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Application of Differential Equations in Thermal Convection of Non-Newtonian Fluids

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Abstract: ***Differential equations are widely used in all branches of Science, Engineering and almost all fields. The application of differential equations towards stability analysis of Non-Newtonian fluids is analyzed. Non-Newtonian fluids are applied in Geological, Biological, Pharmaceutical, Medical, Mechanical and industrial areas. The present work deals with the thermal convection of non-Newtonian fluids in the presence of uniform magnetic field. It has been found that the magnetic field has both stabilizing and destabilizing effects. Stationary and oscillatory modes are checked for allowable range of parameters.***

Keywords: thermal convection, non-Newtonian fluids, magnetic field, visco-elastic fluids. suspended particle

1. Introduction

In recent years, the heat transfer in porous medium has received considerable attention because of its importance to geophysical thermal engineering, geothermal system, crude oil extraction, recovery of petroleum products etc., non-many of industrially important fluids such as molten plastics, polymers, pulps, foods and fossil fuels exhibit Newtonian fluid behaviors. It is noteworthy that many of the authors worked on the thermal convection of Non-Newtonian fluids and they concentrated on the stability nature of Non-Newtonian fluids. Chandrasekhar (1981) has given a detailed account of the theoretical and experimental study of thermal instability in Newtonian fluids, under varying assumption of hydradynamics and hydromagnetics. Chandra (1938) observed a contradiction between the theory for the onset of convection in fluids heated from below and his experiment. Scanlon and Segel (1973) studied the effect of suspended particles on the onset of Benard convection and found that the critical Rayleigh number was reduced slowly because the heat capacity of the pure gas was supplemented by that of the particles. Sharma (1976) has studied the thermal instability of a layer of viscoelastic fluid acted on by a uniform rotation and found that rotation has destabilizing as well as stabilizing effects under certain conditions in contrast to that of a Maxwell fluid where it has a destabilizing effect. Two such classes of fluids are Rivlin-Ericksen and Walter's (model B) fluids. Rivlin and Ericksen (1955) have proposed a theoretical model for such one class of elastico-viscous fluids. Sharma and Kumar (1996) have studied the effect of rotation on thermal instability in Rivlin- Ericksen elastico-viscous fluid whereas the thermal convection in electrically conducting Rivlin-Ericksen fluid in presence of magnetic field has been studied by Sharma and Kumar (1997). Aggarwal (2010) has studied the effect of rotation on thermosolutal convection in a Rivlin-Ericksen fluid permeated with suspended particles in porous medium. The purpose of the present work is to study the application of differential equations in thermal convection of non-Newtonian fluids in the presence of uniform magnetic field and suspended particles. Here we have considered the effect of suspended particles and magnetic field on thermal convection in Rivlin-Ericksen elastico-viscous fluid in hydromagnetics. Here, we have extended the results reported by Sharma and Rana (2002) to include the effect of magnetic field for Rivlin- Ericksen fluids. Sharma et al.(1998) have studied the thermosolutal convection in Rivlin−Ericksen fluid in porous medium in hydromagnetics. Bhatia and Steiner (1972) have studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation. Rayleigh (1916) has analyzed the thermal instability of a fluid layer heated below with maintained adverse temperature gradient. Sharma (1975) studied the stability of a layer of an electrically conducting Oldroydian fluid in the presence of a magnetic field. Sharma (1979) has studied the thermal instability in compressible fluids in the presence of rotation and a magnetic fluid. Sharma and Kumar (1996) have studied the effect of rotation on thermal instability in Rivlin - Ericksen Elastico viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Sharma et al (1999) have considered the thermosolutal instability of walters' B' rotating in porous medium. Spiegel (1965) analyzed the convective instability in a compressible atmosphere. Srivastava and Singh (1988) have studied the unsteady fluid of a dusty elastic-viscous Rivlin-Ericksen fluid in the Presence of a time dependent pressure gradient. Vadasz (2008) analyzed emerging topics in heat and mass transfer in porous medium. Vafai(2010) studied application of Biological system and Biotechnology.

