

# The Channel Capacity of MIMO Systems with Impact of Transceiver Impairments and Varying Antenna Element

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**Abstract:** In this paper, an effort has been made to illustrate the performance of MIMO system under the effect of transceiver impairments on channel capacity with varying antenna element. The capacity of ideal MIMO channels depends upon number of antenna used in antenna array. At small SNR values capacity increases linearly. When there are distortions from physical transceiver impairments MIMO channels have a finite upper capacity limit, for any channel distribution and SNR. This work investigates the impact of inherent transceiver impairments of multiple-input multiple-output (MIMO) systems, channel Capacity saturates at high SNR but capacity can be enhanced by varying antenna elements.

**Keywords:** Channel capacity, fading channel, Inter symbol interference (ISI), High-SNR analysis, MIMO, Multi antenna communication, Transceiver impairments.

## 1. Introduction

In wireless technology biggest challenge is to provide higher and higher data rates and to overcome the effect of fading channel. The multipath nature of the channel leads to inter symbol interference (ISI) and as the bandwidth occupied increases, the ISI severity is pronounced. To cope with this problem of multipath channel, various techniques have been and continuously are being proposed by the researchers of system modelling. MIMO is one of the widely used such techniques. MIMO technology involves multiple antennas for data transmission at receiver and at transmitter side. MIMO technology has attracted attention in wireless communications, because it offers significant array gain diversity gain. Because of these properties, MIMO is an important part of modern wireless communication standards such as IEEE 802.11 b. MIMO system has large potential for maximization of channel capacity and it can perform efficiently under various fading channels. But there are many pragmatic hindrances to the efficient MIMO system[2]. So there is a need to increase the channel capacity of MIMO system and try to formulate the techniques to reduce effect of Rayleigh and other fading. While these results concern large network MIMO systems, there is another non-ideality that also affects performance and manifests itself for MIMO systems of any size is *transceiver impairments*. Physical radio-frequency (RF) transceivers suffer from amplifier non-linearities, IQ-imbalance, phase noise, quantization noise, carrier-frequency and sampling rate jitter/offsets, etc. These impairments are conventionally overlooked in information theoretic studies, but they have a non-negligible and fundamental impact on the spectral efficiency in modern deployments with high SNR.[3]

### 1.2 MIMO System

MIMO systems are more popular because of its enormous capacity enlargement. As a promising technology, multiple input multiple output (MIMO) system which can improve frequency efficiency by occupying more spatial resource

brings light to development of wireless communication MIMO systems uses antenna arrays both at the transmitter and receiver which makes it to support high bandwidth efficiency and spacial multiplexing. So MIMO channel model has high attention for high data rate cellular communications in rich multipath environments. Generally

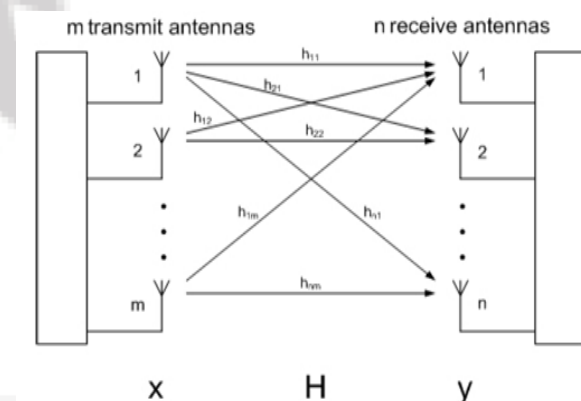


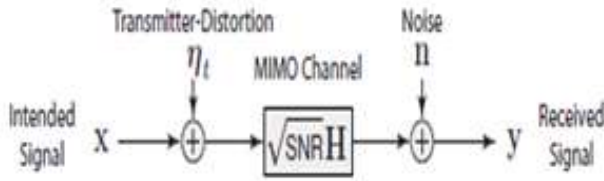
Figure 1: Block Diagram of MIMO

a multipath environment increases the uncertainty in the system. MIMO system depends upon the antenna array used at the receiver, the number of elements in the array and the spacing within the antenna elements. A linear antenna array always gives a high value of signal to noise ratio and also high beam width which further increases the system capacity. But in practice the sub channels of a MIMO system are usually space selective, time selective and frequency selective.

