A Study on Multi Server Queuing Simulation

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Abstract: ***There are occasions where the physical system or real life system becomes rather complex to build a mathematical model. Even if the mathematical model can be constructed with a reasonable degree of accuracy, available quantitative techniques may not be amenable to analyze the system. Simulation can be a valuable tool to get an answer to an engineering problem. Simulation is used to quantify this impact. We develop a Multi-channel queuing system, based on the simulation techniques which can be used to evaluate, a mathematical model of some real system is manipulated and the results and pertinent information are observed. These manipulations and findings are then used to make inferences about the real system. If the model involves random sampling from a probability distribution, the procedure is called Monte Carlo Simulation.***

Keywords: Mathematical model, Quantitative model, Queuing simulation, Multi server model, Probability distribution, Monte Carlo Simulation.

1. Introduction

Simulation is the imitation of reality it is modelled. It is used in performance optimization, safety engineering, testing, training, education, and video games. The main purpose of simulations is to build a scientific model of natural systems or human systems to gain insight into their functioning. [1] Simulation can be used to show the eventual real effects of alternative conditions and courses of action. Mainly it's used when the real system cannot be engaged, because it may not be accessible, or it may be dangerous or unacceptable to engage, or it is being designed but not yet built, or it may simply not exist. [3]

It's an important tool for tackling the complicated problems of managerial decision-making, called as management laboratory because to determine the effect of a number of alternate policies without disturbing the real system and to select the best policy with the prior assurances.

Mr. John Von Neumann and Stanislaw Ulam [5] were given the first important application in the behavior of neutrons in a nuclear shielding problem. With this remarkable success, it became very popular and it's a base for many applications in business and industry Development. The application of this technique varies from simple queuing models to models of large integrated systems of production.

The best examples for simulation studies are scale models of airplanes tested in wind tunnels, a mechanical circuit, the reactions of a certain medicine with animals and then to administer the same to human beings, to build a laboratory model, inventory, queuing, scheduling and forecasting.

In probability theory, simulation means numerical construction of samples of a random process. Monte Carlo simulation is a computerized mathematical technique. In this we need to generate random numbers to obtain random observations from probability distributions, and it's a sequence of numbers whose probability of occurrence is the same as that of any other number in the sequence. We used random phenomena in waiting line model; the arrival rate and the service rate are usually probabilistic rather than deterministic.

Queuing theory originated in telephony with the work of Erlang [4]. His pioneering work stimulated many authors to develop a variety of queuing models. And it's considered a branch of operations research because the results are used when making business decisions with available resources needed to provide service. Main application in customer service situation like, intelligent transportation systems, call enter, networks, telecommunications, server queuing, mainframe computer of telecommunications terminals, advanced telecommunications systems, and traffic flow.

Queuing models provide s analyst with a powerful tool for designing and evaluating the performances of queuing systems [2], [7].Multi server during queuing system, two or more channels (or servers) are available to handle customers who arrive for service for example, there may be more than one runway at an airport for takeoff and landing, there may be more than one doctor in a hospital, and there may be more than one teller in a bank. Commonly it's having a parallel station serving a single queue on a first-come first-served basis.

One of the important applications of simulation is the analysis of waiting line problems and its classified into Analogue model, Continuous model, discrete model. Simulation can be used as a pre-service test to try out new policies and decision rules for operating a system before running the risk of experimentation in the real system and it's the only 'remaining tool' when all other techniques become fail. It is the trial and error approach that produces different solutions in repeated trials. This means it gives only the optimum solution to the industrial problems. The simulation model does not produce answers by itself. The user has to provide all the constraints for the solutions which he wants to examine.

In this paper we analyze the multichannel queuing system through simulation. The purpose of this study is to review Queuing theory and its analysis based on the data from a hospital has given an appointment to consulting a doctor.

The patient's arrival and service time probability distribution as follows:

Arrival (min)	*Probability*	*Service time (min)*	*Probability*
1.0	0.30	1.0	0.20
2.0	0.25	1.5	0.35
3.0	0.20	2.0	0.25
4.0	0.15	2.5	0.15
5.0	0.10	3.0	0.05

The patient's arrival at the hospital is a random phenomenon and the time between arrivals varies from one minute to five minutes. The service time also varies from one minute to three minutes. The queuing process begins at 6.00am and proceeds for nearly 2 hours. If the attendant's wages are Rs.120 per hour and the patient's waiting time costs Rs.150 per hour, would it be economical to engage a second attendant? Now we use a Monte Carlo Simulation, its uses the mathematical models to generate random variables for the artificial events and collect observations [6]. The aim of studying queuing system simulation is trying to find the arrival events are separated by the 'inter-arrival time' and the departure events are specified by the service time in the facility. The fact that these events occur at discrete point is known as "Discrete – event Simulation". [9]Table 1.1, 1.2 gives respectively the tag number tables for arrival time and service time distribution.

Table 1.1

Arrival Time (min)	Probability	Cumulative Probability	Tag No.
1	0.3	0.3	00-29
2	0.25	0.55	30-54
3	0.2	0.75	55-74
4	0.15	0.9	75-89
5	0.1	1	90-99

Table 1.2

Service Time (min)	Probability	Cumulative Probability	Tag No.
1	0.2	0.2	00-19
1.5	0.35	0.55	20-54
2	0.25	0.8	55-79
2.5	0.15	0.95	80-94
3	0.05	1	95-99

The following table gives the detailed simulation of multichannel queuing model with 2 servers.

