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Modern Trends of Mathematical Application in Geographical Thoughts and Its Environment – Increasing Relevance in Geo-scientific Study

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Abstracts: ***A number of ways in which mathematics is used in geography are mentioned. Plane Euclidean geometry is used in surveying small areas in the field, while spherical geometry and trigonometry are required in the construction of map projections, both traditional elements of mathematical geography. In the newer applications of mathematics to geography, topology is being used increasingly in the spatial analysis of networks. Graph theory provides indices to describe various types of network, such as drainage patterns. Differential equations are needed to study dynamic processes in geomorphology. Statistical techniques, such as trend surface analysis, factor analysis, cluster analysis and multiple discriminant analysis, can be applied to the description and analysis of the data of regional geography. Mathematical models are used in various forms to simplify the problems in geography. Examples of analogue models, such as the gravity model, are mentioned. Simulation models and Markov chain stochastic models are of value in studying certain geographical processes. Game theory is mentioned briefly. In the final section planning and prediction are very briefly referred to. In the former linear programming is a useful method, and in the latter trend fitting and extrapolation are applicable. Geography has gained a great deal in quantitative value and precision in adopting mathematical techniques (Cuchlaine A.M. King, 2006).***

Keywords: Plane Euclidean geometry, spherical geometry and trigonometry, mathematical geography, spatial analysis and Mathematical models.

1.Introduction

Each science has an ambition to be grateful for and explain the real world phenomena. Although geography is 'short on theories and long on facts', yet development of theory seems to be vital both to satisfactory explanations and to the identification of geography as an independent field of study. Few would deny the fact that the last few decades have been one of the greatest periods of thinker changes in the trends of geographic development. Most of these changes, the questioning of the past approaches, looking at old problems with new eyes, have been of a methodological nature involving, in virtually every instance, the substitution of quantitative approaches to problems formerly treated in descriptive ways. Today an in fact new standpoint has been opened under the impact of so-called quantitative revolution. Mathematical and Statistical methods have been introduced to attain a desired level of objectivity, and search for models and theories and proceed apace. The works of Hartshorne (1939, 1959) may be considered the last in the chain of traditional writers in geography. The concepts of geography elucidated by Hartshorne and accepted by many practicing geographers began to come under attack from the early 1950s onwards.

Another aspect that has confident this development has been the spread of quantification. A mounting number of geographers became responsive that mathematics and statistics could be applied to geographical problems. These afford precise tools to test theories and analyze data. The process of rational change led geographers to deliberate less and less on recounting the differences between particular areas or places and more and more on the study of uniformities and the production of theories about the spacing of phenomena on the earth's surface. Such an emphasis on nomothetic approach is in the right direction. Besides, during the last few decades the focus has also changed to make the concept of the systems of much greater significance, along with that of models and theories. The search for generalizations based on the whole rather than on individual parts is, therefore, a complementary method of modern science known as systems analysis. Since all systems, whether physical or human or a combination of both, consist of a set of objects and the relationships binding these objects together into some organization, it is not amazing that the approach is especially useful in dealing with functional aggregates. Indeed, now the main focus of scientific enquiry has moved away from the study of objects or substances to the study of relationships and organizations. And, as all organizations are recognized as being particularly complex, systems analysis proves to be a particularly appropriate framework of study in geography. The systems approach is not a replacement for the analytic method, but it is an additional line of modern scientific enquiry designated to break down the barriers between inter-disciplinary enquiries. It represents one of the major current research frontiers in geography.

2.Beginning of Mathematics in Geography

In B.C.334, Alexander the Great led his army south across the sea, then east towards the Persian Empire. Geographer Nearcnos went along with the troops, gathering necessary information for a "World Map". He noticed that along the marching routes, from west to east, the changing of seasons and the sunshine durations were nearly the same. The geographer made an important contribution: for the first time in history, he drew a latitude line on the map. This latitude line started from the Strait of Gibraltar went along the Himalayas and reached the Pacific Ocean. Alexander's empire collapsed very soon, however in the Egyptian city which named after Alexander the Great, a well-known museum was founded. The old curator Eratosthenes was learned and has mastery of mathematics, astronomy and geography. Through calculation he noted that the circumference of the earth is 46250km and drew a world map with 6 latitude lines and 7 longitude lines on it. From then on, latitude and longitude were used to mark locations accurately. Thereafter, an inseparable relationship was built between geography and statistics and also geography and mathematics through latitude and longitude lines.