## 2. Channel Modeling Of MIMO System

Consider a flat-fading MIMO channel with  $N_t$  transmit antennas and  $N_r$  receive antennas. The received signal  $y \in \mathbb{C}^{N_r}$  in the classical affine baseband channel model of [3] is

$$y = \sqrt{SNR} Hx + n, \quad (1)$$



**Figure 2:** Block diagram of the generalized MIMO channel.

Unlike the classical channel model in [1], the transmitter distortion generated by physical transceiver implementations is included in the model.

where SNR is the SNR,  $x \in \mathbb{C}^{N_t}$  is the intended signal, and  $n \sim CN(0, I)$  is circular-symmetric complex Gaussian noise. The channel matrix  $H \in \mathbb{C}^{N_r \times N_t}$  is assumed to be a random variable  $H$  having any multi-variate distribution  $f_H$  with normalized gain  $E\{\text{tr}(HH^H)\} = N_r N_t$  and full-rank realizations (i.e.,  $\text{rank}(H) = \min(N_r, N_t)$ ) almost surely this basically covers all physical channel distributions. The intended signal  $x$  in (1) is only affected by a multiplicative channel transformation and additive thermal noise, thus ideal transceiver hardware is implicitly assumed. Physical transceivers suffer from a variety of impairments that are not properly described by (1) [3]–[9]. A generalized MIMO channel is proposed in [6], [7] and verified by measurements. The combined (residual) influence of impairments in the transmitter hardware is modeled by the transmitter distortion  $\eta_t \in \mathbb{C}^{N_t}$  and (1) is generalized to

$$y = \sqrt{SNR} H(x + \eta_t) + n \quad (2)$$

Where  $\eta_t$  is the mismatch between the intended signal  $x$  and the signal actually radiated by the transmitter see Fig. 1 [3].

Under the normalized power constraint  $\text{tr}(Q) = 1$  with  $Q = E\{xx^H\}$ , the transmitter distortion is

$$\eta_t \sim CN(0, Y_t(Q)) \text{ with } Y_t = \text{diag}(v_1(q_1), \dots, v_{N_t}(q_{N_t}))$$

The distortion depends on the intended signal  $x$  in the sense that the variance  $v_n(q_n)$  is an increasing function of the signal power  $q_n$  at the  $n$ th transmit antenna. For simplicity, we used the leakage as proportional to the average signal power per antenna. To capture a range of cases

$$v_n(q_n) = \kappa^2 (1-\alpha)q_n + \alpha \frac{\sum_{i=1}^{N_t} q_i}{N_t} \quad (3)$$

where the parameter  $\alpha \in [0, 1]$  enables transition from one ( $\alpha = 0$ ) to many ( $\alpha = 1$ ) subcarriers. The parameter  $\kappa > 0$  is the level of impairments. This model is a good characterization of phase noise and IQ-imbalance, while the impact of amplifier non-linearities grows non-linearly in SNR.

### 3. Analysis of Channel Capacity

The transmitter knows the channel distribution  $f_H$ , while the receiver knows the realization  $H$ . The capacity of (2) is

$$C_{N_t, N_r}(SNR) = \sup I(x; y|H) \quad (4)$$

$$f_x: \text{tr}(E\{xx^H\}) = \text{tr}(Q) = 1$$

where  $f_x$  is the PDF of  $x$  and  $I(\cdot; \cdot|H)$  is conditional mutual information. Note that  $I(x; y|H) = E_H\{I(x; y|H = H)\}$ . The capacity  $C_{N_t, N_r}(SNR)$  can be expressed as

$$\sup E_H \{ \log_2 \det (I + SNR H Q H^H (SNR H Y_t H^H + I)^{-1}) \}$$

$$Q: \text{tr}(Q) = 1$$

and is achieved by  $x \sim CN(0, Q)$  for some feasible  $Q \succeq 0$ .

In generalized form capacity  $C_{N_t, N_r}(SNR)$  can be expressed as

$$C(SNR) = \frac{1}{2} \log_2 \left\{ \det \left( I + \frac{SNR}{M} H H^H \right) \left( I + \frac{SNR}{M} H Y_t H^H \right)^{-1} \right\}$$

For any realization  $H = H$  and fixed SNR, (2) is a classical MIMO channel but with noise covariance  $(SNR H Y_t H^H + I)$ . The given expression and the sufficiency of using a Gaussian distribution on  $x$  follow from [1]. Although the capacity expression looks similar to that of the classical MIMO channel in (1) and [2], it behaves very differently particularly in the high-SNR regime.

$$H_t = \begin{pmatrix} h_t^{1,1} & h_t^{1,2} & \dots & h_t^{1,N_r} \\ h_t^{2,1} & h_t^{2,2} & \dots & h_t^{2,N_r} \\ \vdots & \vdots & \ddots & \vdots \\ h_t^{N_r,1} & h_t^{N_r,2} & \dots & h_t^{N_r,N_r} \end{pmatrix}$$

where  $h_{t,i}$  is the channel fading coefficient from the transmit antenna  $i$  to the receive antenna  $j$  at time  $t$ .

