

# Role of Coupled-Channels in Heavy Ions Reactions at the Coulomb Barrier

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**Abstract:** *The effect of the coupled channel on the calculations of the total fusion reactions cross section  $\sigma_{fus}$  and the fusion barrier distribution  $d^2(E\sigma_{fus})/dE^2$  have been studied for the reactions involving heavy ions nuclei for the systems  $^{16}\text{O}$  on  $^{144,154}\text{Sm}$ . The effect of double, octupole and quadrupole phonon excitations in  $^{144}\text{Sm}$  were considered and rotational deformation were included for the nucleus  $^{154}\text{Sm}$  with ground state rotational band up to the  $4^+$  states. The calculated fusion reactions cross section  $\sigma_{fus}$  and the fusion barrier distribution  $d^2(E\sigma_{fus})/dE^2$  are compared with the available experimental with and without the inclusion of the coupled channels effects. The inclusion of the coupled channel in the calculations enhances the fusion cross section below the Coulomb and brings the calculated fusion reaction cross section near to the experimental data.*

**Keywords:** Heavy-ion fusion reactions, Fusion barrier distribution, Coupled-channels calculations.

## 1. Introduction

Heavy ion collisions around the Coulomb barrier offer a very rich variety of phenomena. The coupling of various channels with each other results in the splitting of the barrier and hence, the fusion cross sections are substantially enhanced in the sub-barrier region as compared to the predictions of one dimensional [1]. The fusion of two nuclei at very low energy is an example of tunneling phenomena in nuclear physics. These reactions aren't only of central important for stellar energy production and nucleosynthesis, but they also provide new insights into reaction dynamics and nuclear structure [2]. Heavy-ion fusion cross sections at energies well above the Coulomb barrier can be reproduced by a barrier penetration model in which the one-dimensional fusion barrier results from a combination of the repulsive Coulomb and centrifugal potentials and the attractive, short range nuclear potential. At energies below this single barrier, measured fusion cross-sections are generally enhanced relative to calculations with this model. The role of static deformation effects in enhancing sub-barrier fusion has long been recognized and has been demonstrated experimentally. Here the enhancement occurs because there is a distribution of barrier heights which can be thought of as resulting from different orientations of the deformed target nuclei. Any distribution of barriers around the single barrier leads to enhancement of the cross sections, at energies below that of the single barrier, because passage over the lower barriers is much more probable than penetration through the single barrier. The effects of collective surface vibrations on fusion were also considered in a semi classical picture, again resulting in a distribution of fusion barriers. The term "sub barrier" fusion is conventionally used to describe fusion at energies below the single fusion barrier, even though the cross sections result largely from passage over barriers, whose heights are lower than the bombarding energy [3].

The experimental measurements of the fusion barrier distribution (BD) represent a new stage in the study of heavy ion fusion. The fusion BD analysis is a valuable tool to understand the fusion mechanism of two heavy nuclei and the role of their internal degrees of freedom leading to fusion. The fusion BD has been shown to be sensitive to the data related to the nuclear structure, such as the nuclear shapes, the multiple excitations and the anharmonicity of nuclear surface vibrations etc. For this purpose, high precision measurements of the fusion cross-section data are required and have been reported for many systems. The fusion BD analyses of these data provided impetus to understanding of the fusion mechanism and generated a widespread interest in this study [4].

The aim of the present work is to investigate the effect of the coupled-channels on the calculations of the fusion reaction cross section and the fusion barrier distribution for the systems  $^{16}\text{O}+^{144,154}\text{Sm}$ .

## 2. Coupled-Channels Formalism

The nuclear structure effects can be taken into account in a more quantal way using the coupled-channels method. In order to formulate the coupled-channels method, consider a collision between two nuclei in the presence of the coupling of the relative motion  $r=(r,\hat{r})$  to a nuclear intrinsic motion  $\xi$ . We assume the following Hamiltonian for this system [5],

$$H(r, \xi) = -\frac{\hbar^2}{2\mu} \nabla^2 + V(r) + H_0(\xi) + V_{coup}(r, \xi) \quad (1)$$

Where  $H_0(\xi)$  and  $V_{coup}(r, \xi)$  are the intrinsic and the coupling Hamiltonians, respectively.  $V(r)$  is the standard Woods-Saxon potential which has the form,

$$V(r) = \frac{-V_0}{1 + \exp[(r - r_0)/a]} \quad (2)$$

Where  $a$ , is the diffuseness parameter.

