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Ecological System of a Prey-Predator Model with Prey Reserve

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Abstract: ***In this paper, we propose and analyze a mathematical model to study the dynamics of a fishery resource system in an aquatic environment that consists of two zones, a free fishing zone and a reserve zone where fishing is strictly prohibited. The existence of biological system is discussed. The local and global stability analysis has been carried out. An optimal harvesting policy is given using Pontryagin's Maximum Principle.***

Keywords: Prey-predator, Local and Global stability, Optimal harvesting.

1. Introduction

It is well known that many species have already become extinct and many others are at the verge of extinction due to several natural or manmade reasons like over exploitation, indiscriminate harvesting, over predation, environmental pollution, loss of habitat and mismanagement of natural resources etc. To save the species from getting extinct we are taking measures like improving conditions of their natural habitat, reduce the interaction of the species with external agents which tend to decrease their numbers, impose restrictions on species harvesting, create natural reserves, establish protected areas etc. While creating protected areas for a species, several factors have to be taken into consideration. The competitive cooperative and predator-prey models have been studied by many authors.

The mathematical and bio-economic theories concerning renewable resources for harvesting have been systematically developed by Clark [1, 2] in his two books. he discussed the management of biological population from an analytical point of view. Kizner [11] focused on the stability analysis of a certain class of catch effort controlled discrete stock-production models for optimal management of exploited populations. Kar [6] considered a prey- predator fishery model and discussed the selective harvesting of fishes age or size by incorporating a time delay in the harvesting terms. Kar and Chaudhary [8] studied a dynamic reaction model, in which prey species are harvested in the presence of a predator and a tax. Kronbak [4] set up a dynamic open-access model of a single industry exploiting a single resource stock.Karet.al. [10] considered a prey predator fishery model with influence or a prey reserve.

Mikkelsen [3] investigated aquaculture externalities on fishery, affecting habitat, wild fish stock genetics, or fishery efficiency under open access and rent maximizing fisheries. Zhang et.al.[5] analyzed of a prey predator fishery model with prey reserve. Kar and Matsuda [7] examined the impact of the creation of marine protected areas, from both economic and biological perspectives. Kar and Chakraborty [9] considered a prey predator fishery model with prey dispersal in a two patch environment, one of which is a free fishing zone and other is protected zone.

2. Mathematical Model

We consider a prey –predator model with Holling type of predation. Following Kar and Swarnakamal [10], mathematical formulation for this model is as under

$$\frac{dx}{dt} = r_1 x(1-\frac{x}{K_1}) - \frac{\alpha xz}{(1+mx)} - \sigma_1 x + \sigma_2 y - q_1 E_1 x$$
$$\frac{dy}{dt} = r_2 y(1-\frac{y}{K_2}) + \sigma_1 x - \sigma_2 y \qquad (1)$$
$$\frac{dz}{dt} = -dz + \frac{k\alpha xz}{(1+mx)} - q_2 E_2 z.$$

Here, x(t) and z(t) are biomass densities of prey species and predator species inside the unreserved area which is an open-access fishing zone, respectively, at time t. y(t) is the biomass density of prey species inside the reserved area where no fishing is permitted at time t. All the parameters are assumed to be positive. $r_1$and $r_2$are the intrinsic growth rates of prey species inside the unreserved and reserved areas, respectively. d, α and k are the death rate, respectively. $K_1$and $K_2$are the carrying capacities of prey species in the unreserved and reserved areas, respectively. $\sigma_1$and $\sigma_2$are migration rates from the unreserved area to the reserved area and the reserved area to the unreserved area. $E_1$and $E_2$are the effects applied to harvest the prey species and predator species in the unreserved area. $q_1$and $q_2$are the catch ability coefficients.

Where $0 < m < 1$ is constant, If there is no migration of fish population from the reserved area to the unreserved area i.e. when $\sigma_2 = 0$ and $r_1 - \sigma_1 - q_1 E_1 < 0$, then $x < 0$. Similarly if there is no migration of fish population from the unreserved area to the reserved area i.e. when $\sigma_1 = 0$ and $r_2 - \sigma_2 < 0$. Then $y < 0$.