The channel matrix here depends upon the structure of the antenna array (no of elements) and the spacing within the antenna elements. The channel at time  $t$  is modeled by a  $N_r * N_t$  matrix

### 4. Simulation Results

MIMO channel without transceiver impairments and  $N_t = N_r = 4$  and varying SNR shown in Fig 3. This figure provides information regarding capacity for low SNR and high SNR. At low SNR and high SNR values, capacity increases linearly with SNR values. It is also evident that there is stark difference between Shannon's capacity and MIMO system channel capacity without transceiver impairments.

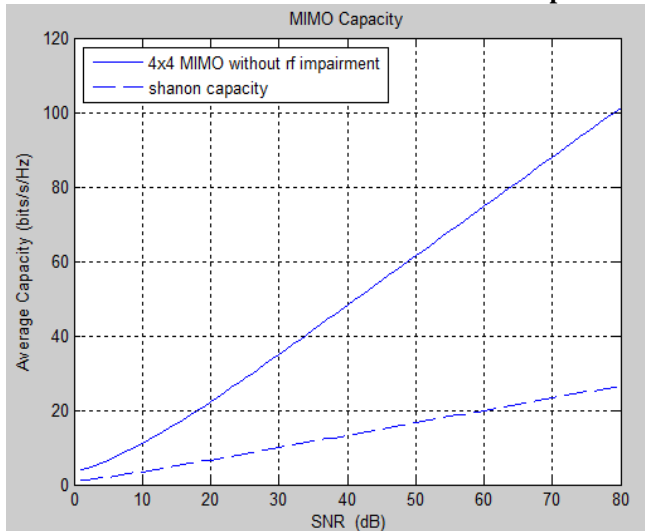


Figure 3: Average capacity of a 4x4 MIMO channel without transceiver impairments and Shannon's capacity.

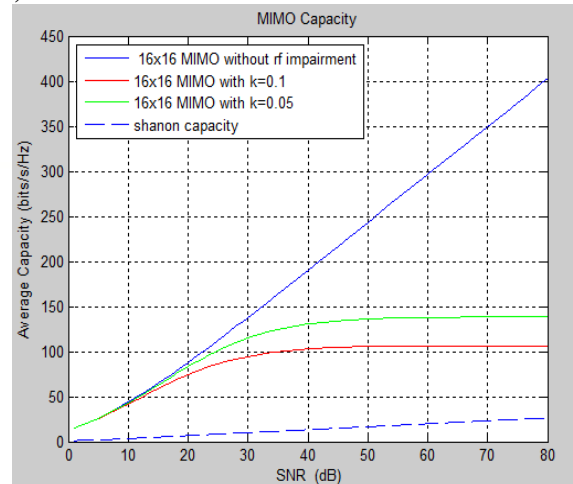


Figure 5: Average capacity of a 16x16 MIMO, with different levels of transceiver impairments i.e  $\kappa=0.05$  and  $\kappa=0.1$

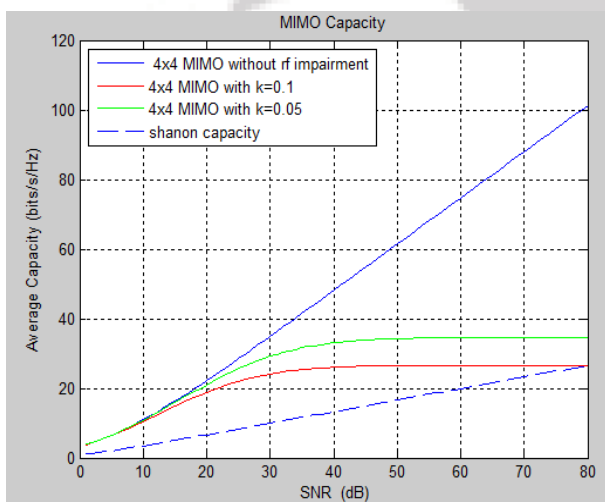


Figure 4: Average capacity of a 4x4 MIMO, with different levels of transceiver impairments i.e  $\kappa=0.05$  and  $\kappa=0.1$

