2}\Gamma\left(\kappa_j - \frac{1}{2}\right)} \times \frac{(\omega_r - kV_{dj})}{kV_{Tj}}\right]$$
$$\left[\frac{(\omega_{pO^+}^2 + \omega_{pO^-}^2)}{\omega_r^3} - 4\sum_{j=ce,se,H^-}\frac{\omega_{pj}^2}{k^2 V_{Tj}^2}\frac{\left(\kappa_j^2 - \frac{1}{4}\right)}{\left(\kappa_j - \frac{3}{2}\right)^2}\frac{(\omega_r - kV_{dj})}{k^2 V_{Tj}^2}\right]^{-1} \qquad (11)$$

with $V_{dce}=0$.

(10) and (11) are, respectively, the expressions for the real frequency and the growth/damping rate for IA waves in the plasma composition under consideration.

4. Discussion

The stability of the IA wave was studied in a pair-ion electron plasma with all constituents being described by Maxwellian distribution by Saleem [7]. Our results are a generalisation of this study since our plasma composition reduces to this one when the solar wind hydrogen and electron densities are set equal to zero. Besides all constituents are described by kappa distributions in our study.

More recently, Arshad et. al. [8] studied the stability of this wave again in a pair-ion electron plasma with all constituents being described by kappa distributions. The wave was damped in the absence of a driving force. In contrast, the plasma composition under consideration has two sources that can drive the IA wave unstable, namely, streaming solar wind hydrogen and electrons. The stability of the IA mode was also studied when one quasi-neutral electron-ion plasma propagated through quasi-neutral electron-ion plasma [4]. Maxwellian distributions were used to model all constituents of the plasma in this study. Our investigation is also an extension of this study since all constituents have been modelled by kappa distributions and the target plasma is a pair-ion electron plasma.

5. Results

We study the stability of IA waves for typical parameters observed at comet Halley. The hydrogen density in the streaming component (the solar wind) is $n_H^+ = 4.95$ cm^{-3}. Since the streaming component is quasi-neutral, $n_{se} = 4.95$ cm^{-3}. The temperature of these electrons is $T_{se} = 2\times10^5$ K while that of hydrogen is $T_H^+ = 8 \times 10^4$ K [9]. We next consider the densities and temperatures of the constituents of the target plasma.

The density of the positively charged oxygen ions is $n_O^+ = 0.5$ cm^{-3}. The density of the negatively charged ions, in the 7-19 amu peak is ≤ 1 cm^{-3} with an energy of $\approx$1 eV. We thus set the temperatures of the oxygen ions as $T_O^+ = T_O^- = 1.16 \times 10^4$ K, while the density of O^- was set at 0.05 cm^{-3} [2]. The temperature of the cometary electrons was set slightly lower at $T_{ce} = 1.2 \times 10^5$ K while its density was calculated from the charge neutrality condition.

We plot, in Figure 1, the growth rate $\dfrac{\omega_i}{\omega_{pse}}$ versus $k\lambda_{De}$ (λ_{De} is the Debye length of the solar wind electrons) for various spectral indices. The parameters are particle densities $n_H^+ = 4.95$ cm^{-3}, $n_O^+ = 0.5$ cm^{-3} and $n_O^- = 0.05$ cm^{-3} while the species temperatures are $T_{se} = 2\times10^5$ K, $T_H^+ = 8 \times10^4$ K, $T_{ce} = 1.2 \times10^5$ K and $T_O^+ = T_O^- = 1.16 \times 10^4$ K with $V_{de} / V_{Te} = 0.001$ and V_{dH}^+ being calculated from the zero current condition. Curve (a) is for spectral indices $\kappa_s = 2$, curve (b) is for $\kappa_s = 5$ and curve (c) for $\kappa_s = 7$. We find that the growth rate decreases with increasing κ_s. Since a kappa distribution reduces to a Maxwellian when $\kappa \to \infty$, the results show that the growth rate is a minimum in a Maxwellian plasma and increases rapidly as κ_s decreases.

