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On Minima α Open Sets

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Abstract: ***In this paper we introduce and investigate a new class of sets in topological spaces called minima α-open sets.***

Keywords and phrases: minima topology, minima α-open set, minima topological spaces.

1. Introduction

In 1983, A. S. Mashhour et al.[1] introduced the supra topological spaces. In 1987, M. E. Abd El-Monsef et al[2] introduced the fuzzy supra topological spaces. Now, we introduce the concept of minima topology, minima α- open set and investigate some of the basic properties for this class of functions.

2. Preliminaries

All topological space considered in this paper lack any separation axioms unless explicitly stated. The topology of a space is denoted by τ and (X, τ) will be replaced by X if there is no chance for confusion. For a subset A of (X, τ), the closure and the interior of A in X with respect to τ are denoted by cl(A) and int(A) respectively. The complement of A is denoted by X – A.

Definition: 1
A subfamily τ * of X is said to be a minima topology on X if,
(i) X, Φ Є τ*
(ii) The intersection of the elements of any finite sub collection of τ * is in τ.

(X, τ*) is called a minima topological space. The elements of τ * are called minima open sets in (X, τ*) and complement of a minima open set is called a minima closed set and it is denoted by $(\tau^*)^c$.

Definition 2:
The minima closure of a set A is denoted by min cl(A) and defined as min cl(A)=∩{B: B is a τ minima open and A ⊇ B}

Definition 3:
Let (X,τ) be a topological space and τ* be a minima topology on X. we call τ* a minima topology associated with τ if τ*⊆ τ.

Definition 4:
Let (X,τ) be a minima topological space. A set A is called minima semi open set if A ⊆ min cl(min int (A)).

3. Basic Properties of Minima A-Open Sets

In this section we introduce one new class of sets.

Definition 5:
Let (X, τ *) be a minima topological space. A set A is called minima α-open set if A ⊆ min int(min cl(min int(A))).

Theorem 3.1:
Every minima open set is minima α-open set

Proof: Let A be a minima open set in (X, τ *). Since A ⊆ min cl(min int (A)), then min int(A) ⊆ min int(min cl(min int(A)))Therefore
A ⊆ min int(min cl(min int(A))).
Hence A is a minima α-open set.
The converse of the above theorem need not be true. This is shown by the following example.

Example 3.1:
let (X, τ *) be a minima topological space, where X={ϕ, X, {b}, {a,b}, {b,c}, {a,b,c}}. Here {b,d} is a minima α-open set, but not a minima open.

Theorem 3.2:
Every minima α-open set is minima semi-open set.

Proof:
Let A be a minima α-open set in (X, τ *). Therefore A ⊆ min int(min cl(min int (A))). It is obvious that, min int(min cl(min int(A))) ⊆ min cl(min int(A)).
Hence A ⊆ min cl(min int(A)).
The converse of the above theorem 3.2 is not usually true.

Example 3.2:
Let (X, τ *) be a minima topological space, where X = {a,b,c} and τ* = {Φ, X, {b}, {a,b}, {b,c}, {a,b,c}}. Here {b,c,d} is a minima semi-open set, but not a minima α-open set.

Theorem 3.3:
(i) Finite union of minima α open sets is always a minima α-open set.
(ii) Finite intersection of minima α-open sets may also a minima α-open set.

Proof:
(i) Let A and B be two minima α-open sets. Then A ⊆ min int(min cl(min int(A))) and B ⊆ min int(min cl(min int(A)))
By [1] this implies AUB ⊆ min int(min cl(min int(A))).
Therefore AUB is min α-open set.
(ii) Let (X, τ *) be a minima topological space. Where X = {a,b,c,d} and τ * = {Φ, X, {a,b}, {b,c}, {a,b,c}}. Here {b,c}, {a,b,c} are minima α-open set.

Definition 6:
Complement of a minima α-open set is a minima α-closed set.

Theorem 3.4
(i) Finite intersection of minima α-closed set is always a minima α-closed set.
(ii) Finite union of minima α-closed set may also minima α-closed set.

Proof :
i) This follows immediately from theorem 3.3
ii) Let (X, τ *) be a minima topological space. Where X = {a,b,c,d} and τ * = { X, Φ, {a,c,d}, {c,d}, {a,d},{d}}. Here {a,d}, {c,d} are minima α-closed sets but their union may also a minima α-closed set.

Definition 7:
The minima α-closure of a set A is denote by minima α cl(A) and defined as, minima α cl(A) = ∩ {B:B is a minima α-closed set and A⊆B}
The minima α- interior of a set is denoted by minima α int(A) = ∪ {B: B is a minima α-open set and A ⊇ B}.

Remark 3.1:
It is clear that minima α int(A) is a minima α-open set and minima α cl(A) is a minima α-closed set.

Theorem 3.5:
(i) X – minima α int(A) = minima α cl(X - A)
(ii) X - minima α cl(A) = minima α int(X - A)

Proof:
(i) and (ii) are clear.

Theorem 3.6:
The following statements are true for every A and B.
(i) Min α int(A) ∪ min α int(B) = min α int(A ∪ B)
(ii) Min α cl(A) ∩ min α cl(B) = min α cl(A ∩ B)

Proof: Obvious.

References

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