International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Impact Factor (2012): 3.358

Temperature Distribution in Human Skin and Subcutaneous Region

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Abstract: ***An attempt has been made to analyze the temperature distribution in the skin of human body. The problem is studied by considering three layers of the skin under variable physiological parameters and atmospheric conditions. The analytical and numerical solutions are discussed and displayed graphically.***

Keywords: Temperature, Skin, Human being, distribution, epidermis

1. Introduction

In human body, skin is not only a protective device but it also plays an important role in the process of body thermoregulation in environmental conditions and physiological functions. The temperature of human body is usually maintained constant irrespective of the temperature of the surroundings. Thermogenesis is the process of heat production by oxidation of food materials in our body. Under most circumstances, heat loss from the body is balanced by equal heat production. In particular, when heat production is increased during exercise heat loss is also increased. The only circumstance when heat production per second is increased to balance the equation is during cold exposure when we are unable to reduce heat loss sufficient to maintain core temperature. Under these circumstances thermoregulatory thermogenesis occurs. A large amount of heat is produced in the skeletal muscles, especially during physical work. When muscle contracts, only about 22% of the energy produced appears as external work and the rest as heat. Heat is also produced in liver, digestive glands, ingestion of hot food during digestion and by action of some internal secretion. Ingestion of hot food during digestion and by action of some internal secretion Ingestion of hot food may lead to conduction of heat to the body. Heat is also produced in response to food due to movement of the gastro intestinal tract and production of secretions in salivary glands, pancreas etc. Hormones do not produce heat, but thyroxine may stimulate heat production in tissues.

Thermolysis is an opposite process of heat production. Heat is reduced from the body through three channels, viz, the skin, lungs and excretion, mainly through the process of radiation, convection and evaporation. In cooler environment, heat is lost from the body by radiation. Amount of heat loss by this process is about 55% of the total loss of heat. About 22% of heat is lost from the body through conduction and convection. The heat loss through these processes depends upon the temperature of the surrounding atmosphere. About 25% of heat is lost by evaporation from the body including lungs. From the skin (i) insensible perspiration occurs due to continuous diffusion of fluid from the capillaries of the deeper layer of skin to its dry surface and (ii) the sweat is vaporized from the skin surface which decreases its temperature.

Thus any abnormality arising in the skin and subcutaneous region or atmospheric conditions can disturb the thermal balance and also the temperature distribution in our body Therefore temperature distribution in human body is of great interest to the researchers.

In 1962, Perl [1] derived a heat flow equation by using differential forms of Fiek's perfusion principle and applied the model in simple problems of heat flow. Cooper and Trezek [2] obtained a solution of the problem for cylindrical symmetry assuming all parametric values of thermal conductivity. Patterson [3] paid attention to an experimental determination of temperature distribution in skin and tissue layers. The problem of heat flow was also studied by the Saxena and Arya [4], Sexena [5], Saxena and Bindra [6].The problem of temperature distribution in human limbs in two dimensional steady state has been investigated by Saxena and Gupta [7] and Pardasani and Adlakha [8]. Jain and Jain [9] considered the problem of cold environment. Sanyal and Maji [1O] discussed solutions for temperature distribution of the bioheat equation in which the role of metabolic heat generation and perspiration has been noted.

In the present chapter, our aim is to analyze the problem of temperature distribution in three layers of human skin, viz. epidermis, dermis and hypoderm which depend on physiological phenomena of metabolic heat generation, blood perfusion and perspiration. The roles of different physiological parameters on skin and subcutaneous regions have been exhibited through graphs.

2. Formulation of the Problem

The temperature variation at a point in skin and subcutaneous region at time t is governed by the bio-heat transfer equation [1].

$$pc\frac{-\partial T}{\partial t}=div(K\,grad\,T)+m_b c_b(T_b-T)+S \qquad (1)$$

when T denotes the temperature at time t: ρ, the density of tissue; $\bar{c}$ the specific heat of the tissue; K, the thermal conductivity of the skin, m_b, the blood mass flow; c_b, the specific heat of blood; S, the metabolic heat generation rate and T_b is the body core temperature.

Let us consider the skin to be composed of three layers, namely the epidermis, the dermis and the subcutaneous fatty tissue region known as hypoderm. The thickness of epidermis varies from 0.5 to 0.8 mm, the dermis upto 4 mm while the hypoderm usually from 2 to 10 mm [54]. We denote the thickness of epidermis, dermis and hypoderm by $a_2 - a_1$ and $a_3 - a_2$ respectively.