We assume that

$$r_1-\sigma_1-q_1E_1>0\,,\; r_2-\sigma_2>0\,. \qquad (2)$$

3. Existence of Equilibria

Equilibria of model (1) can be obtained by equating right hand side to zero. This provides three equilibria $E_0(0,0,0)$, $E_1(\bar{x},\bar{y},0)$, $E_2(\hat{x},\hat{y},\hat{z})$ .The equilibrium E_0 exists obviously and we shall show the existence of E_1 and E_2 as follows:

3.1 Existence of E_1 ($\bar{x},\bar{y},0$)

Here $\bar{x}$ *and* $\bar{y}$ are positive solutions of the following algebraic equations:

$\bar{y}=\dfrac{1}{\sigma_2}\left[(-r_1+\sigma_1+q_1E_1)\bar{x}+\dfrac{r_1\bar{x}^2}{K_1}\right]$ and $z = 0$,then we get algebraic equation $a_1x^3+b_1x^2+c_1x+d_1=0$ (3)

Where

$$a=\frac{r_2r_1^2}{K_2K_1^2\sigma_2^2},$$

$$b=-\frac{2r_1r_2(r_1-\sigma_1-q_1E_1)}{K_1K_2\sigma_2^2},$$

$$c=\frac{r_2(r_1-\sigma_1-q_1E_1)^2}{K_2\sigma_2^2}-\frac{r_1(r_2-\sigma_2)}{K_1\sigma_2},$$

$$d=\frac{(r_2-\sigma_2)}{\sigma_2}(r_1-\sigma_1-q_1E_1)-\sigma_1\,.$$

Equation (3) has a positive solution $x=\bar{x}$ if the following inequalities hold :

$$\frac{r_2(r_1-\sigma_1-q_1E_1)^2}{K_2\sigma_2}<\frac{r_1(r_2-\sigma_2)}{K_1} \qquad (4)$$

$$(r_2-\sigma_2)(r_1-\sigma_1-q_1E_1)>\sigma_1\sigma_2 \qquad (5)$$

And for $\bar{y}$ to be positive, we must have

$$\frac{K_1}{r_1}(r_1-\sigma_1-q_1E_1)<\bar{x} \qquad (6)$$

Here the equilibrium $E_1(\bar{x},\bar{y},0)$ exists under the above conditions.

3.2 Existence of E_2 ($\hat{x},\hat{y},\hat{z}$)

Now again $\hat{x},\hat{y},$ and $\hat{z}$ are positive solutions of

$$r_1x(1-\frac{x}{K_1})-\frac{\alpha xz}{(1+mx)}-\sigma_1x+\sigma_2y-q_1E_1x=0 \qquad (7)$$

$$r_2y(1-\frac{y}{K_2})+\sigma_1x-\sigma_2y=0 \qquad (8)$$

$$-dz+\frac{k\alpha xz}{(1+mx)}-q_2E_2z=0 \qquad (9)$$

From (9) we get $\hat{x}=\dfrac{d+q_2E_2}{k\alpha-m(d+q_2E_2)}$, and substitute value of $\hat{x}$ in equations (7) and (8) ,then

we get

$$\hat{y}=\frac{K_2}{2r_2}\left[(r_2-\sigma_2)+\left\{(r_2-\sigma_2)^2+4r_2\sigma_1\hat{x}/K_2\right\}^{\frac{1}{2}}\right],$$

$$\hat{z}=\frac{(1+m\hat{x})}{\alpha\hat{x}}\{(r_1-\sigma_1-q_1E_1)\hat{x}-\frac{r_1\hat{x}^2}{K_1}+\sigma_2\hat{y}\}$$

It may be noted that for $\hat{z}$ to be positive, we must have

$$(r_1-\sigma_1-q_1E_1)\hat{x}-\frac{r_1\hat{x}^2}{K_1}+\sigma_2\hat{y}>0 \qquad (10)$$

4. Stability Analysis

a) In the absence of predator, the model (1) becomes

$$\frac{dx}{dt}=r_1x(1-\frac{x}{K_1})-\sigma_1x+\sigma_2y-q_1E_1x \qquad (11)$$

$$\frac{dy}{dt}=r_2y(1-\frac{y}{K_2})+\sigma_1x-\sigma_2y\,.$$

For the study of the stability of the equilibrium point the variational matrix of the system (11) is

$$J=\begin{pmatrix} r_1-\dfrac{2r_1x}{K_1}-\sigma_1-q_1E_1 & \sigma_2 \\ \sigma_1 & r_2-\dfrac{2r_2y}{K_2}-\sigma_2 \end{pmatrix} \qquad (12)$$

The characteristic equation of the variational matrix of (12) at $E_0(0,0)$ is

$$\begin{aligned}&\lambda^2-(r_1-\sigma_1-q_1E_1+r_2-\sigma_2)\lambda\\&+(r_1-\sigma_1-q_1E_1)(r_2-\sigma_2)-\sigma_1\sigma_2=0\end{aligned} \qquad (13)$$

Since $\lambda_1+\lambda_2=(r_1-\sigma_1-q_1E_1+r_2-\sigma_2)>0$ and $\lambda_1\lambda_2=(r_1-\sigma_1-q_1E_1)(r_2-\sigma_2)-\sigma_1\sigma_2>0$

Hence $E_0(0,0)$ is unstable.

