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Predicting Global Solar Radiation using Gamma Test and Local Linear Regression Data Models in Bauchi, Nigeria

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Abstract: ***We report the implementation of a new perspective in non-linear modelling resulting in the development of data models for prediction of global solar radiation (G_{sr}) in Bauchi (10'.19' N, 9.51' E) Nigeria. Several studies to find alternative means of generating good estimates of G_{sr} data have been carried out and have continued for locations in Nigeria including Bauchi. Questions relating to the amount of data, determination of possible input combinations and noise variances before modelling has not been answered. Using Gamma Test(GT), weather variables namely, rainfall (R_f), cloud cover C_c, average temperature(T_{av}), average relative humidity (Rh_{av}), wind speed (U), extraterrestrial radiation (G_0) and cleanness index (I_c) as input variables and G_{sr} as the output in the numerical data set named bauchi-solar.csv for a period of five years (2003 – 2007) with 853 unique data points(M) were examined. An optimal near neighbour number ($p_{\max}$) of 32 was used in arriving at a gamma statistic, Γ of 0.0013. The M-test analysis shows that the noise variance becomes relatively stable at between 700 and 750 data points being data required for the modelling process. 127 Local Linear Regression (LLR) models were identified using the Full Embedding Search heuristics with coefficient of determination (R^2) values for both the training set (1-750) and the validation set (751-853) ranging between -18.85 to 0.9923 which guided the selection of models. The average MSE for both training and validation data sets are between 0.0254 and 20.77 for the location. An average value of about 0.99 for the gradient showed that the functions were generally moderate in complexity.***

Keywords: Global Solar radiation, Gamma test, Local Linear Regression, Weather variables, bauchi-solar.csv

1. Introduction

Accurate measurement of global solar radiation(G_{sr}) received on a horizontal surface however essential in a wide range of applications, is still a challenge particularly in developing nations like Nigeria. The search for alternative ways of estimating G_{sr} in such locations in Nigeria is necessary and appears ever ongoing. Bauchi (10'.19' N, 9.51' E) location is the focus of this work. A few efforts have been made in estimating global solar radiation for this location using empirical methods (Burari and Sambo, 2001; Gana and Akpootu, 2013). Knowledge of G_{sr} is required for the design of solar energy conversion systems and a lot of other modelling systems where it constitutes a critical input parameter in the model development process.

This parameter is used in the design and prediction of the performance of solar energy devices, architectural constructs, crop growth models and other applications (Majeed, 2013). Several models ranging from classical/empirical to non-linear numerical/statistical models have been proposed for estimating G_{sr} in several locations in Nigeria. Augustine & Nnabuchi (2009) studied the relationship between global solar radiation and sunshine hours with average coefficient of determination(R^2) of 0.726. Ogolo (2010), evaluated the performance of some predictive models for estimating G_{sr} in Nigeria with average R^2 ranging between 0.172-0.94 and 0.415-0.970. Ewona (2011) modelled some basic climatic parameters in Nigeria using correlation coefficient & artificial neural network with the correlation between measured and predicted values of global solar radiation showing values that range from 0. 28 - 0.88. The results of various studies on the prediction and analysis of solar radiation using both empirical and non-linear, non-parametric techniques like ANN shows that much need to be done to improve on the predictive reliability of prevalent methods. Enhanced statistical indicators characteristic of more reliable smooth models could be derived by an initial determination of, the noise variance in the available data, the exact number of data points required for optimal training and validation and input variable relevance determination before the model building process.