The epidermis is a stratified epithelium with many nerve endling but no blood vessel. So there is no blood mass flow and the rate of metabolic heat generation can be taken to be constant. In dermis layer the blood mass flow, thermal conductivity and the rate of metabolic heat generation are position dependent and the density of blood vessel increases with increase in thickness. Also the rate of metabolic heat generation is assumed to be time-dependent and varies linearly as (T_b-T). This assumption is based on the reversible nature of chemical reactions involving adenosine tri-, di- and monophosphates liberating heat energy in the invitro tissue. It is also assumed that the outer skin takes place due to convection, radiation and evaporation of sweat. The quantitative details are available from the experimental investigation of Hodson [02J] and these are used extensively. These results provide information about the rate of exploration from a naked human body for various environmental temperatures and humidity and for two wind velocities. The thermal conductivity is assumed to be constant but with different values in three different parts.

In one dimensional steady state, equation (1) is reduced to the form

$$\frac{d}{dx}\left(K_i\frac{dT_i}{dx}\right)+M_i(T_b-T_i)+S_i=0;(i=1,2,3) \qquad (2)$$

with boundary conditions

$$K\frac{dT}{dx}=h(T-T_a)+LE \qquad \text{at } x=0, \qquad (3)$$

$$T = T_b \text{ at } x = a_3. \qquad (4)$$

At the interior points $x = a_i$ (i = 1,2,3)

$$K_i\frac{dT_i}{dx}=K_{i+1}\frac{dT_{i+1}}{dx} \qquad (5)$$

and $$T_i = T_{i+1} \qquad (6)$$

where x is the perpendicular distance from the skin surface; T_i, the temperature at different depths (i = 1,2,3); $M_i = m_b c_b$; S_i and K_i are x-dependent parameters; T_a, the atmospheric temperature; h, the heat transfer coefficient; L, the latent heat of evaporation and E, the rate of evaporation of perspiration.

Now the blood circulating system of the skin is characterized by two major types of vessels: (i) the nutritive arteries, capillaries and veins and (ii) vascular structures. The vascular structures are concerned with heating the skin, consisting primarily of (a) an extensive subcutaneous venous plexus which can hold large quantities of blood that can heat the skin surface and (b) some areas of arteriovenous anastomoses which are large vascular communications between arteries and venous plexes. Epidermis has no blood vessel. On the other hand, the population density of blood vessels in the dermis is very thin near the interface of epidermis and dermis but it increases gradually and becomes almost uniform in the sub dermal part. This gives us some idea regarding the variation of different physical quantities like the rate of blood mass flow, the rate of metabolic heat generation and the thermal conductivity of tissue in relation to its position.

Under the assumptions, we arrange the bio-physical and physiological parameters as [5];

Epidermis ($0 \le x \le a_1$) (7)

K = constant k_1 (say)

$M = M_1 = 0$

$S = S_1$ = constant S_0 (say)

Dermis ($a_1 \le x \le a_2$)

$$K=K_2=K_1\left(\frac{x-a_1}{a_2-a_1}\right)$$

$$M=M_2=m\left(\frac{x-a_1}{a_2-a_1}\right) \qquad (8)$$

$$S=S_2=S_0\left(\frac{x-a_1}{a_2-a_1}\right)\left(\frac{T_b-T_2}{T_b}\right)$$

Hypoderm ($a_2 \le x \le a_3$)

K = constant k_3 (say)

$M = M_3 = 2m$

$$S=2s_0\left(\frac{T_b-T}{T_b}\right) \qquad (9)$$

3. Solutions of the Problem

Substituting the values of all parameters as given in equations (7), (8) and (9), the transformed equations for epidermis, dermis and hypodermal layers are

$$\frac{d^2T_1}{dx^2}=0,\ 0\le x\le a_1, \qquad (10)$$

$$y\frac{d^2\theta}{dy^2}+\frac{d^2\theta}{dy^2}-\alpha^2\theta y=0,\ \ a_1\le y\le a_2 \qquad (11)$$