In general the intrinsic degree of freedom  $\xi$  has a finite spin. We therefore expand the coupling Hamiltonian in multipoles as [6],

$$V_{coup}(r, \xi) = \sum_{\lambda > 0} f_{\lambda}(r) Y_{\lambda}(\hat{r}) \cdot T_{\lambda}(\xi) \quad (3)$$

Here  $Y_{\lambda}(\hat{r})$  are the spherical harmonics and  $T_{\lambda}(\xi)$  are spherical tensors constructed from the intrinsic coordinate. The dot indicates a scalar product. The sum is taken over all except for  $\lambda = 0$ , which is already included in the bare potential,  $V(r)$ . For a fixed total angular momentum  $J$  and its  $z$ -component  $M$ , the expansion basis for the wave function in Eq. (2) are defined as [5],

$$\langle \hat{r}, \xi | (\alpha I) JM \rangle = \sum_{m_l, m_i} \langle l m_l I m_i | JM \rangle Y_{l m_l}(\hat{r}) \varphi_{\alpha I m_i}(\xi) \quad (4)$$

Where  $l$  and  $I$  are the orbital and the intrinsic angular momenta, respectively  $\varphi_{\alpha I m_i}(\xi)$  are the wave functions of the intrinsic motion which obey,

$$H_0(\xi) \varphi_{\alpha I m_i}(\xi) = \varepsilon_{\alpha I} \varphi_{\alpha I m_i}(\xi) \quad (5)$$

Here,  $\alpha$  denotes any quantum number besides the angular momentum. Expanding the total wave function with the channel wave functions as [5],

$$\psi_J(r, \xi) = \sum_{\alpha, I, I'} \frac{u_{\alpha I}^J(r)}{r} \langle \hat{r}, \xi | (\alpha I) JM \rangle \quad (6)$$

The coupled-channels equations for  $u_{\alpha I}^J(r)$  read [5],

$$\left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) - E + \varepsilon_{\alpha I} \right] u_{\alpha I}^J(r) + \sum_{\alpha', I', I''} V_{\alpha I; \alpha' I' I''}^J(r) u_{\alpha' I' I''}^J(r) = 0 \quad (7)$$

Where the coupling matrix elements  $V_{\alpha I; \alpha' I' I''}^J(r)$  are given as [7],

$$\begin{aligned} V_{\alpha I; \alpha' I' I''}^J(r) &= \langle (\alpha I) JM | V_{coup}(r, \xi) | (\alpha' I' I'') JM \rangle, \\ &= \sum_{\lambda} (-1)^{I-I'+J} f_{\lambda}(r) \langle l \| Y_{\lambda} \| l' \rangle \langle \alpha I \| T_{\lambda} \| \alpha' I' \rangle \\ &\quad \times \sqrt{(2I+1)(2I'+1)} \begin{Bmatrix} I' & I' & J \\ l & I & \lambda \end{Bmatrix}. \end{aligned} \quad (8)$$

Notice that these matrix elements are independent of  $M$ . For the sake of simplicity of the notation, in the following let us introduce a simplified notation,  $n = \{\alpha, l, I\}$ , and suppress the index  $J$ . The coupled-channels Eq. (7) then reads [5],

$$\left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) - E + \varepsilon_n \right] u_n(r) + \sum_{\alpha', l', I'} V_{nn'}(r) u_{n'}(r) = 0 \quad (9)$$

These coupled-channels equations are solved with the incoming wave boundary conditions of [5],

$$\begin{aligned} u_n(r) &\approx \sqrt{\frac{k_n}{k_n(r)}} \mathfrak{T}_{nn_i}^J \exp\left(-i \int_{r_{abs}}^r k_n(r') dr'\right) \quad r \leq r_{abs}, \\ &= H_{ln}^{(-)}(k_n r) \delta_{n, n_i} - \sqrt{\frac{k_n}{k_n(r)}} S_{nn_i}^J H_{ln}^{(+)}(k_n r) \quad r \rightarrow \infty \end{aligned} \quad (10)$$

