A Fuzzy Mathematical Model for the Effect of Gastrin in Humans

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Abstract: *Gastrin is a potent stimulant of lower esophageal sphincter pressure in human. Injections of gastrin produce a marked increase in lower esophageal sphincter pressure. Both gastrin-17 and gastrin-34 doses significantly increased lower esophageal sphincter pressure. In this paper, we developed a mathematical model for the effects of gastrin-17 and gastrin-34 on lower esophageal sphincter pressure in humans. We calculated the reliability of lower esophageal sphincter pressure on 100pmol/kg doses of gastrin-17 and gastrin-34 using Weibull distribution in fuzzy parameter. Also we calculate the reliability using three parameter Weibull distributions and compared the result with two parameters. The result shows that the alpha cut for the reliability of the effect of gastrin-34 on lower esophageal sphincter pressure is comparatively higher than the effect of gastrin-17 on LESP.*

Keywords: Reliability Analysis, Fuzzy Weibull distributions, Gastrin, LESP

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1. Introduction

One of the important disciplines in engineering is reliability analysis, which is used in the design and development of technical system. Reliability is the probability that the system component will not fail to perform within specified time. Two fundamental assumptions on usual reliability are the system is precisely defined as success or failure and the system behaviour is fully characterized in terms of probability measure.

Due to uncertainties and imprecision of data in the most of the system, the estimation of exact values of probabilities are often difficult. Many researchers focused on using fuzzy set theory for fuzzy system reliability analysis. Cai et al [1], [2], [3] modified the assumptions of the system is precisely defined as success or failure to Fuzzy state assumption: At any time, the system may be either in the fuzzy success state or fuzzy failure state Possibility assumption: The system behaviour can be fully characterized by possibility measure. Cai[1] presented an introduction to system failure and its use of fuzzy methodology. In [2], [3] a method for fuzzy system reliability analysis using fuzzy number was presented. Chen S.M [4] presented a method for fuzzy system reliability analysis and alpha cuts operations of fuzzy numbers. Karpisek et al. [6] described fuzzy reliability function models that are based on Weibull distribution. In [5], Utkin et al presented a system of functional equations for fuzzy reliability analysis of various systems.

The existence of gastrin was firmly established in the early 1960s. Gastrin is the best-studied gastrointestinal hormone. There are presently three known biologically active forms of the gastrin molecule gastrin-14, gastrin-17 and gastrin-34. Gastrin-17 accounts for about 95% of the gastrin in antral G cells; gastrin-34 predominates in serum. Gastrin-17 is presumed to be the physiologically most important form of gastrin. Gastrin is a potent stimulant of lower esophageal sphincter pressure in human [10]. Gastrin has physiological role in the control of sphincter strength [9]. Equimolar rapid injections of synthetic human gastrin-17 and gastrin-34 in doses of 100 pmol/kg resulted an increase in lower esophageal sphincter pressure.

2. Notation

λ - scale parameter
β - shape parameter
t - test termination time
p - probability of failure
$\bar{\lambda}[\alpha]$ - alpha cut of scale value
$\bar{\beta}[\alpha]$ - alpha cut of shape value
$\bar{R}[t]$ - reliability function in fuzzy parameter

3. Fuzzy Weibull Distribution

The Weibull distribution is widely used in statistical model for life data. Among all statistical techniques it may be in use for engineering analysis with smaller sample sizes than any other method. A continuous random variable T with two parameter Weibull distribution $W(\beta, \lambda)$ Where $\beta > 0$ is the shape parameter, $\lambda > 0$ is the scale parameter has the probability density function

$$f(t, \lambda, \beta) = \beta \lambda^{-\beta} t^{\beta-1} e^{-\left(\frac{t}{\lambda}\right)^{\beta}}, \quad t \geq 0, \lambda \geq 0, \beta \geq 0$$

and the reliability function is $R(t) = e^{-\left(\frac{t}{\lambda}\right)^{\beta}}$

The three parameter Weibull distribution has the probability density function

$$f(t) = \beta \lambda^{-\beta} (t - \gamma)^{\beta-1} e^{-\left(\frac{t-\gamma}{\lambda}\right)^{\beta}},$$
$$t \geq 0, \lambda \geq 0, \beta \geq 0, \gamma > 0$$

and the reliability function is $R(t) = e^{-\left(\frac{t-\gamma}{\lambda}\right)^{\beta}}$

The shape parameter gives the flexibility of Weibull distribution by changing the value of shape parameter. However sometimes we face situations when the parameter is imprecise. Therefore we consider the Weibull distribution with fuzzy parameters by replacing the scale parameter λ into the fuzzy number $\overline{\lambda}$ and shape parameter β into $\overline{\beta}$.