2. Mathematical Formulation

Consider an infinite, horizontal, incompressible electrically conducting Rivlin-Erickson visco-elastic fluid layer of thickness d, permeated with suspended particle heated from below so that the temperatures and densities at the bottom surface $z=0$ are T_0 and ρ_0 and at the upper surface $z=d$ are T_d are ρ_d respectively. A uniform temperature gradient $\beta = |dt/dz|$ is maintained. The gravity field $\vec{g}(0,0,-g)$ and a uniform vertical magnetic field

$\vec{H}(0,0,H)$ act on the system. The equations of motion, continuity, heat conduction, and Maxwell's equations governing the flow of Walters'B visco-elastic fluid in the presence of magnetic field.

$$\frac{\partial q}{\partial t}+(q.\nabla)q=-\frac{\nabla p}{\rho_0}+g\left(1+\frac{\delta p}{\rho_0}\right)+\left(v+v'\frac{\partial}{\partial t}\right)\nabla^2 q+\frac{\mu_e}{4\pi\rho_0}(\nabla\times H)\times H$$

$$+\frac{KN}{\rho_0}(q_d-q) \quad (1)$$

$$\nabla.q \quad (2)$$

$$mN\frac{\partial q_d}{\partial t}+(q_d.\nabla)q_d=KN(q-q_d) \quad (3)$$

$$\frac{\partial N}{\partial t}+\nabla.(Nq_d)=0 \quad (4)$$

$$\frac{\partial T}{\partial t}+(q.\nabla)T+\left(\frac{mNC_{pt}}{\rho_0 C_v}\right)\left(\frac{\partial}{\partial t}+q_d.\nabla\right)T=k\nabla^2 T \quad (5)$$

$$\frac{\partial H}{\partial t}=\nabla\times(q\times H)+\eta\nabla^2 H \quad (6)$$

$$\nabla\ .\ H=0 \quad (7)$$

where $q(u,v,w)$, $q_d(l.r.s)$, N, p, ρ, T, v,v' denote the fluid velocity, velocity of the suspended particle, number density of the suspended particle, pressure, density, temperature, kinematic viscosity, and kinematic visco-elasticity respectively. In addition, the magnetic permeability μ_e, thermal diffusivity k and electrical resistivity η are all assumed constant. Here $K=6\pi\mu\varepsilon$,

where ε is the particle radius, K is Stokes drag coefficients, C_v is the specific heat of the fluid at constant volume, C_{pt} is the specific heat of particle, m is the mass of the suspended particle, mN is mass of the particle per unit volume. From equation (1) we made use of the Boussinesq approximation, which states that the densityvariations are ignored in all terms in the equations of motion except the external force term.

Spiegel and Veronis (1960) defined f as any of the state variables (pressure p, density ρ or temperature T) are expressed in the form

$$f(x,y,z,t)=f_m+f_0+f'(x,y,z,t) \quad (8)$$

where f_m is the constant space average of f, f_0 is the variation in the absence of motion and f' is the fluctuation resulting from the motion. The initial state is, therefore, a state in which the density, pressure, temperature and velocity in the fluid are given by

$$\rho=\rho(z),\ p=p(z),\ T=T(z),\ q=(0,0,0) \quad (9)$$

$$T(z)=T_0-\beta z,\ p(z)=p_m-g\int_0^d(\rho_m+\rho_0)dz$$

$$\rho(z)=\rho_m\left[1-\alpha_m(T-T_m)+K_m(p-p_m)\right] \quad (10)$$

3. Perturbation Equations

Assume small disturbances in the basic solution and let δp, $\delta\rho$, N, θ, $q(u,v,w)$, $q_d(l,r,s)$, $h(h_x,h_y,h_z)$ denote the perturbations in the fluid pressure, density, particle number density N_0, temperature, fluid velocity, Suspended particle velocity, magnetic field H respectively.