Similarly, The characteristic equation of the Jacobian matrix of (12) at $E_1(\bar{x},\bar{y})$ is

$$\begin{aligned}&\lambda^2-(\frac{r_1}{K_1}\bar{x}+\frac{r_2}{K_2}\bar{y}+\frac{\sigma_1}{\bar{y}}\bar{x}+\frac{\sigma_2}{\bar{x}}\bar{y})\lambda\\&+\frac{r_2}{K_2}\bar{y}(\frac{r_1}{K_1}\bar{x}+\frac{\sigma_2}{\bar{x}}\bar{y})+\frac{r_1\sigma_1}{K_1\bar{y}}\bar{x}^2\end{aligned} \qquad (14)$$

Since $\lambda_1+\lambda_2=-(\dfrac{r_1}{K_1}\bar{x}+\dfrac{r_2}{K_2}\bar{y}+\dfrac{\sigma_1}{\bar{y}}\bar{x}+\dfrac{\sigma_2}{\bar{x}}\bar{y})<0$ and

$$\lambda_1\lambda_2=\frac{r_2}{K_2}\bar{y}(\frac{r_1}{K_1}\bar{x}+\frac{\sigma_2}{\bar{x}}\bar{y})+\frac{r_1\sigma_1}{K_1\bar{y}}\bar{x}^2>0\,.$$

Thus $E_1(\bar{x},\bar{y})$ is locally asymptotically stable.

b) We assume that system (1) has a positive equilibrium $E_2(\hat{x},\hat{y},\hat{z})$ .The Jacobian matrix of (1) at $E_2(\hat{x},\hat{y},\hat{z})$ is

$$\begin{pmatrix} r_1-\frac{2r_1x^*}{K_1}-\frac{\alpha z^*}{(1+mx^*)^2}-\sigma_1-q_1E_1 & \sigma_2 & \frac{-\alpha x^*}{(1+mx^*)} \\ \sigma_1 & r_2-\frac{2r_2y^*}{K_2}-\sigma_2 & 0 \\ \frac{k\alpha z^*}{(1+mx^*)^2} & 0 & -d+\frac{k\alpha x^*}{(1+mx^*)}-q_2E_2 \end{pmatrix} \quad (15)$$

The characteristic equation of the Jacobian matrix of (15) at E_2 $(\hat{x},\hat{y},\hat{z})$ is

$$\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0 \quad (16)$$

Where

$$a_1=\frac{r_1}{K_1}x^*+\frac{r_2}{K_2}y^*+\frac{\alpha z^*mx^*}{(1+mx)^2}+\frac{\sigma_1}{y^*}x^*+\frac{\sigma_2}{x^*}y^*$$

$$a_2=\left(\frac{r_1}{K_1}x^*+\frac{\sigma_2}{x^*}y^*+\frac{\alpha mz^*x^*}{(1+mx^*)^2}\right)\left(\frac{r_2}{K_2}y^*+\frac{\sigma_1}{y^*}x^*\right)$$

$$+\sigma_1\sigma_2+\frac{k\alpha^2x^*z^*}{(1+mx^*)^3}$$

$$a_3=\frac{k\alpha^2x^*z^*}{(1+mx^*)^3}\left(\frac{r_2}{K_2}y^*+\frac{\sigma_1}{y^*}x^*\right)$$

According to Routh –Hurwitz criteria, the necessary and sufficient conditions for local stability of equilibrium point E_2 are $a_1>0, a_3>0$, and $a_1a_2-a_3>0$