An Application of Geography to Mathematics:

Every student of integral calculus has done battle with the formula:- $\int sec\theta d\theta = \ln|sec\theta + tan\theta| + c$………………………………. **(1)**
This formula can be checked by differentiation or "derived" by using the substitution, $u = sec\theta + tan\theta$, but these ad-hoc methods do not make the formula any more understandable. Experience has taught us that this troublesome integral can be motivated by presenting its history. Perhaps the title seems twisted, but the tale to follow will show that this integral should be presented not as an application of mathematics to geography, but rather as an application of geography to mathematics. The secant integral arose from cartography and navigation (part of geography), and its evaluation was a central question of mid-seventeenth century mathematics. The first formula, discovered in 1645 before the work of Newton and Leibniz, was

$$\int sec\theta d\theta = \ln\left|\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right| + c \quad \text{.................... (2)}$$

This is a trigonometric variant of (1). This was discovered, not through any mathematician's cleverness, but by a serendipitous historical accident when mathematicians and cartographers sought to understand the Mercator map projection. Thereafter, development of integral calculus has been occurred through the different correction and modifications over time.

3.Recent Practice of Quantitative Geography and Trend towards Mathematics

Quantitative practices in geography date back at least to Greek attempts to measure the circumference of the earth, but the term `quantitative geography' was coined in the 1960s. The terminology was deliberate, an attempt to promote the `quantitative revolution' as a new and better scientific geography over an older regional geography. The power of this moment in Anglo-American geography, both in its own terms and also in generating a concern for theory and for philosophical foundations that remains with us today, is no doubt one reason why the term quantitative geography still resonates with what was practiced then. The quantitative revolution brought a change not only in disciplinary language, but also in worldview, each reinforcing the other. In the history of geography, the **quantitative revolution (QR)** was one of the four major turning-points of modern geography – the other three being **environmental determinism, regional geography** and **critical geography**). The quantitative revolution occurred during the 1950s and 1960s and marked a rapid change in the method behind geographical research, from regional geography into a spatial science. The main claim for the quantitative revolution is that it led to a shift from descriptive (idiographic) geography to an empirical law-making (nomothetic) geography. The Quantitative Revolution began in the universities of Europe with the support of geographers and statisticians in both Europe and the United States. First emerging in the late 1950s and early 1960s, the Quantitative Revolution responded to the rising regional geography paradigm. Under the loosely defined banner of bringing 'scientific thinking' to geography, the quantitative revolution led to an increased use of computerized statistical techniques, in particular multivariate analysis, in geographical research. The newly adopted methods reflected an array of mathematical techniques that improved precision.

Some of the techniques that epitomize the quantitative revolution include:

- Descriptive statistics;
- Inferential statistics;
- Basic mathematical equations and models, such as gravity model of social physics, or the Coulomb equation;
- Stochastic models using concepts of probability, such as spatial diffusion processes;
- Deterministic models, e.g. Von Thünen's and Weber's location models.

The common factor, linking the above techniques, was a preference for numbers over words, plus a belief that numerical work had a superior scientific pedigree. Proponents of quantitative geography tended to present it as bringing science to geography. In fact, the particular contribution of the quantitative revolution was the huge faith placed in multivariate analysis and in particular methods associated with econometrics. It was also very strongly aligned with positive science and this would prove a major source of epistemological debate. The overwhelming focus on statistical modelling would, eventually, be the undoing of the quantitative revolution. Many geographers became increasingly concerned that these techniques simply put a highly sophisticated technical gloss on an approach to study that was barren of fundamental theory. Other critics argued that it removed the 'human dimension' from a discipline that always prided itself on studying the human and natural world alike. As the 1970s dawned, the quantitative revolution came under direct challenge.