The density equation of state for the fluid is

$$\rho=\rho_0\left[1-\alpha(T-T_0)\right] \quad (11)$$

Where ρ_o is the density and T_0 is temperature of the fluid at the reference level $z=0$ and α is the coefficient of thermal expansion.

The change in density δp, caused by the perturbation θ in temperature is given by

$$\delta p=-\alpha\rho_0\theta \quad (12)$$

Then the linearized hydromagnetic perturbation equations for thermal convection in a compressible walters'B elastico-viscous fluid particle layer under speigel and veronis et al (1960) assumptions are

$$\frac{\partial q}{\partial t}=-\frac{1}{\rho_0}(\nabla\delta p)+g\frac{\delta p}{\rho_0}+\left(v+v'\frac{\partial}{\partial t}\right)\nabla^2 q+\frac{\mu_e}{4\pi\rho_0}(\nabla\times h)\times H$$

$$+\frac{KN_0}{\rho_0}(q_d-q) \quad (13)$$

$$\nabla.q=0 \quad (14)$$

$$mN_0\frac{\partial q_d}{\partial t}=KN_0(q-q_d) \quad (15)$$

$$\frac{\partial M}{\partial t}+\nabla q_d=0 \quad (16)$$

$$\nabla.h=0 \quad (17)$$

$$\frac{\partial h}{\partial t}=(H.\nabla)q+\eta\nabla^2 h \quad (18)$$

$$(1+h)\frac{\partial\theta}{\partial t}=\left(\beta-\frac{g}{c_p}\right)(w+hs)+k\nabla^2\theta=0 \quad (19)$$

where $k=\frac{v}{\rho_0 C_v}$, $h=\frac{mN_0 c_{pt}}{\rho_0 C_v}$ $\alpha_m=\frac{1}{T_m}=\alpha(say)$,

$v=\frac{\mu}{\rho_0}$ and $\frac{g}{c_p}$, θ denote the adiabatic gradient, temperature T and C_p being specific heat of the fluid at constant pressure.

Eliminating q_d between the equations (13) - (15) and rewriting the above set of equations we have

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[\frac{\partial}{\partial t}\nabla^2 w-g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-\frac{\mu_e H}{4\pi\rho_0}\frac{\partial}{\partial z}\nabla^2 h_z\right] \quad (20)$$
$$+\frac{KN}{\rho_0}\frac{m}{K}\frac{\partial}{\partial t}\nabla^2 w=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)(v+v'\frac{\partial}{\partial t})\nabla^4 w$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z=H\frac{\partial w}{\partial z} \quad (21)$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[(1+h)\frac{\partial}{\partial t}-k\nabla^2\right]\theta=\left(\frac{G-1}{G}\right)\left(\frac{m}{K}\frac{\partial}{\partial t}+H\right)w \quad (22)$$

Equations (20)-(22) yield three-perturbation equation in w,θ,h_z

4. Dispersion Relation

Now analyze the disturbances into normal modes, assuming that the perturbation quantities are of the form

$$[w,\theta,h_z]=[W(z),\Theta(z),K(z),]\ \exp(ik_x x+ik_y y+nt) \quad (23)$$

where k_x, k_y are the wave numbers along x- and y-directions respectively ,$k^2=(k_x^2+k_y^2)$ is the resultant wave number, and n is the growth rate which is in general, a complex constant. Using the equation (4.23) and the non-dimensional parameters

$$a=kd,\sigma=\frac{nd^2}{v},F=\frac{v'}{d^2},H=1+h,h=\frac{mN_0c_{pt}}{\rho_0 c_v},$$

$f=\frac{mN_0}{\rho_0},G=\frac{c_p\beta}{g},\tau=\frac{mk}{Kd^2}$ $pr_1=\frac{v}{k}$ is the prandtl number , $pr_2=\frac{v}{\eta}$ is the magnetic prandtl number,

F is dimensionless kinematic viscoelasitc, G is the dimensionless compressibility.