Where  $n_i$  denotes the entrance channel. The local wave number  $k_n(r)$  is defined by,

$$k_n(r) = \sqrt{\frac{2\mu}{\hbar^2} \left( E - \varepsilon_n - \frac{l_n(l_n+1)}{2\mu r^2} - V(r) \right)} \quad (11)$$

Where  $k_n = k_n(r = \infty) = \sqrt{2\mu(E - \varepsilon_n)/\hbar^2}$ . Once the transmission coefficients  $\mathfrak{T}_{nn_i}^J$  are obtained, the inclusive penetrability of the Coulomb potential barrier is given by,

$$P_J(E) = \sum_n |\mathfrak{T}_{nn_i}^J|^2 \quad (12)$$

The fusion cross section is then given by [5],

$$\sigma_{fus}(E) = \frac{\pi}{k^2} \sum_J (2J+1) P_J(E) \quad (13)$$

The fusion barrier distribution is given by [8],

$$D_f(E) = \frac{d^2(E\sigma_{fus})}{dE^2} = \pi R_b^2 \left[ -\frac{d}{dE} \left( \frac{1}{1 + \exp\left[2\pi \frac{E - V_b}{\hbar\omega}\right]} \right) \right] \quad (14)$$

### 3. Fusion Barrier Distributions

Over recent years precision measurements of “experimental fusion barrier distributions” have led to significant insights into how the collective modes (rotational and vibrational) of the target and projectile influence the dynamics of a nuclear reaction. The simple idea behind these measurements is that since the classical fusion cross  $\sigma_{fus}$  (zero below the Coulomb barrier) is given above the barrier by [5],

$$E\sigma_{fus} = \pi R^2 (E - B) \quad (15)$$

Where  $B$  and  $R$  are the Coulomb barrier heights and radius, and  $E$  is the incident center of mass energy. Then the second derivative  $d^2(E\sigma_{fus})/dE^2$  is simply a delta function of area  $\pi R^2$  located at the energy  $E = B$ . quantum tunneling merely smooth out this function into a symmetric peak with a width of around 2-3 (MeV), but if a range of barriers wider than that value is present in a given reaction then their “distribution” can be readily deduced from [5],

$$D_{fus} = d^2(E\sigma_{fus})/dE^2 \quad (14)$$

The second derivative of  $(E\sigma_{fus})$  was extracted from the excitation functions using a simple point difference method. It is defined at energy  $(E1 + 2E2 + E3)/4$  as,

$$\begin{aligned} \frac{d^2(E\sigma_{fus})}{dE^2} &= 2 \left( \frac{(E\sigma_{fus})_3 - (E\sigma_{fus})_2}{E_3 - E_2} \right. \\ &\quad \left. - \frac{(E\sigma_{fus})_2 - (E\sigma_{fus})_1}{E_2 - E_1} \right) \left( \frac{1}{E_3 - E_1} \right) \end{aligned} \quad (15)$$

where  $(E\sigma_{fus})_i$  are evaluated at energies  $E_i$ .

with equal energy increments  $\Delta E = (E_2 - E_1) = (E_3 - E_2)$

this reduces to,

$$\frac{d^2(E\sigma_{fus})}{dE^2} = \left( \frac{(E\sigma_{fus})_3 - 2(E\sigma_{fus})_2 + (E\sigma_{fus})_1}{\Delta E^2} \right) \quad (16)$$

Then the statistical error ( $\delta_c$ ) associated with the second derivative at energy  $E$  is approximately given by,

$$\delta_c \cong (E / \Delta E^2) \left[ (\delta\sigma_{fus})_1^2 + 4(\delta\sigma_{fus})_2^2 + (\delta\sigma_{fus})_3^2 \right]^{1/2} \quad (17)$$