If a random variable T has a crisp Weibull distribution $W(\beta, \lambda)$ then the corresponding fuzzy random variable $\overline{T}$ with fuzzy Weibull distribution $W(\overline{\beta}, \overline{\lambda})$ has reliability function $\overline{R}(t) = e^{-\left(\frac{t}{\lambda}\right)^{\beta}}$

4. Fuzzy Mathematical Model

The α- cut of a fuzzy number $\overline{A}$ is non –fuzzy set defined as
$M[\alpha] = \{x \in R : \mu_A(x) \geq \alpha\}$.
Hence we have $M[\alpha] = [M_{\alpha_1}, M_{\alpha_2}]$

The interval of confidence defined by alpha cuts can be written as
$M[\alpha] = [(a_2 - a_1)\alpha + a_1, (a_3 - a_2)\alpha + a_3]$.

For $\alpha \in [0,1]$, the alpha cuts of fuzzy Weibull reliability function is
$\overline{R}[\alpha] = [R_1[\alpha], R_2[\alpha]]$
where

$R_1[\alpha] = \inf\{e^{-\left(\frac{t}{\lambda}\right)^{\beta}}, \lambda \in \overline{\lambda}[\alpha] \ \beta \in \overline{\beta}[\alpha]\}$

$R_2[\alpha] = \sup\{e^{-\left(\frac{t}{\lambda}\right)^{\beta}}, \lambda \in \overline{\lambda}[\alpha] \ \beta \in \overline{\beta}[\alpha]\}$

The three parameter fuzzy Weibull distribution has reliability function $\overline{R}(t) = e^{-\left(\frac{t-\gamma}{\lambda}\right)^{\beta}}$

For $\alpha \in [0,1]$, the alpha cuts of fuzzy Weibull reliability function corresponding to three parameters is
$\overline{R}[\alpha] = [R_1[\alpha], R_2[\alpha]]$
where

$R_1[\alpha] = \inf\{e^{-\left(\frac{t-\gamma}{\lambda}\right)^{\beta}}, \lambda \in \overline{\lambda}[\alpha] \ \beta \in \overline{\beta}[\alpha]\}$

$R_2[\alpha] = \sup\{e^{-\left(\frac{t-\gamma}{\lambda}\right)^{\beta}}, \lambda \in \overline{\lambda}[\alpha] \ \beta \in \overline{\beta}[\alpha]\}$

5. Application

Let us consider an example for the effect of lower esophageal sphincter pressure on synthetic gastrin in human. Increases in lower esophageal sphincter pressure were prompt in onset, occurring 2-6 min after gastrin-34 and 1-3min after gastrin-17 injections. Maximal increases in lower esophageal sphincter pressure were similar in amplitude, 18 mmHg after gastrin-34 and 24 mmHg after gastrin-17. Figure 5.1 shows the responses to 100 pmol/kg doses of gastrin-17 and gastrin-34. Both gastrin-17 and gastrin-34 doses significantly increased lower esophageal sphincter pressure. The response to gastrin-34 was more prolonged compared with gastrin-17. Lower esophageal sphincter pressure is returned to baseline by 15 min after gastrin-17 injection, whereas lower esophageal sphincter pressure