$x^*=\frac{x}{d},y^*=\frac{y}{d},z^*=\frac{z}{d},D=\frac{d}{dz}$ for our convenience

Using the expression (23), the equations (20)-(22) in non-dimensional form becomes

$$(1+pr_1\sigma\tau)(D^2-a^2)[(1-F\sigma)(D^2-a^2)-\sigma]W-f\sigma(D^2-a^2)W$$
$$+(1+pr_1\sigma\tau)\frac{\mu_e Hd}{4\pi\rho_0 v}(D^2-a^2)DK=$$
$$(1+pr_1\sigma\tau)\left(\frac{g\alpha d^2}{v}a^2\Theta\right) \quad (24)$$

$$(1+pr_1\sigma\tau)[D^2-a^2-Hpr_1\sigma]\Theta$$
$$=-\left(\frac{\beta d^2}{k}\left(\frac{G}{G-1}\right)\right)(H+pr_1\sigma\tau)W \quad (25)$$

$$[D^2-a^2-pr_2\sigma]K=-\left(\frac{Hd}{\eta}\right)DW \quad (26)$$

Eliminating K and θ from the equations (24)-(26) we obtain

$$(1+pr_1\sigma\tau)(D^2-a^2)(D^2-a^2-pr_2\sigma)(D^2-a^2-Hpr_1\sigma)$$
$$[(1-F\sigma)(D^2-a^2)-\sigma]W-f\sigma(D^2-a^2-pr_2\sigma)$$
$$(D^2-a^2-Hpr_1\sigma)(D^2-a^2)W$$
$$-Q(1+pr_1\sigma\tau)(D^2-a^2-Hpr_1\sigma)(D^2-a^2)D^2W$$
$$=Ra^2(D^2-a^2-pr_2\sigma]\left(\frac{G}{G-1}\right)(H+pr_1\sigma\tau)W \quad (27)$$

where $Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0 v\eta}$ is the Chandrasekhar number,

$R=\frac{g\alpha\beta d^4}{vk}$ is the thermal Rayleigh number. Since both the boundaries are maintained at constant temperature, the perturbations in the temperature are zero at the boundaries. The case of two free boundaries is little artificial but it enables us to find analytical solution and to make some qualitative conclusions.

The appropriate boundary conditions with respect to which the equations (24)-(26) must be solved

The boundary conditions are $W=D^2W=0$, $\Theta=0$, z = 0, 1and K=0 on a perfectly conducting boundary (28)
Using the above boundary conditions, it can be shown that all the even-order derivatives of W must vanish for $z=0$ and $z=1$, and hence the proper solution of w characterizing the lowest mode is

$$W=W_0\sin\pi z,\ W_0 \text{ is constant} \quad (29)$$

Substituting equation (29) in equation (27),we get

$$R_1=\left(\frac{G}{G-1}\right)(1+x)(1+x+ipr_2\sigma_1)(1+x+ip_1\sigma_1)$$
$$[(1+ipr_1\pi^2\tau\sigma_1)\{(1-i\pi^2F\sigma_1)(1+x)\}+i\pi^2f\sigma_1]$$
$$+Q_1(1+x)(1+ipr_1\pi^2\tau\sigma_1)(1+ipr_1H\sigma_1)$$
$$[x(1+ipr_1\pi^2\tau\sigma_1)(1+ipr_2\sigma_1)]^{-1} \quad (30)$$

where $Q_1=\frac{Q}{\pi^2},x=\frac{a^2}{\pi^2},i\sigma_1=\frac{\sigma}{\pi^2},F_1=F\pi^2$

5. Stationary Convection

Let us consider in the case when instability sets in the form of stationary convection. For stationary convection $\sigma_1=0$ and the dispersion relation (30) reduces to

$$R_1=\left(\frac{G}{G-1}\right)\left\{(1+x)^2+Q_1\left(\frac{1+x}{xH}\right)\right. \quad (31)$$

which express the modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters $Q_1,G.H$. Then the non-dimensional number G accounting for compressibility effect is kept as fixed, then we get