The philosophy drawn on to promote quantitative geography over Richard Hartshorne's exceptionalist approach was positivism. Interestingly, the term itself was not widely used by quantitative geographers (Hill, 1981), until they came under attack from post positivist geographers, and quantitative geographers have paid little attention to the finer points of distinction between empiricism, positivism, logical positivism, and critical rationalism. Nevertheless, David Harvey's (1969) Explanation in Geography canonized this approach and clearly was influenced by the same logical positivist philosophers as Schaeffer. The result, by the end of the 1960s, was a hegemonic representation of modern geographic practice, in which quantitative practices are fused with scientific geography, made possible by the myriad of processes and actors, external and internal to geography, marshaled in support of this representation. Useful shorthand for this is Latour's term, techno science.

There are three broad ways in which practices of quantitative geography differ from the representation summarized above. **First,** its emphasis on statistics has eliminated from view the significant mathematical practices of geographers. **Second,** quantitative practices of both a mathematical and a statistical nature cannot be equated with positivism. **Third,** the evolution of practices in all areas has been neglected. If we take seriously the evolutionary nature of the language of mathematics, critics of quantitative geography cannot be satisfied with analyzing past practices as if quantitative geography stopped evolving in the early 1970s (only to rise again from its coffin, in the view of such critics, reincarnated through GIS like a bad Dracula movie).

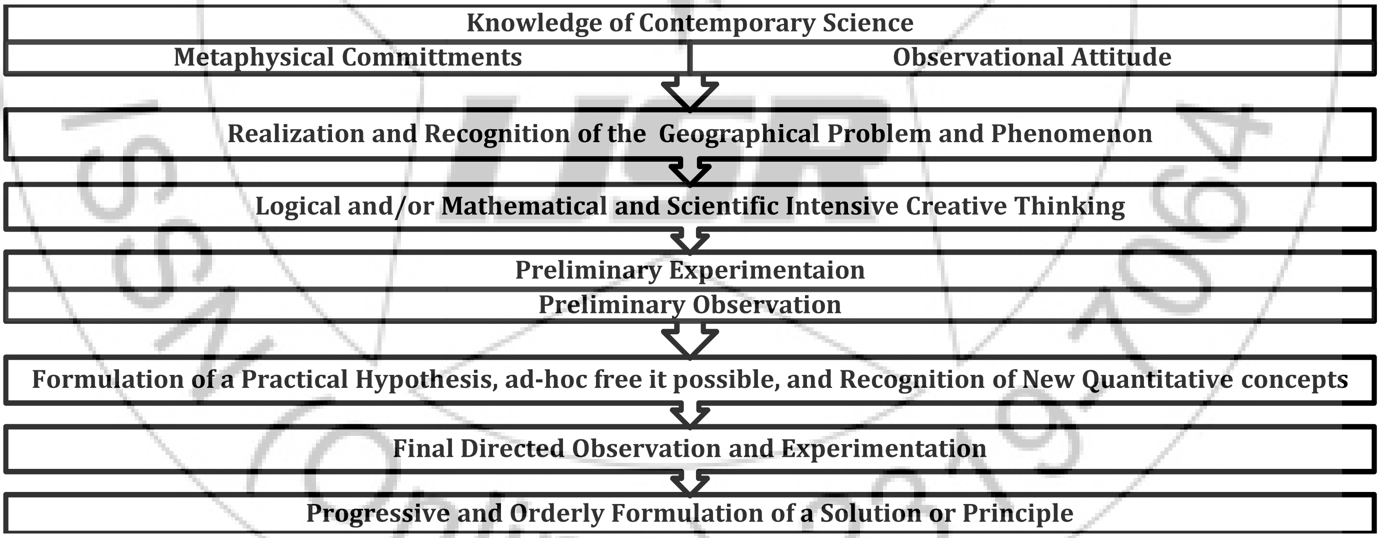
4. The necessity of Mathematics in Modern Geography