Table 1.3

Trail No.	*Random No. (Arrival Time)*	*Inter Arrival Time (Minutes)*	*Random No. (Service Time)*	*Service Time (Minutes)*	*Clock Arrival Time*	*Server – 1*		*Server – 2*		*Customer Waiting time*	*Waiting time*		*Queue length*
						Service begins at	*Service ends at*	*Service begins at*	*Service ends at*		Server. 1	Server. 2	
1.	17	1	45	1.5	6.01	6.01	6.02½	-	-	-	1	-	-
2.	17	1	96	3	6.02	-	-	6.02	6.05	-	-	2	-
3.	77	4	33	1.5	6.06	6.06	6.07½	-	-	-	3½	-	-
4.	66	3	83	2.5	6.09	6.09	6.11½	-	-	-	1½	-	-
5.	14	1	77	2	6.1	-	-	6.1	6.12	-	-	5	-
6.	68	3	5	1	6.13	6.13	6.14	-	-	-	1½	-	-
7.	26	1	15	1	6.14	6.14	6.15	-	-	-	-		-
8.	85	4	40	1.5	6.18	6.18	6.19½	-	-	-	3	-	-
9.	11	1	43	1.5	6.19	-	-	6.19	6.20½	-	-	7	-
10.	16	1	34	1.5	6.2	6.2	6.21½	-	-	-	½	-	-
	26	1	44	1.5	6.21	-	-	6.21	6.22½	-	-	½	-
	95	5	89	2.5	6.26	6.26	6.28½	-	-	-	4½	-	-
	67	3	20	1.5	6.29	6.29	6.30½			-	½		-
	97	5	69	2	6.34	6.34	6.36	-	-	-	3½	-	-
	73	3	31	1.5	6.37	-	-	6.37	6.38½	-	-	14½	-
	75	4	97	3	6.41	6.41	6.44	-	-	-	5	-	-
	64	3	5	1	6.44			6.44	6.45	-	-	5½	-
	26	1	59	2	6.45	6.45	6.47			-	1		-
	45	2	2	1	6.47			6.47	6.48	-		2	-
	1	1	35	1.5	6.48	6.48	6.49½			-	1		-
	87	4	73	2	6.52	6.52	6.54	-	-		-	-	-
	20	1	21	1.5	6.53	-	-	6.53	6.54½	-	-	5	-
	1	1	45	1.5	6.54	6.54	6.55½	-	-	-	-	-	-
	19	1	76	2	6.55	-	-	6.55	6.57	-	-	½	-
	36	2	96	3	6.57	6.57	7	-	-	-	1½	-	-
	45	2	94	2.5	6.59	-	-	6.59	7.01½	-	-	2	-
	41	2	53	1.5	7.01	7.01	7.02½	-	-	-	1	-	-
	96	5	57	2	7.06	7.06	7.08	-	-	-	3½	-	-
	71	3	96	3	7.09	7.09	7.12			-	1	-	-
	98	5	43	1.5	7.14	7.14	7.15½	-	-	-	2	-	-
	77	4	65	2	7.18	-	-	7.18	7.2	-	-	16½	-
	80	4	82	2.5	7.22	7.22	7.24½	-	-	-	6½	-	-
	52	2	91	2.5	7.24	-	-	7.24	7.26½	-	-	4	-
	31	2	3	1	7.26	7.26	7.27	-	-	-	1½	-	-
	87	4	26	1.5	7.3	-	-	7.3	7.31½	-	-	3½	-
	58	3	61	2	7.33	7.33	7.35	-	-	-	6	-	-

	45	2	54	1.5	7.35	-	-	7.35	7.36½	-	-	3½	-
	43	2	77	2	7.37	7.37	7.39	-	-	-	2	-	-
	36	2	13	1	7.39	-	-	7.39	7.4	-	-	2½	-
	46	2	93	2.5	7.41	7.41	7.43½	-	-	-	2	-	-
	46	2	86	2.5	7.43	-	-	7.43	7.45½	-	-	3	-
	70	3	18	1	7.46	7.46	7.47	-	-	-	2½	-	-
	32	2	66	2	7.48	7.48	7.5			-	1		-
	12	1	59	2	7.49			7.49	7.51	-		3½	-
	40	2	1	1	7.51	7.51	7.52			-	1	-	-
	51	2	39	1.5	7.53			7.53	7.54½	-		2	-
	59	3	88	2.5	7.56	7.56	7.58½			-	4	-	-
	54	2	25	1.5	7.58			7.58	7.59½	-	-	3½	-
	16	1	74	2	7.59	7.59	8.01			-	½	-	-
	68	3	5	1	8.02	8.02	7.03	-	-	-	1	-	-
				90.½									

2. Inference

(I). Average length of queue = $\frac{\text{Number of customers in waiting line}}{\text{Number of arrivals}} = 0$

(II). Average waiting time of customer $= \frac{\text{Customer waiting time}}{\text{Number of arrivals}} = 0$

(III). Average service time $= \frac{\text{Total service time}}{\text{Number of arrivals}} = 1.8$ minutes

(iv). Time a customer spends in the system =Average service time + Average waiting time before the service = 1.8 +0 = 1.8 minutes.

(v). Cost with a Multiserver = Cost of waiting time of a customer + cost of the server = 0×150+2×120 = Rs.240/-

3. Conclusion and Discussion

This paper reviews a queuing model for multiple servers. The main idea of this study is to measure the expected queue length in each server. The second idea is to gives a view of queuing process and running the simulation experiments to obtain the required statistical results. This method infers that the multichannel queuing model is better model, since there is no waiting time for customers and hence there is no queue length. So this model is costlier than the single channel queuing model. The overall scope of future research is to balance the cost of waiting with the cost of adding more resources by using the programming languages and to simplify the design of simulation programs.

References

[1] ^ *a* *b* In the words of the Simulation article in Encyclopedia of Computer Science, "designing a model of a real or imagined system and conducting experiments with that model".

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