This paper attempts to achieve all these through the use of a novel technique called the Gamma test (GT) deployed for the pre-analysis of the data and the adoption of Local Linear Regression (LLR) method for the model building process. This GT technique was introduced by Agalbjorn *et al.*,(1997) and Koncar (1997). Before engaging in the LLR data model development, GT performs some pre-analysis on the data by determining the best mean square error (MSE) that can be achieved by any smooth model on unseen data for a given selection of inputs, the best embedding dimension and the data length determined, before using gamma statistic (Γ) and other Gamma Test indicators like, gradient(A), standard error(SE) & the $V-ratio$. In Bauchi and elsewhere in Nigeria, this study is a pioneer attempt to use GT in the estimation of G_{sr} from meteorological parameters by determining first, the noise variance (Γ) and a suitable near neighbour number, secondly, the data length (l_M) and thirdly the different possible embedding dimension (input parameter combinations) before the modelling and evaluation process. The study adopts the information derived from GT analyses to develop/construct non-linear models using LLR algorithm.

2. Materials and Methods

2.1 Study Area and Data

Bauchi is located in the midland area with latitude $10^0 19' N$ and longitude $9^0 51' E$ as coordinates and suspended at an altitude of 622 meters above sea level and covering a total area of 49,119 square kilometres representing about 5.3% of the total Nigeria's land mass. The data used for this study were collected from the Nigeria Meteorological Agency (NIMET) Oshodi, Lagos. The data covered a period of 7 years (2003-2009). The G_{sr} was derived from records of Gunbellani distillates using the conversion factor in equation 1 (Sambo, 1985)

$$G_{sr} = (1.35 \pm 0.176) H_{GB} \quad MJ/m^2 day \qquad (1)$$

The daily mean extraterrestrial radiation (R_0) on a horizontal surface from sunrise-sunset was computed for daily periods using equation 2 (Cai & Mu, 2005)

$$R_0 = \frac{24(60)}{\pi} G_{sc} d_r \left(\omega_s SinLSin\delta + CosLCos\delta Sin\omega_s\right) \qquad (2)$$

$$\omega_s = Cos^{-1}(-\tan L \tan\delta) \qquad (3)$$

$$\delta = 23.45 Sin\left(\frac{360(d_n + 284)}{365}\right) \qquad (4)$$

Where R_0 is extraterrestrial radiation, G_{sc} is solar constant $(4.92 MJ/m^2 day)$, d_r is the inverse relative Earth-Sun distance, d_n is the day number during a given year, L is the latitude of the location, δ is solar declination and ω_s is the sun hour angle. The data base used for the analyses include Global Solar radiation (G_{sr}), average temperature (T_{av}), Rainfall (R_f), cloud cover (C_c), average relative humidity (Rh_{av}), wind speed (U), extraterrestrial radiation (G_0) and clearness index (I_c)

2.2 The Gamma Test (GT)

Atleast for the location under investigation and in the tropics generally, GT is a new approach in nonlinear modelling and analysis which presupposes thus, given M number of unique data with x_i inputs and y_i outputs, representing a set of data observations of the form,

$$\{(x_i, y_i), 1 \le i \le M\} \qquad (5)$$

The under lying relationship between input variables $x_{i(i=1,2,3,\ldots n)}$ and the outputs y_i is of the form,

$$y = f(x_1 .. x_m) + \varepsilon \qquad (6)$$

Where f is a smooth function and ε represents the noise in the data. The GT reported by Koncar (1997) is developed to estimate the noise variance ε, which cannot be accounted for by the smooth model function. The input variables $x_{i(i=1,2,3,\ldots n)}$ and the corresponding outputs y_i are continuous. Given 7-inputs/1-output data set as in *bauchi-solar.csv*, GT finds answers to questions relating to, how the inputs determine the output from an unknown smooth model, the extent to which we can predict the output from a given set of inputs, the number of data points required for modelling to predict the output from a given set of inputs and the inputs relevant in making good prediction. In the search for answers, Stefansson *et al.*, (1997) suggested that as the number of data sample M increase, the gamma statistic (Γ) converges in probability to an asymptotic value which is equal to the variance of the noise ε on the output. The nearest-neighbour lists for the first k-nearest neighbours can be established in $O(M \log M)$ time (Evans & Jones, 2002) using a kd-tree building technique.