It is obvious that $a_1>0, a_3>0$. Thus , for the stability of E_2, we calculate $a_1a_2-a_3$

$$a_1a_2-a_3=\begin{array}{l}\left[\frac{r_1}{K_1}x^*+\frac{r_2}{K_2}y^*+\frac{\alpha z^*mx^*}{(1+mx)^2}+\frac{\sigma_1}{y^*}x^*+\frac{\sigma_2}{x^*}y^*\right] \\ \left[\left(\frac{r_1}{K_1}x^*+\frac{\sigma_2}{x^*}y^*+\frac{\alpha z^*mx^*}{(1+mx)^2}\right)\left(\frac{r_2}{K_2}y^*+\frac{\sigma_1}{y^*}x^*\right) \right.\\ \left.+\sigma_1\sigma_2+\frac{k\alpha^2x^*z^*}{(1+mx^*)^3}\right] \\ -\frac{k\alpha^2x^*z^*}{(1+mx^*)^3}\left(\frac{r_2}{K_2}y^*+\frac{\sigma_1}{y^*}x^*\right)>0\end{array}$$

Hence $E_2(\hat{x},\hat{y},\hat{z})$ is locally asymptotically stable.

Theorem 1. The equilibrium point E_1 is globally asymptotically stable.

Proof- Let us consider the Lyapunov function

$$V(x,y)=\omega_1(x-\bar{x}-\bar{x}\ln\frac{x}{\bar{x}})+\omega_2(y-\bar{y}-\bar{y}\ln\frac{y}{\bar{y}})$$

Where ω_1, ω_2 are positive constants, to be chosen later on.
Differentiating V with respect to time t, we get

$$\frac{dV}{dt}=\omega_1\frac{(x-\bar{x})}{x}\frac{dx}{dt}+\omega_2\frac{(y-\bar{y})}{y}\frac{dy}{dt}$$

Choosing $\frac{\omega_2}{\omega_1}=\frac{\bar{y}}{\bar{x}}\frac{\sigma_2}{\sigma_1}$, a little algebraic manipulation yields

$$\frac{dV}{dt}=-\frac{r_1\omega_2\sigma_1\bar{x}}{K_1\sigma_2\bar{y}}(x-\bar{x})^2-\frac{r_2}{K_2}\omega_2(y-\bar{y})^2$$

$$-\frac{\omega_2\sigma_2}{xy\bar{y}}(\bar{x}y-x\bar{y})^2<0.$$

Therefore, $E_1(\bar{x},\bar{y})$ is globally asymptotically stable.

Theorem2. $E_2(\hat{x},\hat{y},\hat{z})$ is globally asymptotically stable.

Proof. Let us choose the Lynpunov function.

$$V(x,y,z)=\omega_1(x-x^*-x^*\ln\frac{x}{x^*})+\omega_2(y-y^*-y^*\ln\frac{y}{y^*})$$

$$+\omega_3(z-z^*-z^*\ln\frac{z}{z^*})$$

Where ω , ω_2, ω_3 are positive constants, to be chosen later on.
Differentiating V with respect to time t, we get,

$$\frac{dV}{dt}=\omega_1\frac{(x-x^*)}{x}\frac{dx}{dt}+\omega_2\frac{(y-y^*)}{y}\frac{dy}{dt}+\omega_3\frac{(z-z^*)}{z}\frac{dz}{dt}$$

Choosing $\frac{\omega_1}{\omega_3}=k, \frac{\omega_1}{\omega_2}=\frac{x^*}{y^*}\frac{\sigma_1}{\sigma_2}$, a little algebraic manipulation

$$\frac{dV}{dt}=-\frac{r_1\omega_1}{K_1}(x-x^*)^2-\frac{r_2\omega_1\sigma_2y^*}{K_2\sigma_1}(y-y^*)^2$$

$$-\frac{\omega_1\sigma_2}{xyx^*}(x^*y-xy^*)^2<0.$$

Therefore $E_2(\hat{x},\hat{y},\hat{z})$ is globally asymptotically stable.

5. 6. Optimal harvesting policy:

Our objective is to maximize the present value J of continuous time stream of revenue given by

$$J=\int_0^\infty e^{-\delta t}\{(p_1q_1x-c_1)E_1(t)+(p_2q_2z-c_2)E_2(t)\}dt$$

(27)

Where δ is instantaneous rate of annual discount. Thus our objective is to maximize J subject to state equation (1) and to the control constraints

$$0\le E(t)\le E_{max} \quad (28)$$

To solve the optimization problem, we utilize the Pontryagin Maximal Principal.