$\overline{R}_c = \left(\frac{G}{G-1}\right) R_c$ where $\overline{R}_c$, R_c denote the critical Rayleigh number in the presence and absence of compressibility .Thus the effect of compressibility is to postpone the onset of thermal instability. The cases G<1 and G=1 correspond to negative and infinite value of Rayleigh number which are not relevant in the present study .Hence, compressibility has a stabilizing effect on the thermal convection.

To study the effect of magnetic field and suspended particles, we examine the nature of

$\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dH_1}$ analytically

To investigate the effect of suspended particles, from equation (4.31) we obtain

$$\frac{dR_1}{dH} = -\left(\frac{G}{G-1}\right)\left\{(1+x)^2 + Q_1\right\}\left(\frac{1+x}{xH^2}\right) \qquad (32)$$

where the negative sign implies that the effect of suspended particle is to destabilize the system

$$\frac{dR_1}{dQ} = \left(\frac{G}{G-1}\right)\left\{(1+x)^2\right\}\left(\frac{1+x}{xH^2}\right) \qquad (33)$$

which shows that the usual stabilizing effect of magnetic field on thermal convection in a compressible Rivlin-Ericksen visco-elastic fluid in the presence of dust particles, for the stationary convection.

Table 1: The Critical Rayleigh numbers R_c and the wave numbers of the associated disturbances for the onset of instability as stationary convection G=10 and for various value of H and Q_1

H	$Q_1=100$		$Q_1=200$		$Q_1=300$		$Q_1=500$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
10	1	23.08	1	44	1	67.48	1	111.1
20	2	9.07	1.5	19.72	2	25.72	2	42.37
30	2.5	5.8	3	10.65	3	15.5	2	28.24
50	5	3.6	5	6.28	5	7.45	5	14.27

Table 2: The Critical Rayleigh numbers R_c and the wave numbers of the associated disturbances for the onset of instability as stationary convection G=10 and for various value of H and Q_1

Q_1	H=10		H=20		H=30		H=50	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
100	1	23	1	5.7	1	7.6	1	4.6
200	3	31.96	2	17	1.2	13.6	2	6.95
300	3	46.76	2.2	24	3	15.5	2	10.2
400	4	58	3	30	4	19.9	3	12.3
500	5	71	5	35	5	23.7	5	14.2

Table 3: The Critical Rayleigh R_1 numbers and the wave numbers of the associated disturbances for the onset of instability as stationary convection fixed G=10 and for various value of H and Q_1

Q_1	H=10		H=20		H=30		H=50	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
100	3.0	17.80	3.0	8.49	3.0	5.66	3.0	3.43
150	3.5	24.28	3.5	12.09	3.5	7.93	3.5	4.85
200	4.0	31.25	4.0	15.60	4.0	10.38	4.0	6.24
250	4.5	38.25	4.5	19.00	4.5	12.94	4.5	7.60
300	5.0	44.75	5.0	22.00	5.0	14.91	5.0	8.95