Given that $x_{N(i,k)}$ represents the k^{th} nearest neighbour of x_i in the set $\{x_1 \ldots, x_m\}$. The GT is based on the statistics.

$$\partial_M(k) = \frac{1}{M}\sum_{i=1}^{M}\left|x_{N(i,k)} - x_i\right|^2 \quad (1 \le k \le p) \qquad (7)$$

$$\gamma_M(k) = \frac{1}{2M}\sum_{i=1}^{M}\left(y_{N(i,k)} - y_i\right)^2 \quad (1 \le k \le p) \qquad (8)$$

Where $|\ldots|$ is the Euclidean distance, $y_{N(i,k)}$ is the output value associated with inputs $x_{N(i,k)}$. From equations (6) and (7), GT is derived from the delta function of the inputs vectors and the corresponding gamma function of the output values. A least square regression line constructed for p points $(\partial_M(k), \gamma_M(k))$ offering a means for computing the Γ-statistic from the relation,

$$\gamma = A\partial + \Gamma \qquad (9)$$

The plot shows that $\gamma_M(k) \to Var(\varepsilon)$ in probability $(\partial_M(k) \to 0$. The value Γ read on the vertical axis gives the data noise variance. The gradient (A) provides useful information that can enhance our understanding of the complexity of the unknown function. The estimate of Γ depends on the number of data points (M) and/or the nature of the function revealed by A (Kemp, 2006). A mathematical proof of the Gamma test method found in Evans and Jones (2002) elucidates the usefulness of this method as a good noise variance determinant. To prevent under training and over fitting problems associated with arbitrary choice of training and testing data set, the reliability of the $\Gamma - statistic$ can be determined by running a series of GT for increasing M to establish the size of the data set required to produce a stable asymptote. It helps us to avoid the needless effort of fitting a model beyond the point where the MSE on the training data is smaller than the ε (Remesan *et al.,* 2008). The total process of this analysis is referred to as $M-test$

2.3 Local Linear Regression (LLR)

LLR is a widely studied non-parametric repression method it is the most effective method in sessions of high data point density in input space (Remesan *et al*., 2008). To make a prediction for a given query point in the input space, a kd-tree is built through a process that seeks to find the k-nearest neighbours of the query point from the given data set and then builds a model using these k-data points in O(MlogM) time. The kd-tree is used to locate the set of inputs points $x_i(1 \leq i \leq p_{\max})$ which can be accomplished in O(MlogM) time (Jones, 2004). The constructed model is then applied to the query points say, (x_i, y_i) thus producing a predicted output. That is, the procedure uses initial three data points to estimate the initial values and subsequently uses all newly updated data available for further predictions and the challenge associated with the choice of the number of near neighbours $p_{\max}$ is surmounted using influence statistics (Remesan *et al*., 2008). Given $p_{\max}$ for a set of M-number column data and d-dimensional matrix of the nearest neighbour input points $x_i(1 \leq i \leq p_{\max})$ and y column vector of the same length of the corresponding output. A linear matrix equation as represented in equation (10) can be generated

$$Xm = y \tag{10}$$

Equation 10 was deployed in determining the optimal mapping from input data X to the corresponding output y such that the rank r of the matrix X is the number of linearly independent rows which contributes to the uniqueness of the solution for m. For a square and non-singular X, the unique solution to equation 10 is given by $m = X^{-1}y$, whereas for a non-square, non-singular X, we find the vector m which minimizes $|Xm-y|^2$ (Remesan *et al.,* 2008) as derived from Penrose(1955) who proved this showing that the unique solution is provided by $m = X^{\neq}y$ where $X^{\neq}$ is a pseudo-inverse matrix(Penrose, 1956). In this study, a space-partitioning data structure for organizing points in a k-dimensional space called a kd tree (theoretical details in Durrant, 2001: Jones, 2004) is used for efficient search of near neighbours in a given data structure.