The survival of the world is a natural phenomenon which is not easy to characterize In terms of four dimensions. The investigation of the nature of our planet is a human effort, which is uttered in many diverse criteria, one of which is geography. To the geographer, his field is the colorful science of space area. Such a definition gives room for both qualitative and quantitative aspects which are combined necessarily, by nature of the geographer's quest. The intensity, with which the quantitative aspect, that is to say, the scientific method as used in geography, is limited to a level far below that of the qualitative strength. All of the geographers agree with the contemporary Italian and German schools of geography, which stress rightfully the importance of science in Geographical investigations and teachings.

Making an allowance for the method of exploration, the science fields in geography are three:
1) The science of the planet,
2) The science of relationships, (nature to nature, nature to man, and man to nature),
3) The science of distributions (phenomena in cultural or natural occurrences).

In these three levels, science has mathematics as a widespread language because it has a spontaneous response from the physical world that is studied. As is for all sciences, mathematics is desirable by the geographer to assist harmonize those experiences which the qualitative criteria are incapable to bring to a entire logical system. For it is mathematics, the technique par excellence, that implements a comprehensive order in the knowledge of some fields in geography.

Upon assuming a quantitative attitude, the geographer realizes that human scientists are operating in a three dimensional space and therefore must apply themselves to some system of scientific induction, deduction and conclusion, all of which are performed mathematically. To insure the geographer's scientific success a simple practice is here on hand, namely the order of scientific procedure in geography.



This succession of method insures the researcher that the connected principles of scientific achievement are for all time subjects to application; namely: continuity, individual variability, multiplicity of effort and selective development. For geographers there must be a respond to the What, How, Why and the How Much. To synchronize all of the desired answers in its whole, a geographer has to be a methodical and a spontaneous philosopher. This fusion is absolutely not in the way at the relevance of mathematics in systematic thoughts.

So many principles are passed over in geographical research because of a lack of highlighting in a more quantitative study. The earth and its straightforward mathematical relations are not sufficiently well understood by geographers. Literature about the subject is in short supply and far too detached in subject matter at non-geographic fields of study. The mathematical ability of a geographer should include:

Mathematical Abilities to Geographical Study		
1. Algebra		
2. Geometry	(a)Theoretical	(i)Plane
		(ii) Spherical
	(b)Applied	(i)Solid

		(ii)Stereometry
3. Curve Analysis and Determinants		
4. Trigonometry	(a) Plane	
	(b) Spherical	
5. Analytical Geometry		
6. Infinitesimal Arithmetic		
7. Probabilities		

The divisions or portions of geography that require a mathematical background to assure an orderly understanding are:

a) Form and Shape of the Earth
b) Movements of the Earth and its immediate gravitational and electromagnetic relations.
c) Elements of longitude and variables of time determination.
d) Cartography and Map interpretation
e) Climatology
f) Physiography.

5.Mathematics in the World of Modern Geography

A number of ways in which mathematics is used in geography are mentioned. **Plane Euclidean geometry** is used in surveying small areas in the field, while **spherical geometry and trigonometry** are required in the construction of map projections, both traditional elements of mathematical geography. In the newer applications of mathematics to geography, **topology** is being used increasingly in the spatial analysis of networks. **Graph theory** provides indices to describe various types of network, such as drainage patterns. **Differential equations** are needed to study dynamic processes in geomorphology. **Statistical techniques, such as trend surface analysis, factor analysis, cluster analysis and multiple discriminant analysis,** can be applied to the description and analysis of the data of regional geography. **Mathematical models** are used in various forms to simplify the problems in geography. Examples of **analogue models, such as the gravity model, are mentioned. Simulation models and Markov chain stochastic models** are of value in studying certain geographical processes. **Game theory** is mentioned briefly. In the final section planning and prediction are very briefly referred to. In the former **linear programming** is a useful method, and in the latter trend fitting and extrapolation are applicable. Geography has gained a great deal in quantitative value and precision in adopting mathematical techniques.