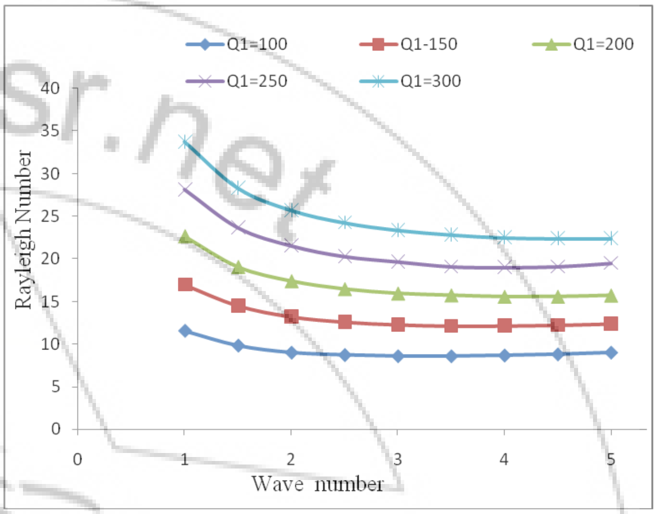

Figure 1: Variation of Rayleigh number R_1 with Wave number x

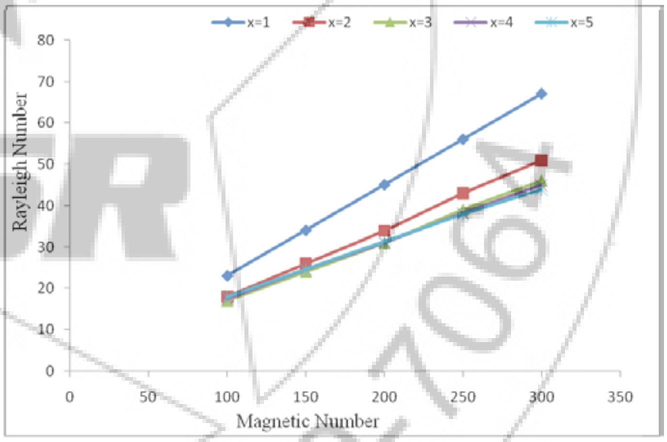

Figure 2: Variation of Rayleigh number R_1 with Magnetic number x

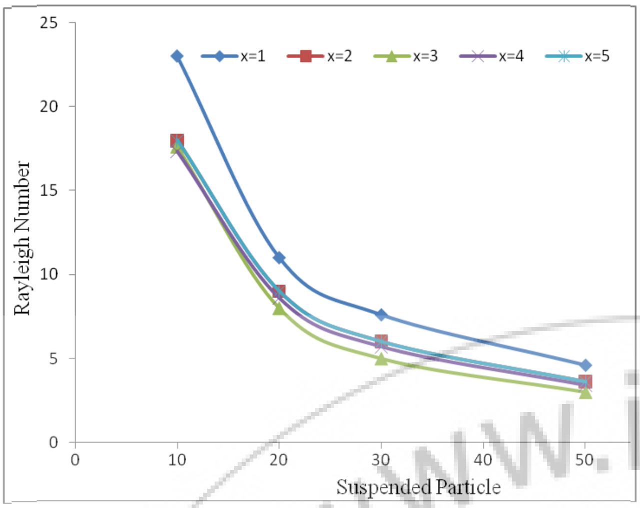

Figure 3: Variation of Rayleigh number R_1 with Suspended particle H

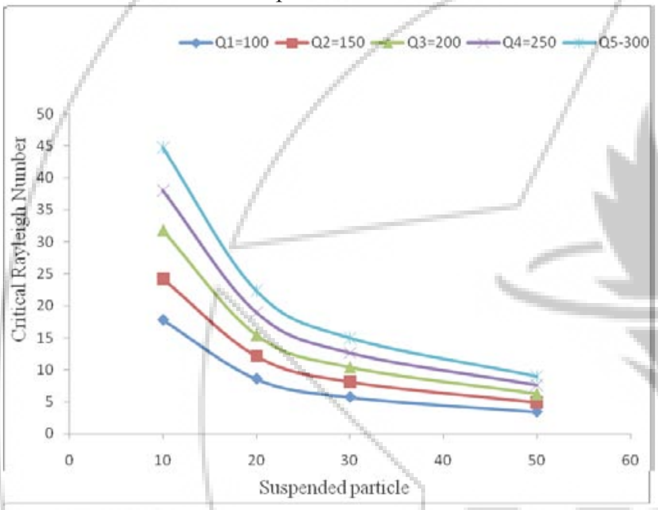

Figure 4: Variation of Critical Rayleigh number R_c with Suspended particle H

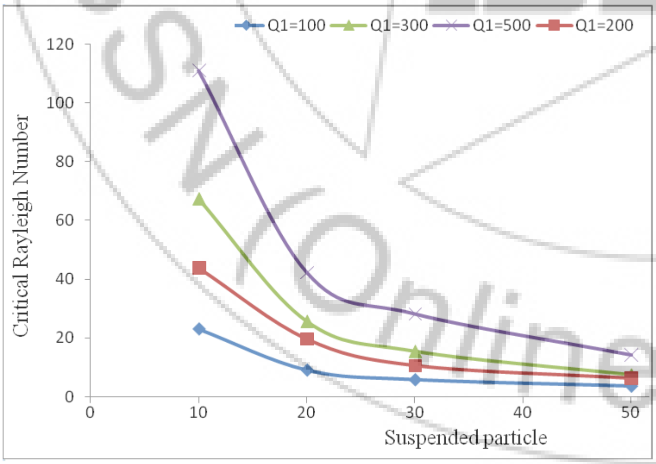

Figure 5: Variation of Critical Rayleigh number Rc with Suspended particle H

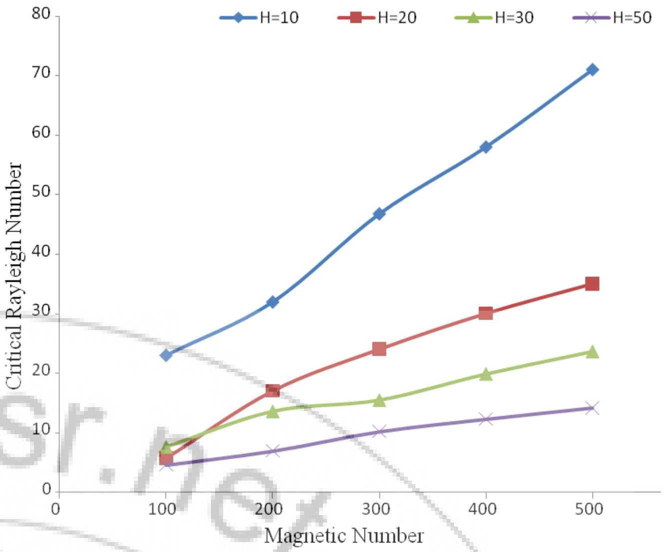

Figure 6: Variation of Critical Rayleigh number R_C with Magnetic number Q_1

6. Stability of the system and oscillatory modes

Multiplying the equation (24), by W^*, the complex conjugate of W, integrating over the range of z