3. Results and Discussion

3.1 Gamma test analyses results for bauchi-solar.csv

Gamma test(GT) analysis of *bauchi-solar.csv* performed on the data with 853 unique data points estimates the best Mean Square Error (MSE) using any nonlinear smooth modelling technique (in this case, LLR). A great deal of very instructive information can be derived from GT analysis. This will reduce guess work and enhance our understanding of the modelling process and consequent characteristic output for a particular selection of input variables. Some determinant indices used in this study are, the choice of the number of near neighbours ($p_{\max}$) for optimal results, the Gamma statistic (Γ) (which is a measure of the noise variance in the data), the gradient (A) derived from the trend line of the scatter plot (which is a measure of the complexity of the function), the Standard Error(SE) (which measures the goodness of fit of the regression line) and the $V-ratio$ (which allows for an independent judgement of how well the output range can be modelled by the smooth function). The GT analyses results for *bauchi-solar.csv* data is as shown in table 1.

Table 1: Gamma test analysis results for different input combinations (I=input inclusion and 0= input exclusion)

Input Parameters Combinations	Gamma(Γ)	Gradient(A)	Standard Error	V-ratio	Mask
$R_f, C_c, T_{av}, Rh_{av}, Wnsp, G_0, C_I$	0.0013	0.0718	0.0027	0.0051	1111111
$C_c, T_{av}, Rh_{av}, Wnsp, G_0, C_I$	0.0001	0.0815	0.0024	0.0004	0111111
$R_f, T_{av}, Rh_{av}, Wnsp, G_0, C_I$	0.0111	0.0951	0.0020	0.0446	1011111
$R_f, C_c, Rh_{av}, Wnsp, G_0, C_I$	0.0027	0.1284	0.0047	0.0110	1101111
$R_f, C_c, T_{av}, Wnsp, G_0, C_I$	0.0036	0.0802	0.0022	0.0145	1110111
$R_f, C_c, T_{av}, Rh_{av}, G_0, C_I$	5.2485E-5	0.0944	0.0032	0.0002	1111011
$R_f, C_c, T_{av}, Rh_{av}, Wnsp, C_I$	0.0015	0.0807	0.0261	0.0062	1111101
$R_f, C_c, T_{av}, Rh_{av}, Wnsp, G_0$	3.5439E-5	0.0919	0.0028	0.0001	1111110
$C_c, T_{av}, Rh_{av}, G_0$	0.00253	0.1596	0.0028	0.0101	0111010

The increasing near neighbour test showed an optimal $p_{\max}$ value of 32 which was adopted for the various input selections. The GT produced an asymptotic convergence for different input combinations represented by the different mask as shown in table 1. Comparably, the significantly smaller Gamma statistic values were observed to be associated with masks or input combinations that alternately excluded rainfall(0111111), Wind speed(1111011) and clearness index(1111110) having respective Γ values of 0.0001, 5.2485E-5 and 3.5439E-5 for $M = 750$. An insight into the precision and accuracy is derived by considering the nearness of the *V-ratio* to zero and the variation of the standard error(SE) with the number of unique data points as graphically exemplified in figure 1. The SE error values for 0111111, 1111011 and 1111110 of 0.002400, 0.003205 and 0.002786 shows that wind speed(U) is the least significant input parameter when predicting global solar radiation from meteorological parameters for this location (This is in agreement with the findings of Remesan *et al.*, 2008 while studying the Brue Catchments in the United Kingdom). The response of Γ for different input combinations to varying and increasing M=853 values ($p_{\max} = 32$) using M-test analysis as shown in figure 1, indicates that the minimum Γ values for different combinations asymptotes and becomes stable around M=700-750.