$$\text{and}\quad \frac{d^2u}{dx^2}-q^2u=0,\ \ a_2\le x\le a_3 \qquad (12)$$

with $y=\dfrac{x-a_1}{a_2-a_1},\theta=\dfrac{T_2-T_b}{T_b}$

and $\alpha^2=\dfrac{(mT_b+s_0)(a_2-a_1)^2}{K_1T_B}$

$u=\dfrac{T_3-T_b}{T_b},q^2=\dfrac{2(mT_b+s_0)}{k_3T_b}$

The solutions of equations (10)-(12) are respectively given by

$$T_1=C_1x+C_2-\frac{S_0x^2}{2k_1} \qquad (13)$$

$$T_2=T_b[1+C_3I_0(\alpha y)+C_4K_0(\alpha y)], \qquad (14)$$

$$T_3=T_b[1+C_5\cosh(qx)+C_6\sinh(qx)], \qquad (15)$$

I_0 and K_0 being modified Bessel functions of order zero.
The arbitrary constants C_i (i = 1,…,6) determined from the boundary conditions are

$$C_1 = \frac{S_0 a_1}{k_1} + \frac{T_b \alpha}{D}\left(C - \frac{S_0 a_1^2}{k_1} - \frac{S_0 a_1}{h}\right)\left[\{AI_0(2\alpha) - BI_1(2\alpha)\}K_1(\alpha) + \{AK_0(2\alpha) + BK_1(2\alpha)\}I_1(\alpha)\right]$$

$$C_2 = \frac{hT_0 - LE}{h} + \frac{k_1}{h}\left[\frac{S_0 a_1}{k_1} + \frac{T_b \alpha}{D}\left(C - \frac{S_0 a_1^2}{k_1} - \frac{S_0 a_1}{h}\right)\right]\left[\begin{matrix}\{AI_0(2\alpha) - BI_1(2\alpha)\}K_1(\alpha) \\ + \{AK_0(2\alpha) + BK_1(2\alpha)\}I_1(\alpha)\end{matrix}\right]$$

$$C_3 = \frac{a_2 - a_1}{D}\left(C - \frac{S_0 a_1}{k_1} - \frac{S_0 a_1}{h}\right)\left[AK_0(2\alpha) + BK_1(2\alpha)\right],$$

$$C_4 = \frac{a_2 - a_1}{D}\left(C - \frac{S_0 a_1^2}{k_1} - \frac{S_0 a_1}{h}\right)\left[AI_0(2\alpha) + BI_1(2\alpha)\right],$$

$$C_5 = \frac{2k_1 \alpha \sinh(qa_3)}{k_3 qD \cosh q(a_2 - a_1)}\left(C - \frac{S_0 a_1^2}{k_1} - \frac{S_0 a_1}{h}\right) \times$$
$$\left[\{AI_0(2\alpha) - BI_1(2\alpha)\}K_1(\alpha) + \{AK_0(2\alpha) + BK_1(2\alpha)\}I_1(\alpha)\right]$$

and

$$C_6 = \frac{2k_1 \alpha \cosh(qa_3)}{k_3 qD \cosh q(a_2 - a3)}\left(C - \frac{S_0 a_1^2}{k_1} - \frac{S_0 a_1}{h}\right) \times$$
$$\left[\{AI_0(2\alpha) - BI_1(2\alpha)\}K_1(\alpha) + \{AK_0(2\alpha) + BK_1(2\alpha)\}I_1(\alpha)\right]$$

where

$$A = k_3 q(a_2 - a_1)\cosh(a_2 - a_3)$$
$$B = 2k_1 \alpha \sinh q(a_2 - a_3)$$
$$C = LE - h(T_a - T_b)$$

$D = T_b[\{AI_0(2\alpha) - BI_1(2\alpha)\}\{k_1 + ha_1)\ \alpha K_1(\alpha) + (a_2 - a_1)\ hK_0(\alpha) + \{AK_0(2\alpha) + BK_1(2\alpha)\}\{k_1 + ha_1)\ \alpha I_1(\alpha) - (a_2 - a_1)\ hI_0(\alpha)\}]$

4. Numerical Results and Discussions

For numerical calculations, the following parametric values have been used as prescribed by Chao et al. [11] and Hodgson [12]:
m_1 = 0.003 Cal cm^{-3} min °C, S = 0.037 Cal cm^{-3} min, k_1 = 0.04 Cal cm^{-1} min °C, k_3 = 0.06 Cal cm^{-1} min °C, h = 0.09 Cal cm^{-1} min °C, L = 580 Cal g^{-1}, T_b = 37°C, a_1 = 0.05 cm, a_2 = 0.12 cm, a_3 = 0.3 cm.