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A Comparative Study of Fabry-Perot and Ring Cavity

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Abstract: ***Fabry-Perot and Ring Cavities are treated critically. Gain and dispersion relations in the semiclassical theory of Laser have been worked out using the complex conjugate terms. It has been shown that the use of the complex conjugate terms in the expression for electric field and polarization gives rise to additional terms for dispersion relations having physical significance. Mode pulling is affected.***

Keywords: Fabry –Perot cavity, power spectrum, Ring cavity, wave analysis

1. Introduction

The semi classical theory as presented by Lamb[1,2] and coworkers has explained a number of laser behaviours, the important one is undoubtedly the so called 'Lamb –dip" that has led to a variety of stabilization schemes [3] and to the new field-lamb dip spectroscopy[4]. The basic equations of semiclassical theory of laser were derived on the basis of geometrical model where the incident electric field perturbs the i^{th} atom according to the laws of quantum mechanics and induces an electric dipole moment P_i expectation value. Values for atoms localized at r are added to yield macroscopic polarization p. This polarization acts as source in Maxwell's equation for a field E. The loop is completed by the self consistency requirement that the field assumed E is equal to the field produced E[1]. In the semiclassical theory the electromagnetic field equations inside the cavity are described by the Maxwell's equations. Further by ignoring the variation of electromagnetic field transverse to the laser axis and on the basis of the fact that only certain discrete modes achieve appreciable magnitude, the basic equations of the theory were derived. There were few approximations which were originally introduced in the semiclassical model. As for example the complex conjugate terms in the expression for electric field and polarization were ignored and the medium was considered as homogeneous. The aim of present work is to work out the basic equations of the theory including the complex conjugate terms which were originally excluded. It is shown that the effect of introducing complex conjugate terms is to produce additional relations which have physical significance.

2. The Electromagnetic Field Equations

Consider a homogeneously broaden medium with two level atoms and a Fabry-Perot cavity with Brewster windows which ensures linearly polarized field and a scalar field. We also describe the electromagnetic radiation in the laser cavity by Maxwell's equations in MKS unit. The wave equation obtained from the Maxwell's equations for the scalar Electric field $E(z,t)$ is

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}+\mu_0\,\sigma\frac{\partial\vec{E}}{\partial t}+\mu_0\,\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}=-\mu_0\frac{\partial^2\vec{P}}{\partial t^2} \quad \text{--- (1)}$$

Where σ is the conductivity introduced in the lossy medium and this conductivity can be adjusted to give damping due to diffraction and the reflection transmission. Considering, $\vec{\nabla}.\vec{D}=0$ and $\vec{\nabla}.\vec{P}=0$ for laser medium. On the basis of the fact that only certain discrete modes have achieved appreciable magnitude the variation in the electromagnetic field transverse to the laser axis is ignored, thus

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}=-\frac{\partial^2\vec{E}}{\partial z^2} \quad \text{--- (2)}$$

Where, z is the direction along the optical path of the laser. The transverse variations of the field are neglected. Inside the cavity only certain discrete modes achieve appreciable magnitude whose circular frequency is

$$\Omega_n=\frac{n\pi c}{L}=K_n c \quad \text{--- (3)}$$

Where L is the length of the cavity, c is the velocity of light, n is a large integer in our discussion we take normal modes to have sinusoidal z dependence. The electric field can be expressed as a sum of modes, i.e.

$$U_n(z)=\exp(\mathrm{i}\,K_n z)\quad ,K_n=\mathrm{n}\frac{\pi}{L}$$

The single polarization component of the electric field is written as

$$E(z,t)=\frac{1}{2}\sum_n \mathbf{E}_n(t)\exp\{-\mathrm{i}(\upsilon_n t+\phi_n)\}U_n(z)+c.c \quad \text{--- (4)}$$

$$P(z,t)=\frac{1}{2}\sum_n \mathbf{P}_n(t)\exp\{-\mathrm{i}(\upsilon_n t+\phi_n)\}U_n(z)+c.c \quad \text{--- (5)}$$