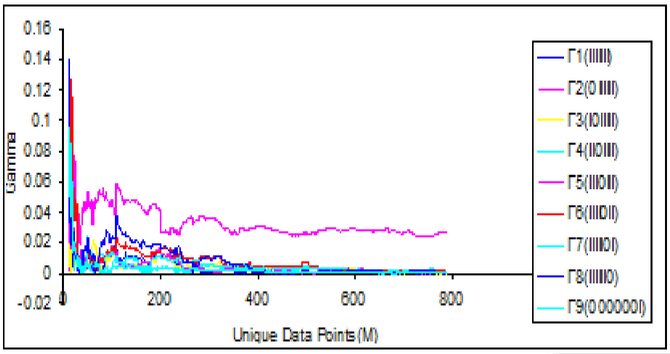

Figure 1:Variation of Gamma Statistic with M=853

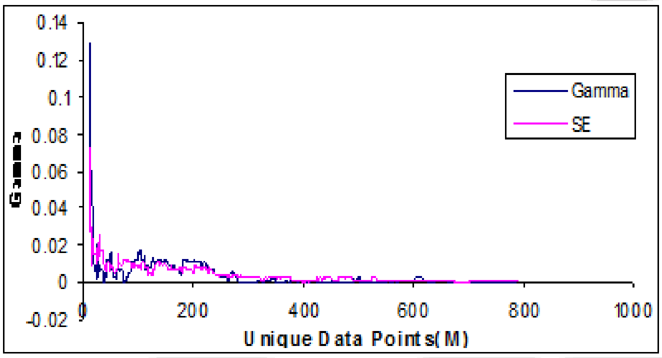

Figure 2: Variation of Gamma Statistic and Standard error with for different embedding dimensions M=853 for bauchi-solar.csv

The value of Γ gives the estimate of the best MSE which can be reached for the unknown smooth function of continues data. It varies for different embedding dimensions. For the input used for the study in Bauchi, a minimum value of Γ=3.5439E-5 was observed using all available inputs except the clearness index (C_I). A low Γ and A offers a good promise for the model development process. The V-ratio is defined as the $Gamma/_{Var(Output)}$ and this standardizes the measure of the Γ and enables a judgement to be made on how well the output can be predicted by available inputs and a given smooth function. $V-ratio$ values close to zero indicates a high degree of predictability of the particular output and close to 1, indicate a poor predictive capability of the unknown smooth function . For the ten different embedding types shown in table 1, the lowest $V-ratio$ of 0.0001 with an associated low Γ=3.5439E-5 for mask 1111110 is identified which is indicative of its reliability in the optimal prediction of G_{sr} .

3.2 Model construction using LLR

The LLR predicted and measured daily global solar radiation (G_{sr}) for three sampled embedding dimensions (0111010, 0111111, 0000010) using *bauchi-solar.csv* with 853 unique data points are shown in figures 3, 4, 5, 6, 7 & 8. The LLR algorithm was used to predict G_{sr} for the available days in the month during a two-year period (2006 and 2007) with 853 unique data points and a possible test range of 751-853. 750 data points (1-750) consisting of data records between 2003-2006 was used for the initial supervised training and computation processes. Model identification analyses using a full embedding search showed that they were 127 possible input combinations that can be used in developing data models that could predict G_{sr} from a smooth function with different degrees of predictability. For the purpose of this paper, despite the numerous possibilities, we concentrated analysis on about 10 possible embeddings to draw a few inferences.

Table 2: LLR models (M=750, $p_{\max}$ =32) for predicting G_{sr} and their statistical measureso of performance

Input Parameters Combinations	Gamma (Γ)	Gradient (A)	Standard Error	V-ratio	Mask
$R_f, C_c, T_{av}, Rh_{av}, Wnsp, G_0, C_I$	0.0013	0.0718	0.0027	0.0051	1111111
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$R_f, T_{av}, Rh_{av}, Wnsp, G_0, C_I$	0.0111	0.0951	0.0020	0.0446	1011111
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$R_f, C_c, T_{av}, Rh_{av}, Wnsp, C_I$	0.0015	0.0807	0.0261	0.0062	1111101
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$C_c, T_{av}, Rh_{av}, G_0$	0.00253	0.1596	0.0028	0.0101	0111010