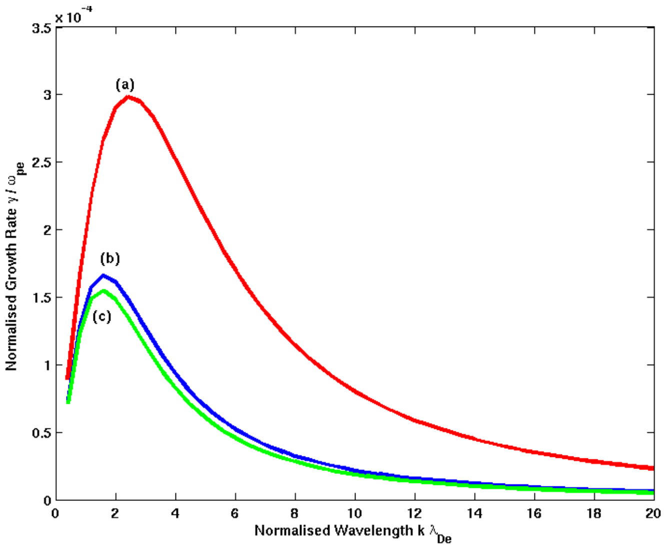
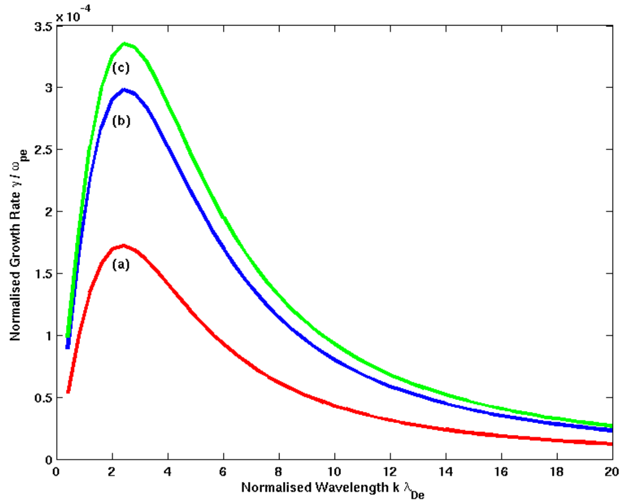


Figure 1: Plot of the growth rate versus $k\lambda_{De}$ for various spectra indices

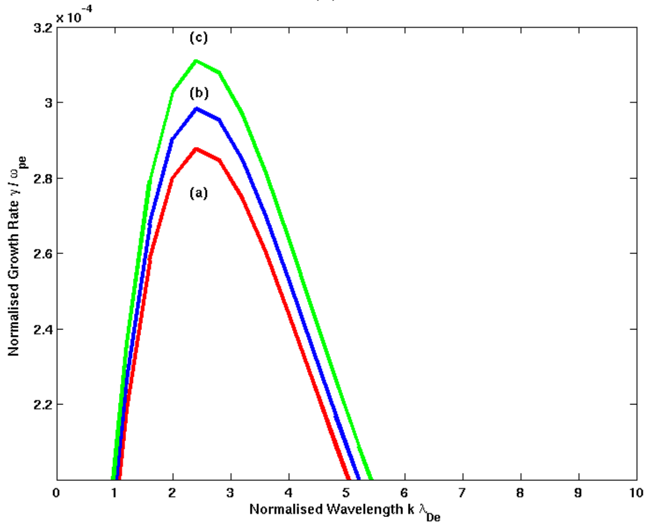
The dependence of the growth rate on the number densities of ions in the target plasma is investigated next. In the two plots that follow $\kappa_s = 2$ while the temperatures are the same as in figure 1. $V_{de} / V_{Te} = 0.001$ while V_{dH}^+ was calculated from the zero current condition.