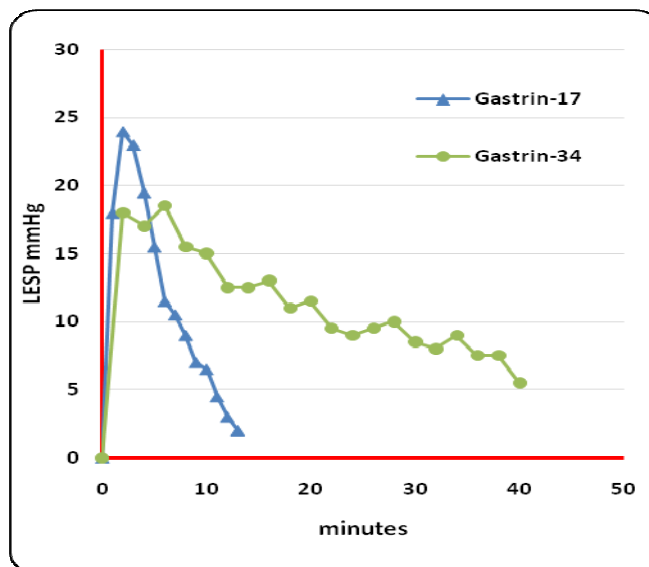values were just approaching baseline 45 min after gastrin-34 injection.

Figure 5.1: LES response over time to equimolar 100 pmol/kg doses of gastrin-17 and gastrin-34 given as rapid intravenous injections in a human.

5.1 Solution by fuzzy Weibull distribution with two parameters

From Figure 5.1, the scale and shape parameter for the effect of gastrin-17 on LESP respectively are 13.42 and 1.46. Similarly the scale and shape parameter of gastrin-34 are 12.61 and 3.71. The fuzzy reliability of the effect of gastrin on LESP for various values of alpha is shown in table 5. 1.

Table 5.1: Reliability for the effect of gastrin – 17 and gastrin – 34 on lower esophageal sphincter pressure using fuzzy Weibull distribution with two parameters
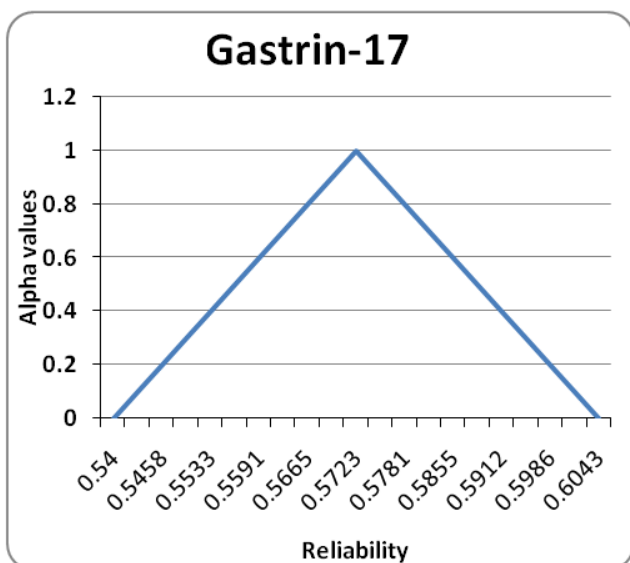
α	Gastrin-17		Gastrin-34	
	$R_1[\alpha]$	$R_2[\alpha]$	$R_1[\alpha]$	$R_2[\alpha]$
0	0.54	0.6043	0.6939	0.801
0.1	0.5425	0.602	0.6998	0.7964
0.2	0.5458	0.5986	0.7064	0.7913
0.3	0.5495	0.5949	0.7121	0.7866
0.4	0.5533	0.5912	0.7178	0.7818
0.5	0.5554	0.5892	0.7234	0.777
0.6	0.5591	0.5855	0.7297	0.7715
0.7	0.5628	0.5818	0.7345	0.7672
0.8	0.5665	0.5781	0.7399	0.7621
0.9	0.5686	0.576	0.7459	0.7564
1	0.5723	0.5723	0.7512	0.7512

5.2 Solution by fuzzy Weibull distribution with three parameters

From Figure 5.1, the scale, shape and location parameter for the effect of gastrin – 17 on LESP respectively are 12.83, 1.34 and 0.5. Similarly the scale, shape and location parameter for gastrin-34 are 8.62, 2.21 and 3.81. The fuzzy reliability of the effect of gastrin on LESP for various values of alpha are shown in table 5. 2