From the details of the LLR modelling statistics presented in Table 2 , models with mask 0111111(rainfall excluded) , mask 1110111(Relative humidity excluded), mask 1111011(wind speed excluded) and an all inclusive mask 1111111 with significantly low RMSE and appreciate less complex smooth function models have potentials of high predictability. A RMSE of 0.1269 obtained by masking the wind speed (U) parameters shows that the wind speed is the least significant input parameter in any G_{sr} prediction model development work. Model with mask 1111101 (which excludes G_0) having slope value of 1.0007 and RMSE of 0.2651 (for training data) and 0.3584 (for validation data) reveals that G_0 is very critical parameter in the model development process when estimating G_{sr}. However, results for the model with mask 0000010 reveals that the G_0 index parameter alone, will predict G_{sr} poorly if the contribution of other intervening variables are not taken into consideration. Low coefficient of determination R^2 values for both training and validation data set with high error indices are clear indications of this claim.

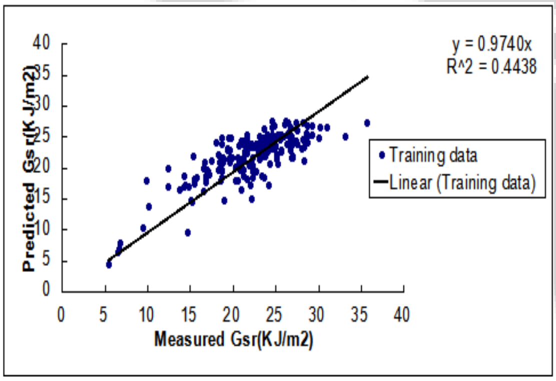

Figure 3: Predicted and measured Gsr for mask 0000010

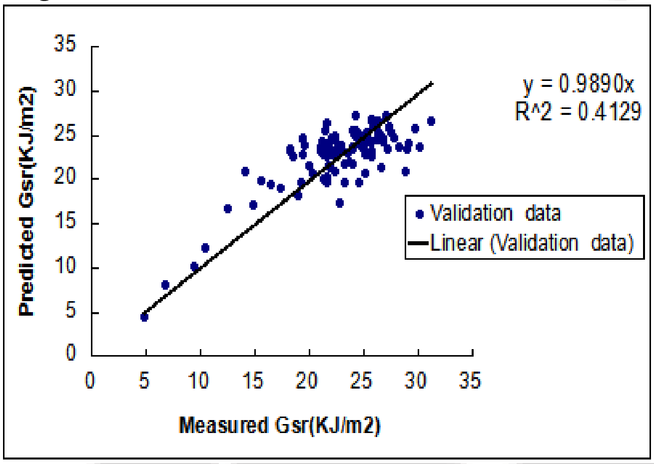

Figure 4: Predicted and measured Gsr for mask 0000010 using the training data using the validation data set.

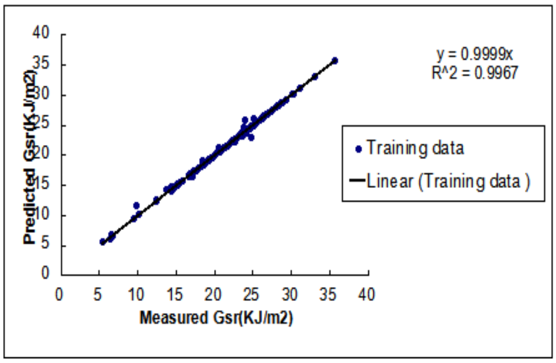

Figure 5: LLR predicted and measured Gsr for mask 0III0I0

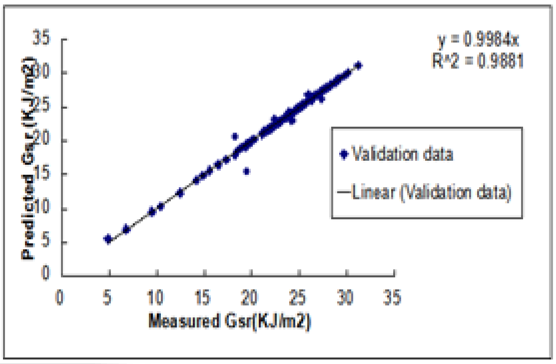

Figure 6: LLR predicted and measured Gsr for mask 0III0I0 using the training data set. using the validation data set.

