Study of Five Dimensional Models in Kaluza-Klein Space-Time in the Frame Work of Saez and Ballester

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Abstract: ***In this paper we discuss five dimensional spherically sysmetric space-time in the presence of cosmic string source in the frame work of scalar tensor theory of gravitation proposed by Saez and Ballester. Exact cosmological models which represents Geometric strings, P-strings and Reddy strings are studied. Some physical and kinematical properties of the models are also taken into account.***

Keywords: Five dimensional cosmological models, scalar-tensor theory

1. Introduction

As the evolving early universe was much smaller than today, the present four dimensional space-time of the universe could have been preceded by higer dimensional space-time. The five dimensional space-time is particularly attractive because both 10D and 11D super gravity theories admit solutions which spontaneously reduced to 5D. At the very early stages of the universe, it is generaly assumed that during the phase transition, as the universe passes through its critical temperature the symmetry of the universe is broken spontaneously. It is still a challenging problem to know the physical situation at very early stages of formation of universe. The present day configuration of the universe are not contradictory by the large scale network of the early universe.

In recent years, the scalar-tensor theories of gravitation proposed by Brans and Dicke (1961), Nordtvedt (1970), Ross (1972) and Donn (1974). Saez and Ballester (1985) have formulated a scalar -tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This theory also suggests a possible way to solve missing matter problem in non-flat FRW cosmologies.

The study of higher dimensional cosmological models is motivated mainly by the possibility of geometrical unification of the fundamental interactions of the universe. In the context of the Kaluza-Klein and super string theories higher dimensions have, recently, acquired much significance. Also, the higher dimensional theory is important at the early stages of the evoluation of the universe (Applequist et al. 1987). Rahaman et al. (2002), Chatterjee (1993) and Khadekar et al. (2005) have investigated higher dimensional string cosmological models in general relativity. In particular, Reddy (2003a, 2006) and Reddy et al. (2006, 2007) have discussed some string cosmological models in Saez-Ballester scalar-tensor theory of gravitation in five dimensions.

In the present investigation, we have reproduced the exact cosmological models in the spherically symmetric five dimensional space-time in the frame work of Saez and Ballester (1985) scalar-tensor theory of gravitation. We have solved the field equations for three different cases and obtained exact solutions. We have also discussed some physical and kinematical properties of the models.

2. Field Equations

Here we consider the five dimensional spherically symmetric space-time in the form

$$ds^2 = dt^2 - e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) - e^{\mu} dy^2 \quad , (2.1)$$

where λ and μ are functions of cosmic time t only.

Consider the field equations given by Saez and Ballester (1982) for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\,\phi_{,j} - \frac{1}{2} g_{ij}\,\phi_{,k}\,\phi^{,k}\right) = T_{ij} \qquad (2.2)$$

and the scalar field satisfies the equation

$$2\phi^n\,\phi^{i}_{;i} + n\phi^{n-1}\,\phi_{,k}\,\phi^{,k} = 0 \qquad (2.3)$$

where $G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$ is the Einstein tensor, ω and n are constants, T_{ij} is the energy tensor of the matter. Comma and semicolon denotes partial and covariant differentiation respectively.

Also,

$$T^{ij}_{;j} = 0 \qquad (2.4)$$

is a consequence of the field equations (2.2) and (2.3).

The energy-momentum tensor for cosmic strings is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \,. \qquad (2.5)$$

Here ρ is the rest energy density of the system of strings with massive particles attached to the strings and λ the tension density of the system of strings. As pointed out by Letelier (1983), λ may be positive or negative, u^i describes the system four velocities and x^i represents direction of anisotropy, i.e. the direction of strings which is taken to be along fifth dimension.

We have

$$u^i u_i = -x^i x_i = +1 \text{ and } u^i x_i = 0. \tag{2.6}$$

We consider

$$\rho = \rho_p + \lambda \tag{2.7}$$

where ρ_p is the rest energy density of particles attached to the strings. Here, we consider ϕ, ρ and λ are functions of t only.