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Table 5.2: Reliability for the effect of gastrin – 17 and gastrin – 34 on lower esophageal sphincter pressure using Weibull fuzzy distribution with three parameters
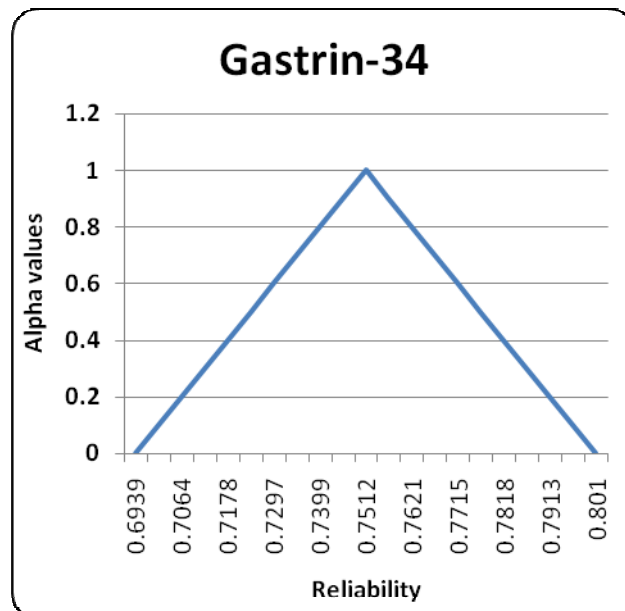
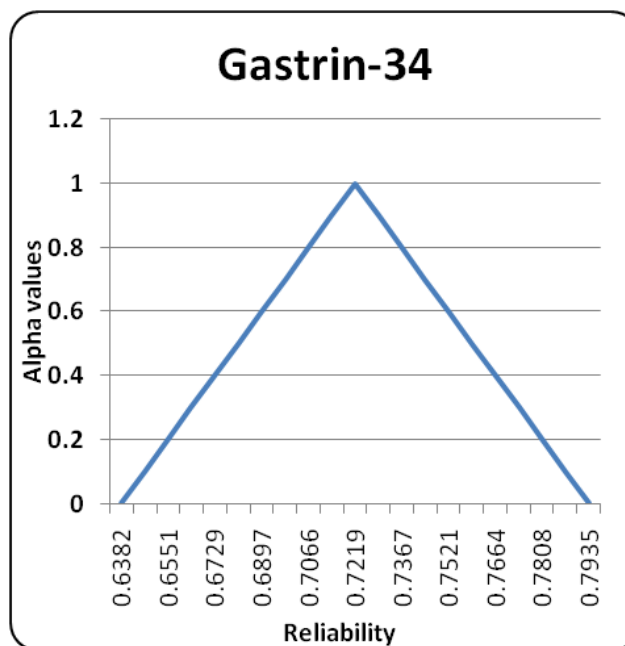| α	Gastrin-17		Gastrin-34	
	$R_1[\alpha]$	$R_2[\alpha]$	$R_1[\alpha]$	$R_2[\alpha]$
0	0.5016	0.6221	0.6382	0.7935
0.1	0.5073	0.6165	0.6463	0.7875
0.2	0.5127	0.6113	0.6551	0.7808
0.3	0.5197	0.6043	0.665	0.7729
0.4	0.5254	0.5987	0.6729	0.7664
0.5	0.5312	0.593	0.6814	0.7594
0.6	0.5382	0.586	0.6897	0.7521
0.7	0.544	0.5803	0.6973	0.7454
0.8	0.5494	0.5749	0.7066	0.7367
0.9	0.5564	0.5679	0.714	0.7296
1	0.5622	0.5622	0.7219	0.7219