The associated Hamiltonian is given by

$$H = e^{-\delta t}\{(p_1q_1x - c_1)E_1 + (p_2q_2z - c_2)E_2\}$$
$$+\lambda_1\left\{r_1x(1-\frac{x}{K_1}) - \frac{\alpha xz}{(1+mx)} - \sigma_1x + \sigma_2y - q_1E_1x\right\}$$
$$+\lambda_2\left\{r_2y(1-\frac{y}{K_2}) + \sigma_1x - \sigma_2y\right\} \qquad (29)$$
$$+\lambda_3\left\{-dz + \frac{k\alpha xz}{(1+mx)} - q_2E_2z\right\},$$

Where $\lambda_1, \lambda_2, \lambda_3$ the adjoint variables and the Hamiltonian function H are is linear in control variables E_1 and E_2. The optimal control are not a binding, we have singular control.

According to Pontrygin's maximal principal

$$\frac{\partial H}{\partial E_1} = 0; \frac{\partial H}{\partial E_2} = 0; \frac{\partial \lambda_1}{dt} = \frac{\partial H}{\partial x};$$
$$\frac{\partial \lambda_2}{dt} = -\frac{\partial H}{\partial y}; \frac{\partial \lambda_3}{\partial t} = -\frac{\partial H}{\partial z} \qquad (30)$$

Solving from (29) and (30) then we get

$$\frac{\partial H}{\partial E_1} = 0 \Rightarrow \lambda_1 = e^{-\delta t}(p_1 - \frac{c_1}{xq_1}), \quad (31)$$
$$\frac{\partial H}{\partial E_2} = 0 \Rightarrow \lambda_3 = e^{-\delta t}(p_2 - \frac{c_2}{zq_2}) \quad (32)$$
$$\frac{\partial \lambda_1}{dt} = -\{(e^{-\delta t}p_1q_1E_1)$$
$$+\lambda_1(r_1 - \frac{2r_1}{K_1}x - \frac{\alpha z}{(1+mx)^2} - \sigma_1 - q_1E_1) \quad (33)$$
$$+\lambda_2\sigma_1 + \lambda_3\frac{k\alpha z}{(1+mx)^2}\}$$
$$\frac{\partial \lambda_2}{dt} = -\left\{\lambda_1\sigma_2 + \lambda_2(r_2 - \frac{2r_2}{K_2}y - \sigma_2)\right\}, \quad (34)$$
$$\frac{\partial \lambda_3}{dt} = -\{e^{-\delta t}p_2q_2E_2 - \frac{\lambda_1\alpha x}{(1+mx)^2}$$
$$+\lambda_3(-d + \frac{k\alpha x}{(1+mx)^2} - q_2E_2)\} \quad (35)$$

From (34) we get $\frac{\partial \lambda_2}{dt} - A_1\lambda_2 = -A_2e^{-\delta t}$, whose solution is given by

$$\lambda_2(t) = \frac{A_2e^{-\delta t}}{A_1 + \delta} \quad (36)$$

Where $A_1 = \frac{r_2}{K_2}y^* + \frac{\sigma_1x^*}{y^*}$, $A_2 = (p_1 - \frac{c_1}{q_1x^*})\sigma_2$

From (33) we get $\frac{\partial \lambda_1}{dt} - A_3\lambda_1 = -A_4e^{-\delta t}$, whose solution is given by

$$\lambda_1(t) = \frac{A_4e^{-\delta t}}{A_3 + \delta} \qquad (37)$$

Where $A_3 = \frac{r_1}{K_1}x^* + \frac{\sigma_2y^*}{x^*} + \frac{\alpha z^*mx^*}{(1+mx)^2}$,

$$A_4 = p_1q_1E_1 + \frac{A_2}{A_1+\delta} + (p_2 - \frac{c_2}{qz^*})\frac{k\alpha z^*}{(1+mx^*)^2}$$

From (31) and (37), we get the singular path

$$\frac{A_4e^{-\delta t}}{A_3+\delta} = (p_1 - \frac{c_1}{x^*q_1}) \quad (38)$$

Using

$$y^* = \frac{K_2}{2r_2}\left[r_2 - \sigma_2 + \left\{(r_2-\sigma_2)^2 + \frac{4r_2\sigma_1x^*}{K_2}\right\}^{1/2}\right]$$

and

$$z^* = \frac{\delta c_2q_1}{p_1q_1q_2(\delta + d - \frac{k\alpha x^*}{(1+mx)}) + \alpha q_2(p_1q_1x^* - c_1)},$$