$$(1+pr_1\sigma\tau)(1-F\sigma)\int_0^1 W^*\left(D^2-a^2\right)dz$$

$$-(1+pr_1\sigma\tau)\sigma\int_0^1 W^*\left(D^2-a^2\right)dz$$

$$-f\sigma\left(D^2-a^2\right)\int_0^1 W^*\int_0^1 W^* dz + (1+pr_1\sigma\tau)\frac{\mu_e Hd}{4\pi\rho_0 v}$$

$$\int_0^1 W^*(D^2-a^2)DKdz =$$

$$(1+pr_1\sigma\tau)\left(\frac{g\alpha d^2}{v}a^2\right)\int_0^1 W^*\Theta dz \tag{34}$$

Integrating equation (4.34) and using the boundary conditions (28) and (29) together with equations (25-26), we obtain

$$(1+pr_1\sigma\tau)(1-F\sigma)I_1+\sigma(1+F+pr_1\sigma\tau)I_2$$

$$+\frac{\mu_e\eta}{4\pi\rho_0 v}(1+pr_1\sigma\tau)(I_3+p_2\sigma^* I_4)-$$

$$\frac{g\alpha ka^2}{v\beta}\frac{G}{G-1}\frac{(1+pr_1\sigma^*\tau)(1+pr_1\sigma\tau)}{(H+pr_1\sigma^*)}(I_5+Hpr_1\sigma^* I_6)=0 \tag{35}$$