Figure 2(a) is a plot of the growth rate $\dfrac{\omega_i}{\omega_{pse}}$ versus $k\lambda_{De}$ as a function of n_O^+, the density of the positively charged oxygen ions. The other densities are $n_H^+ = 4.95$ cm^{-3} and $n_O^- = 0.05$ cm^{-3}; curve (a) is for $n_O^+ = 0.1$ cm^{-3}, curve (b) is for $n_O^+ = 0.5$ cm^{-3} and curve (c) is for $n_O^+ = 0.7$ cm^{-3}. We find that the growth rate increases with increasing O^+ densities. Similarly in figure 2(b) curve (a) is for $n_O^- = 0.01$ cm^{-3}, curve (b) is for $n_O^- = 0.05$ cm^{-3} and curve (c) is for $n_O^- = 0.1$ cm^{-3} while $n_H^+ = 4.95$ cm^{-3} and $n_O^+ = 0.5$ cm^{-3}. Again, the growth rate increases with increasing O^- densities. However, what is interesting is the extreme sensitivity of the growth rate to the density of the O^- ions. This could be due to the decreased Landau damping due to cometary electrons since the target plasma is independently quasi-neutral and the cometary electron density decreases with increasing O^- densities. The growth rate was also studied as a function of the hydrogen ions in the streaming plasma; we find that the growth rate decreases with increasing hydrogen densities.

Finally, Figure 3 is a plot of the growth rate $\dfrac{\omega_i}{\omega_{pse}}$ versus $k\lambda_{De}$ as a function of V_{de} / V_{Te} (V_{dH}^+/V_{Te} being calculated from the zero current condition) with $\kappa_s = 2$, $n_H^+ = 4.95$cm^{-3}, $n_O^+ = 0.5$ cm^{-3} and $n_O^- = 0.05$ cm^{-3} with temperatures remaining unchanged. Curve (a) is for V_{de}/V_{Te}= 0.001, curve (b) is for V_{de}/V_{Te}= 0.005 and curve (c) is for V_{de}/V_{Te}= 0.009. As expected, the growth rate increases with increasing speeds of the streaming species.



(a)



(b)

Figure 2: Plot of the growth rate versus $k\lambda_{De}$ as a function of the number densities of ions in the target plasma.

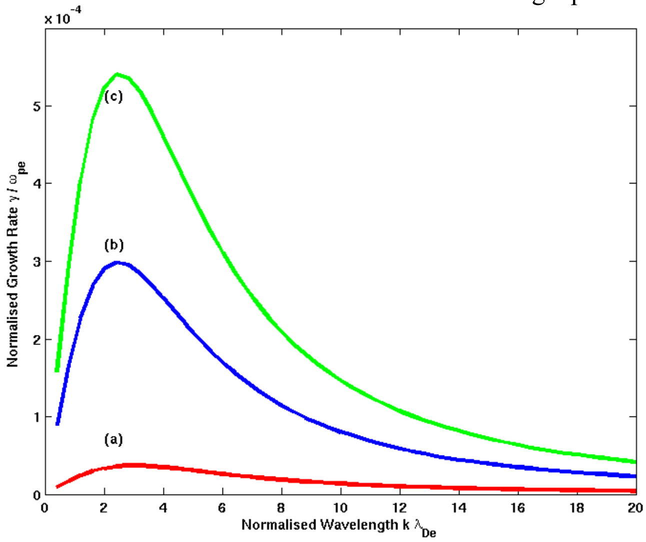


Figure 3: Plot of the growth rate versus $k\lambda_{De}$ as a function of V_{de}/V_{Te}

6. Conclusions

We have, in this paper, studied the stability of the ion-acoustic wave in a multi-ion plasma, all of them being described by either a kappa distribution or a streaming kappa distribution. The flow of the solar wind (composed of streaming hydrogen and electrons) past a comet (composed of positively and negatively charged oxygen ions and cometary electrons) is viewed as an interaction between two permeating plasmas. The streaming components can drive the IA wave unstable; the growth rate increasing with increasing streaming speeds, increasing ion densities in the target plasma and decreasing with increasing spectral indices of the species distribution functions.

At this juncture we would like to emphasize that the moving plasma is current-less since it was assumed quasi-neutral ($n_H^+ = n_{se}$) and the hydrogen streaming speed was calculated from the zero-current condition. Also since the target plasma contains a number of different species of ions, all of them would contribute to the growth of the wave.

Acknowledgments

Financial assistance from Kerala State Council for Science, Technology & Environment, Thiruvananthapuram, Kerala, India and the University Grants Commission (EF and SAP), Department of Science and Technology (FIST and PURSE Programs) is gratefully acknowledged.

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