$$A_1 = \frac{1}{2}(r_2-\sigma_2) + \frac{1}{2}\left[(r_2-\sigma_2)^2 + \frac{4r_2\sigma_1x^*}{K_2}\right]^{\frac{1}{2}}$$
$$+\sigma_1xr_2/[(r_2-\sigma_2) + \{(r_2-\sigma_2)^2 + \frac{4r_2\sigma_1x^*}{K_2}\}^{\frac{1}{2}}$$
$$A_3 = \frac{r_1}{K_1}x^* + \frac{\sigma_2K_2}{2r_2x^*}\{(r_2-\sigma_2) + [(r_2-\sigma_2)^2 + \frac{4r_2\sigma_1x^*}{K_2}]^{\frac{1}{2}}\}$$
$$A_4 = p_1q_1E_1 + (p_1 - \frac{c_1}{q_1x})\sigma_2/[1/2(r_2-\sigma_2)$$
$$+\frac{1}{2}\{(r_2-\sigma_2)^2 + \frac{4r_2\sigma_1x^*}{K_2}\}^{\frac{1}{2}}$$
$$+\sigma_1r_2x/(r_2-\sigma_2) + [(r_2-\sigma_2)^2 + \frac{4r_2\sigma_1x^*}{K_2}]^{\frac{1}{2}} + \delta]$$
$$+\frac{\delta k\alpha q_1c_2p_2}{(1+mx)}/[q_1c_2p_2(\delta + d - \frac{k\alpha x^*}{(1+mx)})$$
$$+(p_1q_1x^* - c_1)\alpha q_2] - \frac{k\alpha c}{q_2(1+mx)}$$

Thus (38) can be written as

$$F(x^*) = (p_1 - \frac{c_1}{x^*q_1}) - \frac{A_4e^{-\delta t}}{A_3+\delta} = 0.$$

There exists a unique positive root $x^* = x_\infty$ of $F(x^*) = 0$ in the interval $0 < x^* < K_1$, if the following inequalities hold :

$F(0) < 0$, $F(K_1) > 0$, $F'(x^*) > 0$ for $x^* > 0$. For $x^* = x_\infty$, we get $z^* = z_\infty$

Then we have

$$y_\infty = \frac{K_2}{2r_2}\left[r_2 - \sigma_2 + \left\{(r_2-\sigma_2)^2 + 4\sigma_1\frac{r_2c_1x}{K_2}\right\}^{1/2}\right]$$

$$E_{1\infty} = \frac{r_1}{q_1}(1 - \frac{x_\infty}{K_1}) - \alpha\frac{z_\infty}{q_1} - \frac{\sigma_1}{q_1} + \frac{\sigma_2y_\infty}{x_\infty}$$ And

$$E_{2\infty} = \frac{k\alpha x_\infty}{q_2(1+mx_\infty)} - \frac{d}{q_2}$$

Hence the optimal equilibrium ($x_\infty, y_\infty, z_\infty$) and optimal harvesting effort ($E_{1\infty}$ and $E_{2\infty}$) can be determined.

6. Numerical Simulation and Conclusion

If we consider the values of the parameters as

$$\begin{aligned} &r_1 = 3, r_2 = 1.6, K_1 = 100, K_2 = 150, \sigma_1 = 1.5, \\ &\sigma_2 = 1.4, \alpha = 8, q_1 = 0.5, q_2 = 0.3, E_1 = 1, E_2 = .5, . \\ &k = .01, d = .01, 0 < m < 10) \end{aligned}$$

We find the equilibrium point using the parameters; the following table shows the stability of system with the variation of m.

This table shows the bifurcation behavior of system (1) with m as the bifurcation parameter. Let us take m = 0.1 to 0.5, then the corresponding interior equilibrium point is unstable and we show that increase the value of m from 0.5 to 0.9 then, x(t) and z(t) are biomass densities of prey species and predator species inside the unreserved area, y(t) is the biomass density of prey species inside the reserved area are stable. We have discussed the local and global stability of the system, whether in the absence or in the presence of predators. The optimal harvesting policy has been also discussed.

Table 1: Prey predator density with m changing

m	x	y	z
0.1	2.50	37.44	17.22
0.2	3.33	39.04	24.44
0.3	5.00	42.24	41.032
0.4	0.16	29.18	0.8428
0.5	0.17	29.18	0.8467
0.6	0.17	29.05	0.8506
0.7	0.17	28.99	0.8544
0.8	0.17	28.93	0.8583
0.9	0.17	28.86	0.8621

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