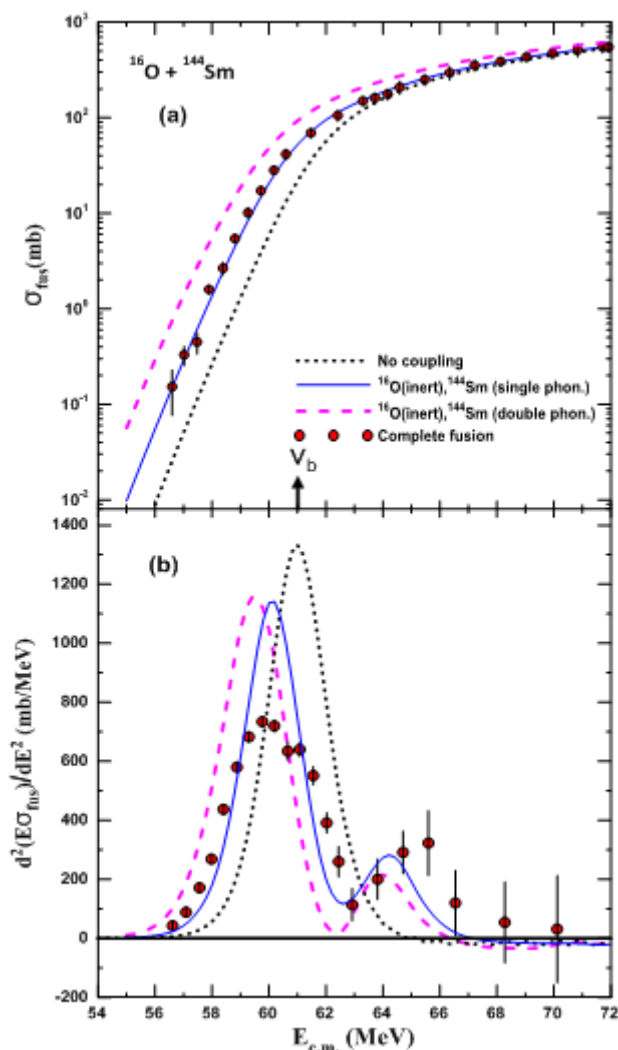
where the  $(\delta\sigma_{fus})$  are the errors in the cross sections. They have dimensions of cross sections and are not percentage errors. Thus when, as is common, the  $(\sigma_{fus})$  are measured with a fixed percentage error,  $(\delta_c)$  is proportional to the value of  $(\sigma_{fus})$  and increases with increasing energy [9].

## 4. Results and Discussion

### 4.1 $^{16}\text{O} + ^{144}\text{Sm}$ System

The coupled-channels calculations of fusion cross section as well as fusion barrier distribution for the heavy ion reaction of  $^{16}\text{O} + ^{144}\text{Sm}$  System by including the lowest state of  $^{144}\text{Sm}$ , that is,  $3^-$  (octupole) (single phonon) and  $3^-$  (octupole) (double phonon).

The results of coupled-channels calculations are performed by using the CCFULL code [10] are compared with the experimental data in Fig. 1. Fig. 1(a) and 1(b) shows the fusion cross section and the fusion barrier distribution, respectively. The dotted line represents the calculations without including the coupling effects, i.e., the target and the projectile are assumed to be inert in which the calculations of the fusion cross section underestimate the experimental data at and below the Coulomb barrier. The solid line represents the results of the calculations taking the single phonon state of the octupole excitations into account, where the calculated fusion cross section are in excellent fit with the experimental data below and above the Coulomb barrier. The calculations including the double octupole phonon states in addition to single octupole vibration is shown by dashed line overestimated the experimental data. The Woods-Saxon parameters are taken to be,  $V_0=105.1$  MeV,  $r_0=1.06$  fm.,  $a_0=0.75$  fm. The fusion barrier distribution are extracted from the experimental data and from the theoretical calculations of the fusion cross section by using the simple three point difference method in which Matlab code was written to do these calculations. Our calculations agrees very well with the previous work (see. Refs. [9,11,12]).



**Figure 1:** Comparison of coupled-channels calculations with the experimental data for  $^{16}\text{O} + ^{144}\text{Sm}$  for (a) the fusion cross and (b) the fusion barrier distribution. The dotted line is the result without coupling. The coupled-channels calculations which take into account the coupling to single quadrupole and octupole excitations in  $^{144}\text{Sm}$  is given by solid line. The dashed line is obtained by including the coupling to double quadrupole phonon excitations in addition to single octupole state in target nucleus. Experimental data are taken from Leigh et al. [9].

