Design and Characterization of a 3\textsuperscript{rd} Order Low-Pass Butterworth Filter

Shashank Soi
B. Tech, Dr. B. R. Ambedkar National Institute of Technology, Jalandhar, India

Abstract: This paper presents a detailed analysis and design of a 3\textsuperscript{rd} order low pass Butterworth filter. The frequency response of the filter (magnitude and gain) is also measured and analyzed and the 3 dB frequency is calculated. Simulation Results are presented in PSpice.

Keywords: Butterworth, 3 dB frequency, Frequency response, PSpice.

1. Introduction

Filters are the circuit that allows the frequencies in one range to pass while blocking the frequencies in all other ranges. There are several type of filters such as “Low Pass”, “High Pass” and “Band Pass” filters. As the name implies a “Low Pass” filter is a circuit that passes low-frequency signals and blocks high-frequency ones. A “High Pass” filter on the other hand passes high-frequency signals and blocks low-frequency ones. A “Band Pass” filter passes signals whose frequency lies in a certain frequency band. Filters are essential components of electronic and communication systems. For instance, in audio applications a filter can be used to emphasize and de-emphasize certain frequencies. It can also be used to block out noise signal.

2. Butterworth Filter: A Review

The principle behind filters is quite simple, although the actual implementation can become complicated, depending on the specifications of the filter. A Butterworth filter has the maximally flat response in the pass-band. At the cut-off frequency, $f_c$, the attenuation is -3dB. Above the -3dB point the attenuation is relatively steep with a roll off of -20dB/decade/pole. Figure 1 shows the frequency response of the Butterworth filter of order n.

![Figure 1: Frequency response of Butterworth filter of order n](image)

The magnitude equation of the 3\textsuperscript{rd} order low pass filter is given by the expressions

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_c)^{2n}}}$$

which n is the order of the filter (number of poles). Figure 2 shows the poles of the Butterworth filter.

![Figure 2: Poles of a Butterworth filter of order n](image)

3. Analysis of 3\textsuperscript{rd} order Low Pass Butterworth filter

Figure 3 shows a 3rd order low pass Butterworth filter that uses an RC network for filtering. The op amp used is in the non-inverting configuration. Resistors $R_1$ and $R_F$ determine the gain of the filter.

![Figure 3: Low pass Butterworth filter of order 3](image)

The magnitude equation of the 3\textsuperscript{rd} order low pass filter is given by the expressions

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_c)^{2n}}}$$
\[ V_{OUT} = \frac{A_F}{\sqrt{1 + \left(\frac{f}{f_C}\right)^6}} \]  

(1)

where

\[ A_F = A_1 \times A_2 = \text{Pass band gain of the filter} \]

\[ A_1 = 1 + \frac{R_F}{R_1} = \text{Gain of 1\textsuperscript{st} stage} \]  

(2)

\[ A_2 = 1 + \frac{R_F}{R_1} = \text{Gain of 2\textsuperscript{nd} stage} \]  

(3)

\[ f = \text{Frequency of the input signal (Hz)} \]

\[ f_C = \frac{1}{2\pi RC} = \text{Corner frequency} \]  

(4)

4. Design Specifications and Simulation Results for Frequency Domain Analysis:

A low pass 3\textsuperscript{rd} order Butterworth filter is designed by applying a 1V peak-to-peak sinusoidal input to the circuit of Figure 3. Following values are taken into consideration for the design procedure:

Table 1: Circuit Parameters for Figure 4

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A_F</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>R</td>
<td>72 \text{kilo-ohm}</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>2.2 \text{nF}</td>
</tr>
<tr>
<td>4</td>
<td>R_F</td>
<td>0.414 \text{kilo-ohm}</td>
</tr>
<tr>
<td>5</td>
<td>R_1</td>
<td>1 \text{kilo-ohm}</td>
</tr>
<tr>
<td>6</td>
<td>R_L</td>
<td>10k</td>
</tr>
</tbody>
</table>

Figure 4: Schematic of Low pass Butterworth filter of order 3 for parameters given in Table 1

From Equation 4 the cut off frequency (also known as the -3 dB frequency) is calculated to have a value of 1 KHz. The same value can be observed from the graph of Figure 5.

Table 2: Circuit Parameters for Figure 7

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A_F</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>R</td>
<td>20 \text{kilo-ohm}</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>0.01 \text{\mu F}</td>
</tr>
<tr>
<td>4</td>
<td>R_F</td>
<td>0.414 \text{kilo-ohm}</td>
</tr>
<tr>
<td>5</td>
<td>R_1</td>
<td>1 \text{kilo-ohm}</td>
</tr>
<tr>
<td>6</td>
<td>R_L</td>
<td>10k</td>
</tr>
</tbody>
</table>

Figure 5: Frequency Response of low pass Butterworth filter of order 3 for circuit parameters given in Table 1

Figure 6: Schematic of Low pass Butterworth filter of order 3 for parameters given in Table 2

The cut off frequency using the circuit parameters given in Table 2 is found to be 0.795 KHz. The frequency response plot of Figure 7 verifies it.
Observation: From Figure 5 and Figure 7 it is observed that the 3 dB frequency $f$ varies as the value of $R$ and $C$ varies.

5. Conclusion

In this paper the low pass Butterworth filter of order 3 has been successfully explained as well as implemented. The simulation has been performed in PSpice. The effect of $R$ and $C$ on the 3 dB frequency has been explained and results shown.

6. Scope for Future Study

Fine tuning of the filter’s frequency response can be focused on. Various stability analysis techniques can be applied to the filter for its characterization. Comparisons can be made among various filters according to the type as well as according to the order and results can be shown.

References


Author Profile