Here, the amplitude coefficient E_n and complex polarization component P_n very little in an optical frequency period. The real part of polarization is in phase with the electric field leads to dispersion due to medium. The imaginary part is in quadrature with the electric field and results in gain or loss. Now using values in equation (1), (after doing calculation by algebraic method)

$$\Rightarrow \Omega_n^2 \mathbf{E}_n - i\frac{\sigma}{\varepsilon_0}\upsilon_n \mathbf{E}_n - 2i\upsilon_n \dot{\mathbf{E}}_n - (\upsilon_n + \dot{\phi}_n)^2 \mathbf{E}_n = \upsilon_n^2 \varepsilon_0^{-1} \mathbf{P}_n \quad (6)$$

Adjusting the fictional conductivity σ to create the desired value of Q of the mode

$$\sigma = \varepsilon_0 \frac{\upsilon_n}{Q_n}$$

From equation (6) equating real and imaginary part we finally get

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = -\frac{1}{2}\upsilon_n \varepsilon_0^{-1} \text{Im part of } \mathbf{P}_n \quad \text{--- (7)}$$

$$\upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\upsilon_n \varepsilon_0^{-1} \mathbf{E}_n^{-1} \text{Re part of } \mathbf{P}_n \quad \text{--- (8)}$$

These are basic equations of the semiclassical theory of laser. In absence of active medium $P_n = 0$

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = 0$$

and $\upsilon_n + \dot{\phi}_n = \Omega_n$

Using the same procedure for the complex conjugate terms, we get the gain and dispersion theorem as

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = \frac{1}{2}\upsilon_n \varepsilon_0^{-1} \text{Im part of } \mathbf{P}_n \quad \text{--- (9)}$$

$$\{2\upsilon_n - \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\} = \Omega_n + \frac{1}{2}\upsilon_n \varepsilon_0^{-1} \mathbf{E}_n^{-1} \text{Re part of } \mathbf{P}_n \quad \text{--- (10)}$$

3. Result and Discussion

From above equations (7) and (9), both are the same except the negative sign. But equations (8) and (10) are different with representing dispersion of the medium. In absence of active medium

$$\dot{\mathbf{E}}_n = \frac{\upsilon}{2Q_n}\mathbf{E}_n \text{ and } \{2\upsilon_n - \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\} = \Omega_n$$

From equation (10) we get two relations, one part is

$$\upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t) \text{ Re } \mathbf{P}_n(t) \quad \text{--- (11)}$$

This equation is same as the real part of the original basic equation of Laser derived by Lamb. The another part of the equation is,

$$[\upsilon_n - \{\dot{\phi}_n + \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\}] = \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t) \text{ Re } \mathbf{P}_n(t) + \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t) \text{ Re } \mathbf{P}_n(t)$$

$$\text{Or, } \upsilon_n - \dot{\phi}_n = \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n + \frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t) \text{ Re } \mathbf{P}_n(t) \quad \text{--- (12)}$$

representing dispersion but with different form.

Physical significance:

The complex polarization for complex susceptibility

$$\mathbf{P}_n(t) = \varepsilon_0 \chi_n = \varepsilon_0(\chi'_n + i\chi''_n)\ \mathbf{E}_n(t) \quad \text{--- (13)}$$

Hence from equation (13), equations (7) and (8)

$$\dot{\mathbf{E}}_n(t) = -\frac{\upsilon}{2Q_n}\mathbf{E}_n(t) - \frac{1}{2}\upsilon\chi''_n \mathbf{E}_n(t) \quad \text{--- (14)}$$

$$\text{and } \upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\upsilon\chi'_n \quad \text{--- (15)}$$

Equation (14) expresses energy conservation. The frequency determining Equation (15) can be summed

$$\therefore \upsilon_n + \dot{\phi}_n = \frac{\Omega_n}{\eta(\upsilon_n)} \quad \text{--- (16)}$$

Equation (15) or (16) shows completely difference between the gain problem and the classical absorption problem.