<u>Modern Trend of Mathematical Application in Geographical Information System(GIS), the Latest Tool of Geography:-</u>

a) <u>Applications of Propositional Logic in GIS</u>: - In GIS applications, we find logical operators mainly in spatial analysis and database queries.
b) <u>Applications of Predicate Logic in GIS</u>:- In relational database technology, we use the select operator to select a subset of tuples t (or records) in a relation that satisfies a given selection condition. In general, we can denote the select operator as σ selection condition (relation name) or σφt (R) when we substitute selection condition with φ (t) and R for the relation name. The selection condition is a predicate, i.e., it designates a property of the tuples, and we can thus write the general selection as a predicative set expression {t∈R |φ (t)}.
c) <u>Applications of Logical Inference in GIS</u>:- Rule-based systems apply rules to data provided using an inference engine. These systems are also called expert systems, and are widely applied in the geosciences. Spatial decision support systems (SDSS) are rule-based systems that are designed and tuned for spatial data. Rules are stored as implications in the form "if <premise> then <consequence>". The inference engine examines the given data in the database and determines if they match a given premise. If this is the case, the consequence is applied accordingly. This is a straightforward application of the modus ponens.
d) <u>Applications of Set Theory in GIS</u>: - Overlay operations are among the most common functions that a GIS provides for spatial analysis. Since spatial features such as points, arcs and polygons can be regarded as sets, overlay operations correspond to set intersection, union, difference, and complement.

Table shows the basic Arc Info overlay commands and the corresponding set operations in mathematical notation. Other Arc Info functions such as CLIP, UPDATE, and IDENTITY are based on combinations of overlay and graphical clip operation.

Table: Arc Info overlay commands

Command	**A**	**B**	**Set Operation**
ERASE	in cover	erase cover	$A- B$
INTERSECT	in cover	intersect cover	$A \cap B$
UNION	in cover	union covers	$A \cup B$

Normally, it does not matter in which sequence we apply overlay operations of the same type. The associative and commutative laws for set operations allow the application of intersection and union in arbitrary order. The distributive laws can be used to simplify spatial overlay operations by reducing the number of operations. For example, if we have three data sets A, B and C. We need the intersection of A and B and the intersection of A and C, and finally, compute the union of the results. These operations amount to the following set operations $(A \cap B) \cup (A \cap C)$.

This would need three overlay operations. However, the distributive law of set operations allows us to reduce the number of operations to two as $A \cap (B \cup C)$. When we deal with polygon features in a GIS, we always have an embedding polygon that contains all features of our data set (or coverage). Often it is called world polygon. In set theoretic terms, this corresponds to the universe of discourse.

a) Applications of Relations and Functions in GIS: - Relations are a very important concept in mathematics. Based on the fundamental principle of the Cartesian product we will introduce relations as the foundation of mappings and functions. Relations are based on a common understanding of relationships among objects. These relationships may refer to a comparison between objects of the same set, or they involve elements of different sets. Two special types of relations, the equivalence relation and the order relation, play an important role in mathematics. The first is used to classify objects; the latter one is the basis for the theory of ordered sets. In this chapter, we deal only with binary relations only. They are relations between two sets.

b) Applications of Coordinate System and Transformation in GIS: - In GIS, we apply geometric transformations in many different ways. One use of transformations is in the graphic editing functions of every GIS. When we edit spatial features, we need to shift, rotate, skew and scale them. Another important application lies in the transformation of coordinates of datasets as it occurs, for instance, in manual digitizing. Here, we have to set up a transformation from the device coordinates, i.e., the coordinates produced by the digitizing device – usually in millimeters or inches – to the world coordinates, i.e., the coordinates of the map projection. Every GIS software package should provide this functionality for manual digitizing or general coordinate transformation. The TRANSFORM command in Workstation Arc/INFO is one example of such a function.

c) Applications of Algebraic Structure in GIS:- Perhaps the most prominent application of algebras in GIS is the map algebra. The carrier set of the map algebra is the set of the "maps", i.e., data sets that are often referred to coverage, shape file, grid or layer.