where

$$I_1=\int_0^1\left(\left|D^2W\right|^2+2a^2|DW|^2+a^4|W|^2\right)dz$$

$$I_2=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz$$

$$I_3 = \int_0^1 \left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K^2| \right) dz$$

$$I_4 = \int_0^1 \left(|DK^2| + a^2|K|^2 \right) dz, \; I_5 = \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2 \right) dz$$

$$I_6 = \int_0^1 \left(|\Theta|^2 \right) dz$$

and σ^* is the complex conjugate of σ^*.The integrals $I_1 - I_6$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and then equating real and imaginary parts of equation (4.35), we obtain

$$\{1 + pr_1\tau\sigma_r - F\sigma_r - Fpr_1\tau\sigma_r^{\,2} + F\sigma_i^{\,2} pr_1\tau\} I_1 + \{\sigma_r(1 + pr_1\sigma_r\tau) - pr_1\sigma_i^{\,2}\tau\} I_2 + \frac{\mu_e\eta}{4\pi\rho_0\nu} \left[(1 + pr_1\tau\sigma_r) I_3 + \{p_2\sigma_r(1 + p_1\tau\sigma_r) + p_1p_2\tau\sigma_i^{\,2}\} I_4 \right]$$

$$- \frac{g\alpha ka^2}{\nu\beta} \left(\frac{G}{G-1} \right) \left\{ \frac{(1 + pr_1\sigma_r\tau)^2 + pr_1^{\,2}\tau^2\sigma_i^{\,2}}{(H + pr_1\sigma_r\tau)^2 + pr_1^{\,2}\tau^2\sigma_i^{\,2}} \right\}$$

$$\{(H + pr_1\tau\sigma_r)(I_5 + Hpr_1\sigma_r I_6) + pr_1^{\,2} H\tau\sigma_i^{\,2} I_6\} = 0 \quad (36)$$

and

$$i\sigma_i \{1 - pr_1\tau F\sigma_r - F(1 + pr_1\tau\sigma_r)\} I_1 + (1 + f + pr_1\sigma_r\tau) I_2 + (pr_1\tau I_3 - pr_2 I_4)$$

$$- \frac{g\alpha ka^2}{\nu\beta} \left(\frac{G}{G-1} \right) \left(\frac{(1 + pr_1\tau\sigma_r)^2 + pr_1^{\,2}\sigma_i^{\,2}\tau^2}{(H + pr_1\sigma\tau)^2 + pr_1^{\,2}\sigma^2{}_i\tau^2} \right)$$

$$(-pr_1 I_5 + pr_1 H^2 I_6) = 0 \quad (37)$$

From equation (36) that σ_r is positive or negative this means that the system may be stable or unstable. It is clear from the equation (37) may be zero or non-zero, meaning that the modes may non -oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of magnetic field and suspended particle, which were non-existence in their absence.

7. Conclusions

In this paper, the combined effect of suspended particles and magnetic field on thermal instability of a Rivlin-Ericksen fluid has been considered. The effect of magnetic field and suspended particles has been investigated analytically as well as numerically. The main results from the analysis are as follows:

1) In order to investigate the effects of magnetic field and suspended particles we examine the behavior of $\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dH_1}$ analytically.
2) It is found that suspended particles have a destabilizing (stabilizing) effect whereas magnetic field has a stabilizing (destabilizing) effect on the system. Figures 1-3 supports the analytic results graphically. It is found that the system has both stabilizing and destabilizing effects.
3) The critical Rayleigh numbers and the associated wave numbers are found for stationary convection for magnetic field and suspended particle involved and it has been found that it increases with the increase in magnetic field parameter and decreases with the increase in suspended particle parameter confirming the stabilizing role of magnetic field and destabilizing role of suspended particles Figures 4-6.
4) It is clear that the effect of compressibility is to postpone the onset of instability.

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