### 4.2 $^{16}\text{O} + ^{154}\text{Sm}$ System

The calculation of the total fusion cross fusion and the fusion barrier distribution are presented in Fig.2 (a) and Fig.2(b). Since  $^{154}\text{Sm}$  is a well deformed nuclei the code defus[10] are employed with deformation parameters  $\beta_2=0.322$  and  $\beta_4=0.05$  to account for the rotational deformation for  $^{154}\text{Sm}$ , while the projectile is kept inert. The dotted line in Fig. 2 (a) and (b) represents the calculations without including coupling, i.e., both projectile and target are inert we can see from the figure that without coupling the calculation of the fusion cross section underestimate the experimental data around and below the Coulomb barrier and the calculations of the fusion barrier distribution centered around the Coulomb barrier are unable to reproduce the experimental data. The solid line represents the calculations by considering the rotation in the target nucleus with

deformation parameters mentioned above which agrees very well with the experimental data and enhance the calculations of the fusion cross section around and below the Coulomb barrier. The woods-Saxons parameters are taken to be  $V_0=165$  MeV,  $r_0=0.95$  fm,  $a_0=1.05$  fm.

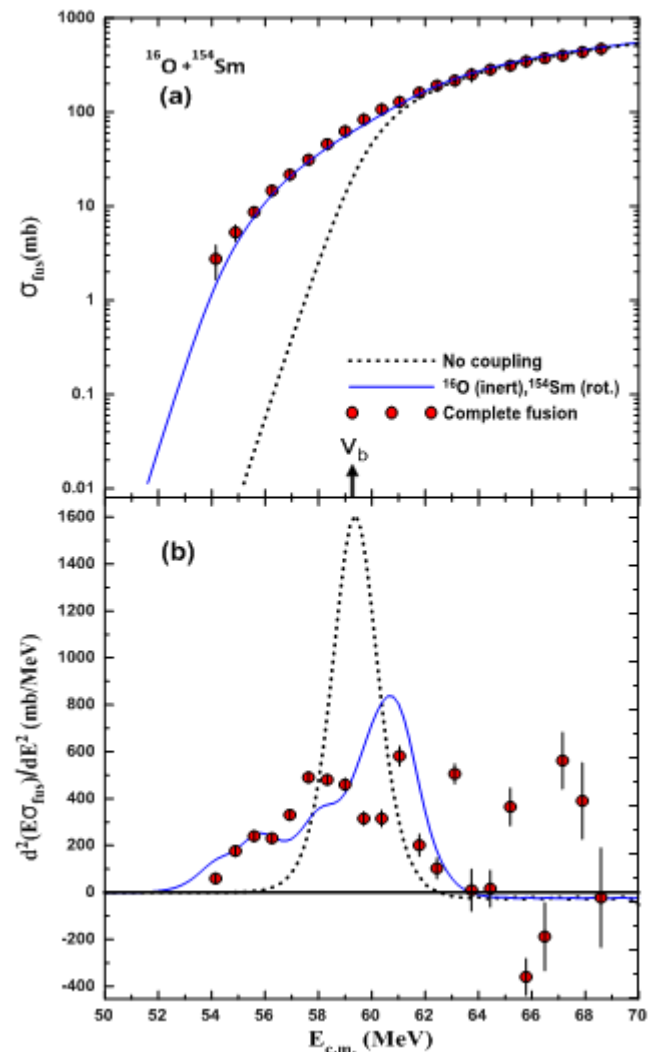
## 5. Conclusions

The effect of coupled channels are investigated for the systems  $^{16}\text{O}+^{144}\text{Sm}$  and  $^{16}\text{O}+^{154}\text{Sm}$  and we concluded that the coupling of the octupole state in  $^{144}\text{Sm}$  target nucleus are very essential and leads to enhance the total fusion cross section calculations and also leads to reasonable agreement with the experimental data for the fusion barrier distributions.

It has been found that the coupling of low lying vibrational states with the relative motion of interacting nuclei enhances the sub-barrier fusion cross-section to large extent as compared with the one dimensional barrier penetration model calculations. Further, the extent of enhancement of the sub-barrier fusion cross section has been found to be very sensitive to the deformation parameter and the energy of the states included in the analysis. This work can be extended to study heavier nuclei and also to study the effect of the breakup channel for halo nuclei.

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**Figure 2:** Comparison of coupled-channels calculations with the experimental data for  $^{16}\text{O}+^{154}\text{Sm}$  for (a) the fusion cross and (b) the fusion barrier distribution. The dotted line is the result without coupling. The coupled-channels calculations which take into account the rotational coupling for the target nucleus  $^{154}\text{Sm}$  is given by solid line. Experimental data are taken from Leigh et al.[9].

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