Figure 5.2: alpha cut for the reliability for the effect of gastrin – 17 on LESP using Weibull distribution with two parameters.
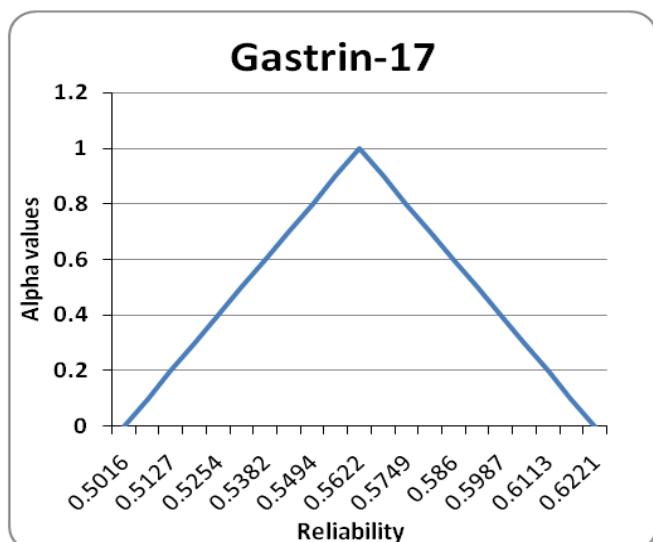
Figure 5.3: alpha cut for the reliability of the effect of gastrin – 17 on LESP using Weibull distribution with three parameters.

Figure 5.4: alpha cut for the reliability of the effect of gastrin – 34 on LESP using Weibull distribution with two parameters.

Figure 5.5: alpha cut for the reliability of the effect of gastrin – 34 on LESP using Weibull distribution with three parameters.
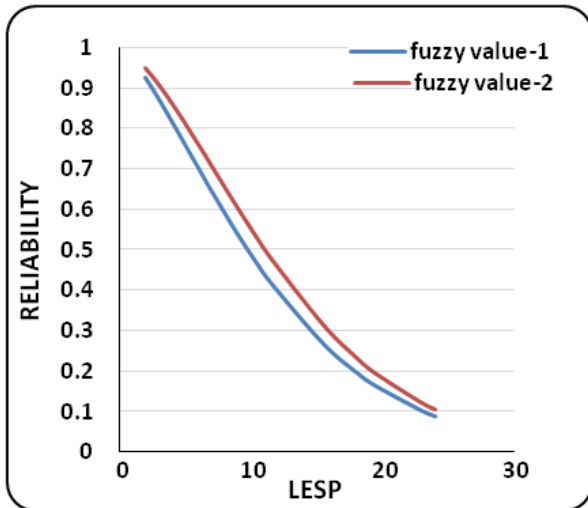
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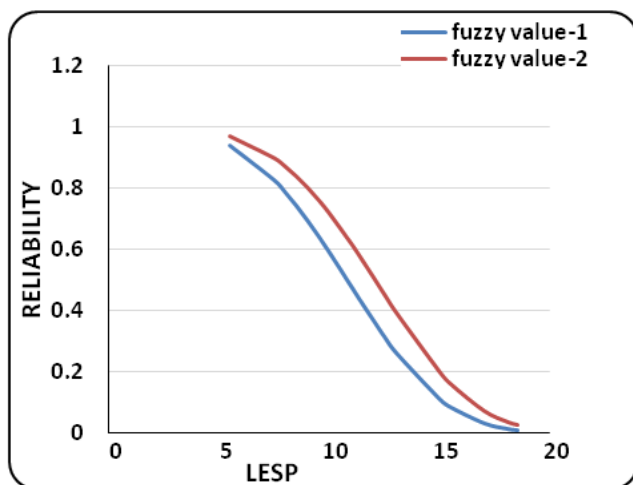**Figure 5.6:** reliability curve for the effect of Gastrin – 17 on LESP

Figure 5.7: reliability curve for the effect of Gastrin – 34 on LESP

6. Conclusion

In this paper, we have reported that the comparative study for the reliability for the effect of gastrin-17 and gastrin-34 on lower esophageal sphincter pressure using Weibull fuzzy distribution with two and three parameters. From the table it is clear that the alpha cut for the reliability for the effect of gastrin-34 on lower esophageal sphincter pressure is comparatively higher than the effect of gastrin-17 on LESP. We hope that this work may be used to analyze the reliability for the effect of gastrin on lower esophageal sphincter pressure in humans. Since our model is a time bounded, it will be supportive to medical professionals to improve the diagnosis of their patients by change in the time of doses with due consideration of the hormone gastrin.

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Author Profile

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