

# Long Memory Volatility of Stock Markets of India and China

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**Abstract:** *The paper examines the existence of long memory in volatility of stock markets of India and China using FIGARCH models. The data set consists of daily return of BSE and SSE stock indices from January 1, 2009 to June 24, 2014 and long memory tests are carried out for the volatilities of these series. The results of FIGARCH model indicate strong evidence of long memory in conditional variance of the stock indices. The long memory property of the BSE market is revealed to be stronger than SSE. JEL classification: C22, C50*

**Keywords:** long memory, FIGARCH

## 1. Introduction

Long memory dynamics are important pointers for identifying presence of nonlinear relations in conditional mean and variance of financial time series. The modelling of stock market volatility has been a considerable field of research after the introduction of ARCH and GARCH classes of models by Engle(1982) and Bollerslev(1986). It has been found that stock market volatility is time varying and exhibits positive serial correlation (volatility clustering). This implies that changes in volatility are non-random. However, these models do not account for long memory in volatility. The long memory is often found in conditional mean and variance of a financial time series at the same time. Slow mean-reverting at hyperbolic rate decay in autocorrelation functions of return and volatility is defined as long memory in return and volatility. Based on this idea, the empirical research has focussed on analysing dual long memory property in conditional mean and variance (Baillie, Han and Kwon, 2002, Beine, Laurent and Lecourt, 2003). The empirical findings act as evidence for presence of long memory in return and volatility of returns.

Modelling long memory properties in stock market return and volatility has become an interesting research area in recent years. The existence of long memory in returns and volatility suggests the presence of dependencies among observations. Long memory in these series is related with the high autocorrelation function which decays hyperbolically and finally dies out (Kasman, Kasman and Torun, 2009). In contrast, if correlation between distant observations is negligible, the series possesses short memory and exhibits exponential decaying observations.

Granger and Joyeux(1980) and Hosking(1981) found that fractionally integrated series could capture long memory property and proposed fractionally integrated autoregressive moving average model. It is characterized by hyperbolic decaying of autocorrelation function. The model has been used extensively in the literature to investigate the presence of long memory in stock market returns (Lo 1991; Jacobson 1996; Crato and Lima 1994 and Tolvi 2003). Besides numerous studies examine long memory in stock return, the long memory in volatility has also been investigated (Ding and Granger 1996; Lobato and Savin 1998; Comte and

Renaut 1998 and Andreano 2005). They showed that the autocorrelation function of the squared daily return decay very slowly. Baillie et al.(1996) developed fractionally integrated generalized conditional heteroscedasticity (FIGARCH) model to allow for fractionally integrated process of conditional variance.

The primary aim of this paper is to investigate the long memory property in the volatility of Stock Markets of India and China using FIGARCH model. FIGARCH model can provide a useful way of examining the relationship between conditional variance of a process exhibiting the long memory property (Kang and Yoon, 2007). Moreover, these models offer greater flexibility to analyze long memory property in volatility with fractionally differencing process (See Kasman, Kasman and Torun, 2009).

The rest of the paper is organized as follows. Section 2 discusses FIGARCH model. Section 3 provides the statistical properties of data, the estimation results of the FIGARCH models. Section 4 summarizes.

## 2. Methodology

### 2.1 FIGARCH model

Nelson (1990b) explains that the specification of mean equation bears a little impact on ARCH models when estimated in continuous time. We follow a classical approach of assuming the first order autoregressive structure for conditional mean as follows:

$$R_t = a_0 + a_1 R_{t-1} + \varepsilon_t \quad \text{Equation 1}$$

where  $R_t$  is a stock return,  $a_0 + a_1 R_{t-1}$  is a conditional

mean and  $\varepsilon_t$  is the error term in period t. The error term is further defined as:

The extension of the ARMA representation in squared errors ( $\varepsilon^2$ ) is FIGARCH model of Baillie et al. The FIGARCH (p,d,q) can be expressed as follows:

$$\phi(L) (1 - L)^d \varepsilon^2 = \omega + [1 - \beta(L)] v_t \quad \text{Equation 2}$$

$V_t = \varepsilon_t^2 - \sigma_t^2$  is mean zero serially uncorrelated error,  $\varepsilon_t^2$  is the squared error of the GARCH process. The  $\{V_t\}$  process is integrated as the “innovations” for the conditional variance ( $\sigma_t^2$ ). If  $d=0$ , the FIGARCH (p, d, q) process reduces to a GARCH (p,q) process and if  $d=1$ , the FIGARCH process becomes an integrated GARCH process. Rearranging the terms in Eq.(3), one can write the FIGARCH model as follows:

$$[1 - \beta(L)]\sigma_t^2 = \omega + [1 - \beta(L)(1 - L)^d]\varepsilon_t^2.$$

**Equation 3**

The conditional variance equation of  $\varepsilon_t^2$  is obtained by:

$$\sigma_t^2 = \frac{\omega}{[1 - \beta(L)]} + \left[1 - \frac{\phi(L)}{[1 - \beta(L)]}(1 - L)^d\right]\varepsilon_t^2$$

**Equation 4**

That is

$$\sigma_t^2 = \frac{\omega}{[1 - \beta(1)]} + \lambda(L)\varepsilon_t^2$$

**Equation 5**

where  $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + \dots$ . Baillie et al. (1996) mention that the impact of a shock on conditional variance of FIGARCH(p,d,q) processes decrease at a hyperbolic rate when  $0 \leq d \leq 1$ . Hence, the long term dynamic is taken into account by the fractional integrated parameter d and the short dynamic is captured through traditional GARCH model parameters.