The concept of structure preserving mapping finds an application in spatial modeling, where we map a subset of the real world to a representation in feature spatial model.

a) Applications of Topology in GIS:- Topology is a central concept in every GIS. It deals with the structural representation of spatial features and their properties that remain invariant under certain transformations

b) Applications of Ordered Sets in GIS: - The intuitive interpretation of order relations as "is contained in" or, dually, as "contains" can be used for relationships among spatial features such as polygons, lines and points. The structure of a poset accommodates both strict hierarchies (every object has exactly one parent object) and relationships where one object possesses more than one parent object.

c) Applications of Graph Theory in GIS: - Graphs have played an important role in GIS right from the early beginnings. The reason is that in the early days of GIS the storage of map data (or cartographic data) was the focus of interest. Early data structures for the representation of spatial data (predominantly two-dimensional) are almost exclusively based on planar graphs.

d) Applications of Fuzzy Logic in GIS: - Many phenomena show a degree of vagueness or uncertainty that cannot be properly expressed with crisp sets of class boundaries. Spatial features often do not have clearly defined boundaries, and concepts like "steep", "close", or "suitable" can better be expressed with degrees of membership to a fuzzy set than with a binary yes/no classification. Many spatial phenomena are inherently fuzzy or vague or possess indeterminate boundaries. Fuzzy logic has been applied for many areas in GIS such as fuzzy spatial analysis, fuzzy reasoning, and the representation of fuzzy boundaries

e) Applications of Spatial Modeling in GIS: -Geographical information systems process spatial information. The information is derived from spatial data in a database. To sensibly work with these systems, we need models of spatial information as a framework for database design. These models address the spatial, thematic and temporal dimensions of real world phenomena. An understanding of the principle concepts of space and time rooted in philosophy, physics and mathematics is a necessary prerequisite to develop and use spatial data models. We know two major approaches to spatial data modeling, the analogue map approach, and spatial databases. Today, the function of maps as data storage (map as a database) is increasingly taken over by spatial databases. In databases, we store representations of phenomena in the real world. These representations are abstractions according to selected spatial data models. We know two fundamental approaches to spatial data modeling, field-based and object-based models. Both have their merits, advantages and disadvantages for particular applications. Consistency is an important requirement for every model. Topology provides us with the mathematical tools to define and enforce consistency constraints for spatial databases, and to derive a formal framework for spatial relationships among spatial objects. Spatial data not only possess spatial and thematic attributes, but extend also into the temporal domain. A model of time for spatial information is an important ingredient for any spatial data model, thus leading to what is called spatiotemporal data models.

6.Conclusion

Generally it is acknowledged, that mathematics is the trickiest allotment of geography, but it is becoming a very crucial discipline. Qualitative analysis alone cannot answer all the questions posed by a modern society with a modern industry and a dynamic activism. More needs to be written about mathematics in geography. More needs to be taught and disseminated. The lack of applied mathematics in geography and the progressive complexity of the problems posed may in the future overwhelm the geographer's ability to deal with people interested in the world he studies. A reflexive knowledge of mathematics is not enough; it must be activated and brought up to date and in accordance with the modern contemporary scientific concepts of the physical world. Let it be clear that this is not an attempt to make a mathematical science out of geography, for it cannot be done as it is understandable to any geographer. Any science that generates new thoughts and theories will make felt its influence on the other sciences, in the long run. Geography is inflowing a period where it will improve its entire system of study and exploration and a greater prominence in mathematics is bound to be one of the points of curiosity in this amendment. Finally, it may be said that ***"Mathematics needs Geography; Geography needs Mathematics"*** to complete the picture and target interest and ***"Mathematical geography or geographical Mathematics is an expression of democracy"***. Mathematics, Statistics and Geography cannot be regarded as separate entities since the combination is fundamental as we move forward.

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