The field equations (2.2), (2.3) and (2.4) for the metric (2.1) with the help of (2.5) and (2.6) can, explicitly, be written as

$$-\frac{3}{4}\lambda_4^2 - \frac{3}{4}\lambda_4\mu_4 - \omega\phi^n \frac{1}{2}\phi_4^2 = \rho, \tag{2.8}$$

$$\lambda_{44} + \frac{3}{4}\lambda_4^2 + \frac{1}{2}\lambda_4\mu_4 + \frac{1}{2}\mu_{44} + \frac{1}{4}\mu_4^2 - \omega\phi^n \frac{1}{2}\phi_4^2 = 0, \tag{2.9}$$

$$\frac{3}{2}\lambda_{44} + \frac{3}{2}\lambda_4^2 - \omega\phi^n \frac{1}{2}\phi_4^2 = -\lambda, \tag{2.10}$$

$$\phi_{44} + \frac{3}{2}\phi_4\lambda_4 + \frac{1}{2}\phi_4\mu_4 + \frac{n}{2}\frac{\phi_4^2}{\phi} = 0, \tag{2.11}$$

$$\rho_4 + \frac{3}{2}\rho\lambda_4 + (\rho - \lambda)\frac{\mu_4}{2} = 0. \tag{2.12}$$

where a suffix 4 denotes differentiation with respect to t.

3. Models and their Physical Features

The field equation (2.8) to (2.12) can be solved by assuming physical or mathematical conditions. In the literature, we have equations of state for string model (Letelier 1983),

$\rho = \lambda$ (geometric string or Nambu string)

$\rho = (1+\omega)\lambda$ (p-string or Takabayasi string)

In addition to above, recently, Reddy (2003a, 2003b); Reddy and Rao (2006); Reddy and Naidu (2007) have obtained inflationary string cosmological models in Brans and Dicke (1961), Saez and Ballester (1985) and Lyra (1951) scalar-tensor theories of gravitation assuming a relation

$$\rho + \lambda = 0 \text{ (Reddy string)} \tag{3.1}$$

i.e. the sum of rest energy density and tension density for a cloud of strings vanishes. The relation (3.1) is analogous to $\rho + p = 0$ in general relativity with perfect fluid as source which represents false vacuum case.

Here we find string cosmological models corresponding to

(i) $\rho = \lambda$, (ii) $\rho = (1+\omega)\lambda$ and (iii) $\rho + \lambda = 0$

in five dimensions in Saez-Ballester scalar-tensor theory.

Case(I): Geometric string $(\rho = \lambda)$

Here we also assume the relation between metric coefficients, i.e. $\mu = n\lambda$ because of the fact that field equations are highly non-linear. Using this relation, the field equatios (2.8-2.12) admits the exact solution.

$$\rho = \lambda = N\log[k_3(t+k_2)] \tag{3.2}$$

$$\mu = nN\log[k_3(t+k_2)] \tag{3.3}$$

$$\phi = \left[\frac{k_3(n+2)}{2(k_4+1)}(t+k_2)^{k_4+1} + k_5\right]^{\frac{2}{n+2}} \tag{3.4}$$

where $k, k_1, k_2, k_3, k_4, k_5$ are constants of integrations.

So, the metric (2.1) can be written as

$$ds^2 = dt^2 - [k_3 t]^N (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) - [k_3 T]^{nN} dy^2, \tag{3.5}$$

where $N = \frac{2}{1+3n}, T = t + k_2$.

The line element (3.5) represents five dimensinoal geometric or Nambu string in Saez-Ballester scalar-tensor theory of gravitation.

The physical and kinematical parameters for the model are

$$\rho = \lambda = N\log[k_3 T] \tag{3.6}$$

$$\phi = \left[\frac{(n+2)}{4}T^2\right]^{\frac{2}{n+2}}, \text{ where } k_4 = 1, k_5 = 0 \tag{3.7}$$

$$\text{Spatial Volume: } V^3 = r^2 \sin\theta \, e^{(\frac{3+n}{2})N\log(k_3 T)} \tag{3.8}$$

$$\text{Scalar expansion: } \theta = \frac{1}{3T} \tag{3.9}$$

Shear scalar: $\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{81T^2}$ (3.10)

Deceleration parameter (Feinstein et al. 1995)

$$q = -\frac{3}{\theta^2}\left[\theta_\alpha u^\alpha + \frac{1}{3\theta^2}\right] > 0 \quad (3.11)$$

The model (3.5) has no initial singularity, while the energy density, the tension density of the string given by (3.6). However as T increases this singularity vanish. The spatial volume of the model given by (3.8) shows the anisotropic expansion of the universe (3.5) with time. For the model (3.5) the expansion scalar θ and shear scalar σ tend to zero as $T \to \infty$. The positive value of the deceleration parameter indicates that the model decelerates in the standard way.

Also, since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq o.$ (3.12)

The model does not approach isotropy for large values of T.