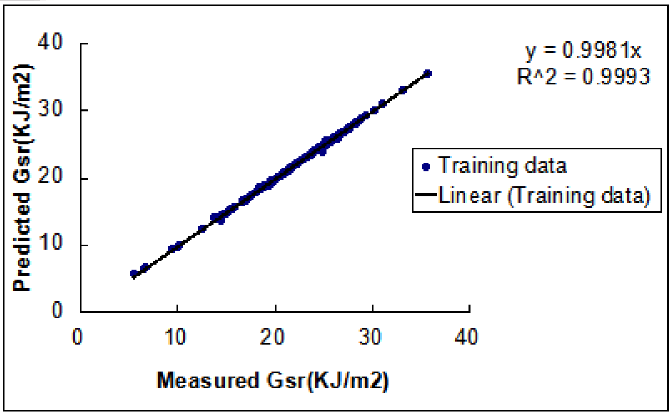

Figure 7: LLR predicted and measured Gsr for mask 0IIIIII

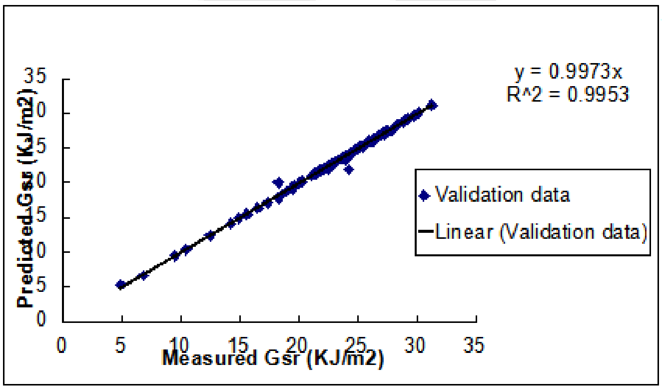

Figure 8: LLR predicted and measured Gsr for mask 0IIIIII using the training data set using the validation data set.

The variation for different input combinations with increasing data points from 1-853 was done through an

analysis process referred to as M-test. The results obtained were used to determine the asymptotic Gamma estimate where the data is sufficient for developing a smooth model or otherwise at a precise cut-off point (m). The results of the $M-test$ on *bauchi-solar.csv* with seven (7) inputs of varying embedding dimensions using $p_{\max} = 32$ is shown in figure 2. From figure 2**,** it is observed that the M-test produced an asymptotic convergence of the Gamma statistic to a value of 3.5439E-5 at around 750 data points (i.e M=750). The variation of standard Error (SE) and with unique data points shown in figure 3 with SE = 0.0011 lends credence to the suitability of the data and equally gives an insight into the precision of the Gamma statistic.
From the graph, it could be deduced that the combination of all inputs excluding R_f and/or wind speed (U) will make good models that are almost comparable to the one with all inputs (1111111) and also, that which excludes the clearness index parameter (C_I). However, the best result of 0.9999 for the location was obtained with $C_c, T_{av}, Rh_{av}, G_0$ only as inputs.

4. Conclusion

It was found that using GT in combination with a nonlinear modelling method like LLR provides as a quick and simple means of estimating G_{sr} using different generated possible combinations of the available inputs. Before the modelling process, the noise variances and general suitability of the data was determined using the different Gamma test indicators. The study adopted the use of 1-750 unique data points for the analysis and developing the data model and validated the result using data between 751- 853 which was not used in developing the data model. The result of the LLR technique used are found to be reliable with coefficient of determination (R^2) for measured and predicted Gsr for the validation data set ranging from 0.4129 for 0000010 to 0.9954 for mask 1111111. From the results it could be inferred that rainfall, wind speed and clearness index are the least significant input parameters, while G_0 is the most significant for optimal result in the estimation of G_{sr} . From the 10 different models shown in table 2, aside the model with mask 0000010 which was noted to have slowly falling *M-test* and SE graphs and high V-ratio value indicative of a low degree of predictability, and relatively high gradient (A) and SE values reflective of high complexity, the other embeddings can produce reliable smooth data models depending on the available input parameter at a given time.