4. Ring Laser

A **ring laser**[8, 9] is a laser in which the laser cavity has the shape of a ring. Light in ring lasers has two possible directions of propagation: clockwise and counter-clockwise. Ring lasers have several applications. Currently the most widespread application is the ring laser gyroscope. If a ring laser is rotating, the two counter-propagating waves are slightly shifted in frequency and a beat interference pattern is observed, which can be used to determine the rotational speed. Semiconductor ring lasers have potential applications in all-optical computing. One primary application is as an optical memory device where the direction of propagation represents either 0 or 1. They can maintain the propagation of light in exclusively the clockwise or counterclockwise direction as long as they remain powered. We consider the case of a three mirror Ring cavity to see the effect of the complex conjugate terms in the amplitude and frequency determining equations as in the case of a Fabry-Perot cavity. The main difference between ring lasers and usual Fabry-Perot lasers is in the possibility to sustain the oscillation of traveling waves rather than standing waves. Now consider the case to see effect of complex conjugate terms in the amplitude and frequency determining equations. Proceed as above considering complex conjugate terms only for electric field and polarizing component we have derived equations in a ring cavity as similar as the equations derived above.

$$\dot{\mathbf{E}}_+ + \frac{\upsilon_+}{2Q_+}\mathbf{E}_n = -\frac{1}{2}\upsilon_+ \varepsilon_0^{-1} \text{Im part of } \mathbf{P}_+ \quad \text{--- (17)}$$

$$\upsilon_+ + \dot{\phi}_+ = \Omega_+ - \frac{1}{2}\frac{\upsilon_+}{\varepsilon_0}E_+^{-1} \text{ Re } P_+ \quad \text{--- (18)}$$

$$\dot{\mathbf{E}}_- + \frac{\upsilon_-}{2Q_-}\mathbf{E}_n = -\frac{1}{2}\upsilon_- \varepsilon_0^{-1} \text{Im part of } \mathbf{P}_+ \quad \text{--- (19)}$$

$$\upsilon_- + \dot{\phi}_- = \Omega_- - \frac{1}{2}\frac{\upsilon_-}{\varepsilon_0} E_-^{-1} \operatorname{Re} P_- \qquad \text{--- (20)}$$

These equations with the complex conjugate terms for (17) and (19) are same. But for (18) and (20) we get

$$2\upsilon_+ - \frac{\Omega_+}{\upsilon_+}\dot{\phi}_+ = \Omega_+ - \frac{1}{2}\frac{\upsilon_+}{\varepsilon_0} E_+^{-1} \operatorname{Re} P_+ \qquad \text{--- (21)}$$

$$2\upsilon_- - \frac{\Omega_-}{\upsilon_-}\dot{\phi}_- = \Omega_- - \frac{1}{2}\frac{\upsilon_-}{\varepsilon_0} E_-^{-1} \operatorname{Re} P_- \qquad \text{--- (22)}$$

With some mathematical analogy from equation (21),

$$\upsilon_+ + \dot{\phi}_+ = \Omega_+ - \frac{1}{2}\frac{\upsilon_+}{\varepsilon_0} E_+^{-1} \operatorname{Re} P_+ \qquad \text{--- (23)}$$

That is same as (18) and the second part of the equation is

$$\upsilon_+ - \dot{\phi}_+ = \frac{\Omega_-}{\upsilon_+}\phi_+ \qquad \text{--- (24)}$$

5. Conclusion

The equation for mode amplitude

$$\dot{\mathbf{E}}_n(t) + \frac{\upsilon}{2Q_n}\mathbf{E}_n(t) = -\frac{1}{2}\upsilon\chi''_n \mathbf{E}_n(t)$$

concerns about the conservation of energy. The Equation (15) determines the associated frequency. For small values of susceptibility, it can rewrite as Equation (16), shows dispersive phenomenon of the laser medium. Thus a difference between the gain and the classical absorption value of this Equation is noticed as the oscillation frequency $\upsilon_n + \dot{\phi}_n$ is shifted by the medium instead of wavelength. This results from the self-consistency nature of the laser field which requires an integral number of wavelengths in a round trip regardless of the medium characteristics. Again from the frequency determining Equation (15) we get a dispersion relation which is different as compared to Equation (16). Thus mode pulling is affected. Similar equations as (24) also can be derived from (22). Thus complex conjugate terms affect the frequency determining equations representing dispersion in the medium. Hence mode pulling is affected.

6. Future Scope of this Study

- Calculation of intensities for both the transverse components clockwise and anticlockwise in case of Ring laser.
- Calculation of polarization of medium for quantum Electronic devices.
- To study variation of coupling factor with other parameters.

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