Baillie et al (1996) through simulations demonstrated that Quasi maximum likelihood (QMLE) estimation method performs better in case of high frequency financial data. Therefore, we use QMLE method to estimate the results of ARFIMA-FIGARCH model.

### 3. Empirical Results

#### 3.1 Preliminary Analysis of Data

We consider daily returns of Sensex of BSE and SSE of China. The dataset consists of daily closing prices starting from January 1, 2009 to June 24, 2014 covering 1359 observations. The daily stock returns are defined as logarithmic difference of the daily closing price of respective indices. The descriptive statistics of these two indices are reported in Table 1.

**Table 1:** Descriptive statistics of sample return series

Descriptive Statistics	BSE	SSE
Mean	0.000706	0.00006
Standard Deviation	0.013416	0.013327
Skewness	1.141962	-0.372981
Kurtosis	18.95906	5.461300
Jarque-Bera	14706.45	374.2680
Q(20)	35.56(0.00)	22.18
Qs(20)	81.67(0.00)	294.69(0.00)

All the return series reveal that they do not correspond with normal distribution assumption. Jarque-Bera statistics

suggest that there are significant departures from normality. We examine the null hypothesis of white noise using the Box-Pierce statistics of the return residuals(Q(20) and squared return residuals Qs(20)). From the results, we certainly reject the null hypothesis of white noise. It also indicates that the series are autocorrelated.

#### 3.2 Unit Root Tests

Before investigating the long memory in return and volatility, we check the series for a presence of unit root. We have employed three unit root tests--ADF(Augmented Dickey Fuller), PP(Philips-Peron) and KPSS(Kwiatkowski, Phillips, Schmidt and Shin) to determine if the individual return series are stationary or not. Three tests differ in the null hypothesis. The null hypothesis of the ADF and PP test is that a time series contains unit root while KPSS test has the null hypothesis of stationarity. The empirical results of all the three tests are presented in Table 2.

**Table 2:** Unit Test Results

Test	BSE	SSE
ADF	-34.59(0.000)*	-36.52(0.00)
PP	-34.55(0.000)*	-36.54(0.00)
KPSS	0.183**	0.333

Notes: \* Mackinnon's 1% critical value is -3.435 for ADF and PP tests.

\*\* A KPSS critical value is 0.739 at 1% significant level.

Large negative values for ADF and PP tests for both return series reject the null hypothesis of a unit root at the 1% significant level. Additionally, the statistics of the KPSS test indicate that return series are stationary. Thus both the series are stationary and suitable for subsequent long memory tests in this study.

#### 3.3 Estimation results of FIGARCH models

We used FIGARCH(1,1) model for variance equation. The results of the models are presented in Table 4. The estimated model is tested for its adequacy using ARCH-LM test. The parameters Q- Statistics on standardized residuals and squared standardized residuals are statistically insignificant. The insignificant value of ARCH(5) test also implies that the model is a good fit. The long run memory is captured by parameter d.

**Table 3:** Estimation Results of FIGARCH models

(p,d,q)	BSE	SSE
$\omega$	0.044 (0.72)	0.104 (0.29)
$\beta_1$	0.794 (0.00)	0.589 (0.00)
$d$	0.669 (0.00)	0.295 (0.00)
$\phi_1$	0.129 (0.13)	0.283 (0.02)
Q(20)	13.3 (0.82)	19.05 (0.45)
Qs(20)	6.875 (0.99)	31.48 (0.96)
ARCH(5)	0.131 (0.99)	0.20 (0.31)

Notes: P(60) is the Pearson goodness-of-fit statistic for 60 cells. The ARCH(5) and P(60) tests are computed on the standardized residuals. \* and \*\* indicate rejection at 1% and 5% significance level, respectively.

It is statistically significant in the both markets. The parameter value is high in Indian stock. Comparing the degree of parameter  $d$  between the BSE and SSE stock markets, the long memory property in volatility in SSE market is less than that BSE.

#### 4. Conclusions

The study examined the long memory property in the Indian stock markets. We investigate the long memory property in conditional variance series of stock markets of India and China using FIGARCH model. The results suggest that there is a prevalence of long memory property in volatility of Indian stock markets. In addition, the long memory property of volatility in the BSE market is revealed to be much stronger than SSE. The evidence of long memory in volatility, however, shows that uncertainty or risk is an important determinant of the behaviour of daily stock data in India and China.

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