References

[1] Agalbjorn S, Koncar N. Jones AJ. (1997) "A note on gamma test" *Neural Computing and Applications* 5(3):131-133
[2] Augustine, C. and Nnabuchi, MN. (2009). "Empirical Models for the correlation of Global Solar Radiation with meteorological Data for Enugu, Nigeria" *Pacific journal of Science and Technology* 10(1):693-700
[3] Burari, F. N. and Sambo, A. S. (2011). "Model for the prediction of Global Solar Radiation for Bauchi using meteorological data" *Nigeria Journal of Renewable Energy* 91:30-33
[4] Cai, J and Mu, J. (2005). Estimation of daily reference evapotranspiration from weather forecast messages. Paper presented at *ICID 21st European Regional Conference,*15-19 May, 2005- Frankfurt(Oder) and Slubice-Germany and Poland.
[5] Durrant, P. J (2001) Win Gamma: A nonlinear data analysis and modelling tool with applications to food production. PhD thesis, department of Computer Science, Cardiff University, Wales, Uk.
[6] Evans, D. and Jones, AJ. (2002), "A proof of the gamma test" *Proceedings of the Royal Society. Series* A 458 (2027):2759-2799
[7] Ewona, I.O. (2011). "Modelling of Basic Computer Climatic Parameters in Nigeria using correlation analysis and Artificial Neural Networks" PhD thesis. Department of Physics University of Calabar, Calabar, Nigeria.
[8] Gana, N.N. and Akpootu, D.O. (2013). Angstrom type empirical correlation for estimating global solar radiation in North-Eastern Nigeria. The International Journal of Engineering and Science(IJES). Vol.2(1): pp. 58-78.
[9] Igbal, M. (1983) "An introduction to Solar Radiation. *Academy Press: New* York, NY
[10] Jones, AJ. (2004). "New tools in non-linear modelling and prediction" *Computational Management Science* 1:109-149
[11] Kemp, S.E. (2006) Gamma test analysis tools for non-linear time series. PhD thesis, department of computing and mathematical sciences, Faculty of Advance Technology, University of Glamorgan, Wales UK.
[12] Koncar, N. (1997 "Optimization methodologies for direct inverse neuro-control. PhD thesis, Department of Computing. Imperial College of Science Technology & Medicine, University of London.
[13] Majeed, A.J. (2013). Empirical models for correlation of clearness index with cloud index at Rutha, Iraq State. International Journal of Engineering Research and Applications. Vol.3(1):pp. 1189-1192.
[14] Ogolo, E.O. (2010) Evaluating the performance of some predictive models for estimating global solar radiation across varying climatic conditions in Nigeria. *Indian Journal of Radio and Space Physics.* Vol. 39. pp. 121-131.
[15] Penrose R. (1955). 'A generalized inverse for matrices "*Proceedings of the Cambridge Philosophical Society"* 51:406-413
[16] Penrose R. (1956) "On the best approximate solution to linear matrix equations. *Proceedings of the Cambridge Philosophical Society* 52:17-19
[17] Remesan, R. Shamin, M. A. and Han, D (2008). Model Data Selection using gamma test for daily solar radiation estimation*; Hydrological processes* 22:4301-4309
[18] Remesan, R. Shamin, M. A. and Han, D (2008). Nonlinear modelling of daily solar radiation using the Gamma Test Data.BHS 10th National Hydrological Symposium, Exeter.
[19] Sambo, A. S. (1985) "Solar Radiation in Kano. A correlation with meteorological data". *Nigeria Journal of Solar Energy* 4:59-64

[20] Stefansson, A., Koncar, N. & Jones AJ. (1997). A note on the gamma test. Neural Computing and Application, 5:131-133.

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