Case (II): Takabayasi string $[\rho = (1+\omega)\lambda, \omega \geq 0]$

In this case, again, assuming the relation between metric coefficients i.e. $\mu = n\lambda$. We obtained the five dimensional Takabayasi string (p-string) model in Saez-Ballester scalar tensor theory as

$$ds^2 = dt^2 - (Z_3T)^{-\frac{1}{Z}}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) - (Z_3T)^{-\frac{n}{Z}} dy^2 \quad , (3.13)$$

where $T = (t + Z_2), Z_3 = -\frac{Z}{Z_1}$.

Models given by (3.5) and (3.13) are similar except for the constants.

The string density, tension density and the scalar field in the universe (3.13) are

$$\rho = (1+\omega)\lambda = (1+\omega)\frac{-\log(Z_3T)}{Z}, \quad (3.14)$$

$$\phi = \left[\frac{(n+2)Z_4}{2(1+Z_5)}T^{(1+Z_5)}\right]^{\frac{2}{n+2}}. \quad (3.15)$$

It can be easily seen that the physical and kinematical parameters of the universe given by (3.13) have the identical behavior as in case (I).

Case (III): Reddy string $(\rho + \lambda = 0)$

In this case, again, $\mu = n\lambda$, the five dimensional Reddy string model can be writter as

$$ds^2 = dt^2 - [R_4T]^{\frac{1}{R}}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) - [R_4T]^{\frac{n}{R}} dy^2 \quad . (3.16)$$

which is similar to the models (3.5) and (3.13) with similar behavior of the physical and kinematical parameters.

In this model, the string density, tension density and the scalar field are

$$\rho = -\lambda = -\frac{3(1+n)}{4R^2T^2} - \omega\phi^n\phi_4^2, \quad (3.17)$$

$$\phi = \left[\frac{4RR_5}{(n+2)(2R-3-n)}T^{-\frac{2R-3-n}{2R}}\right]^{\frac{2}{n+2}}. \quad (3.18)$$

where $R = \frac{3+n}{2}, R_5$ is the constant of integration.

On comparing the models (3.5), (3.13) and (3.15) with the cosmological models obtained in four dimensions by Reddy (2003a) in Brans-Dicke Theory and Reddy and Rao (2006) in Lyra manifold, also Reddy and Naidu (2007) in a scalar-tensor theory of gravitation, we observe that the behaviour of these models is similar as the models obtained in string cosmology. Also the physical quantities like energy density, tension density and scalar field diverge in this theory whereas they do not diverge in Brans-Dicke theory.

4. Conclusions

We have studied five dimensional models generated by spherically symetric space-time in Kaluza-Klein space-time in the frame work of Saze and Ballester (1985) scalar tensor theory of gravitation. These models represents geometric string, p string and Reddy string in five dimension, which are free from initial singularities and they are expanding with respect to space coordinates while the extra coordinate contract with increase in time, they are non-rotating and decelerate in the standard way. All the physical quantities like energy density, tension density and the scalar field diverge at the initial moment of creation. However these behaviour is similar to that of the five dimensional cosmic string model obtained by Reddy (2007).

5. Future Scope of this Study

This paper enables future work on the mathematical methods presented here as well as on analytical treatments of the range of physical problems. Possible further developments and applications are suggested as in recent years, there has been a multitude of efforts to construct alternative theory of gravitation. Saez and Bellester (1985) have formulated scalar tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory discription of weak fields. Inspite of the dimensionless caracter of the scalar field an antigavity regime appears. This theory also suggest a possible way to solve missing matter problems in non-flat FRW cosmologies. The scalar tensor theories of gravitation are important in unified theories of gravitation and to remove the posssible graceful exit problem in an inflationary era.

It is still a challenging problem to know the exact physical situation at very early stages of the formation of our universe. Thus the study of higher dimensional cosmological models is motivated mainly by the possibility of geometrically unifying the fundamental interaction of the universe. Kaluza- klein theory in higher dimension have recently acquired much significance which is important at the early stages of the evolution of the universe. As far as our information goes there has not been much work in literature on five dimensional models in scalar tensor theories of gravitation.

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Author Profile

Kalpana Pawar completed her B. Sc. in 1995, M.Sc. (Mathematics) in 1997 and received Ph.D. degree award in 2003. Her research is mainly includes the area of General Theory of Relativity and Cosmology. Recently she had started research in the area of Fuzzy Mathematics for the new and advanced aveneus in the field. Presently, she is working as an Assistant Professor of Mathematics in Shivaji Science College affiliated to R.T.M. Nagpur University, Nagpur, India. Under her supervision two students achieved the award of M.Phil. degree and two students have submitted their thesis for the award of Ph.D. degree.

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