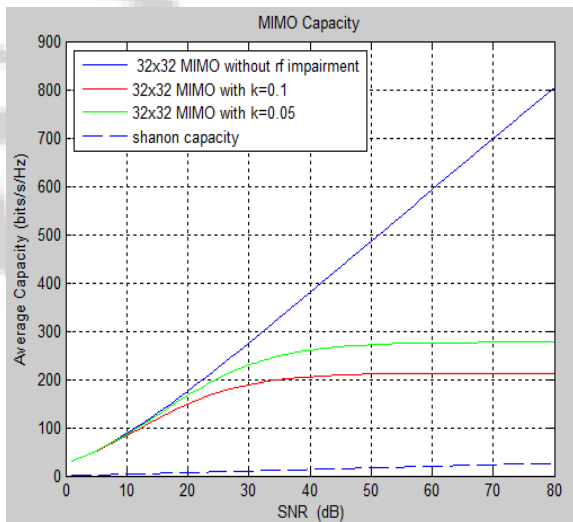


Figure 6: Average capacity of a 32x32 MIMO, with different levels of transceiver impairments i.e  $\kappa=0.05$  and  $\kappa=0.1$

A MIMO channel with  $N_t = N_r = 4$  and varying SNR. Fig. 4 shows the average capacity over different deterministic channels, either generated synthetically with independent  $CN(0, 1)$  entries or taken from the measurements in [3]. The level of impairments is varied as  $\kappa \in \{0.05, 0.1\}$ . Ideal and physical transceivers behave similarly at low and medium SNRs in Fig. 2, but fundamentally different at high SNRs. While the ideal capacity grows unboundedly, the capacity with impairments approaches the capacity limit. The difference between the uncorrelated synthetic channels and the realistically correlated measured channels vanishes asymptotically. Therefore, only the level of impairments,  $\kappa$ , decides the capacity limit.

MIMO channel with transceiver impairments and  $N_t = N_r = 16$  and varying SNR shown in Fig 5 and MIMO channel with transceiver impairments and  $N_t = N_r = 32$  and varying SNR shown in Fig 6. In both the ideal Channel capacity increases linearly while with impairments the capacity saturates at high SNR region. By increasing the antenna element, the channel capacity with impairments can be increased.

Table 1: Table showing channel capacity with and without Impairment for SNR values for different antenna array

SNR (in dB)	Capacity (Bits/S/Hz) 4 Antenna Elements		Capacity (Bits/S/Hz) 16 Antenna Elements		Capacity (Bits/S/Hz) 32 Antenna Elements	
	Without k	With K=0.05	Without k	With K=0.05	Without k	With K=0.05
10	10	10	40	40	90	90
20	22	20	80	70	180	170
30	35	25	140	120	280	230



40	50	26	190	135	390	260
50	62	26	240	140	490	270

#### 4.1 Comparison of Different Antenna Elements

Referring fig 4, fig 5 and fig 6, as well as Table 1, it can be concluded that as no. of antenna elements increases, there is a stark increase in the channel capacity of MIMO system and when this channel capacity is compared with Shannon's capacity, it becomes clear that MIMO system employs such techniques which increase capacity as compared to theoretical values. When the transceiver impairments come into picture, the channel capacity limit to some extent but by increasing the number of antenna elements, the capacity can be enhanced up to a limit. The graphs show the effect of transceiver impairments on SNR, channel capacity and number of antennas used in each antenna array.

#### 5. Conclusion

In this work, it was revealed that (even small) transceiver impairments severely degrade the channel capacity of MIMO systems. In conventional capacity analysis, the capacity of physical MIMO systems saturates in the high-SNR regime and the finite capacity limit is independent of the channel distribution. This fundamental result is explained by the distortion from transceiver impairments. It is also evident that there is a stark difference between Shannon's capacity and a practical MIMO system. Despite the practical importance of these impairments, little was known about their impact on the achievable performance. Technological advances can reduce transceiver impairments, but there is currently an opposite trend towards small low-cost low-power transceivers where the inherent *dirty RF effects* are inevitable and the transmission is instead adapted to them. The point-to-point MIMO capacity limit is an upper bound for scenarios with extra constraints. The capacity in such scenarios therefore saturates in the high-SNR regime even in small networks where the analysis is not applicable. So to enhance the capacity of MIMO systems, the antenna elements need to be increased for the higher channel capacity and data rates.

#### 6. Future Scope

By increasing the antenna element MIMO channel capacity can be increased. Now days smart antennas are efficiently increasing the channel capacity and reducing the power dissipation in transmission of signal in fading environments. This work will be extended to making MIMO systems compatible with smart antennas to obtain higher efficiency and mitigating the noise effect. Channel capacity enhancement will be carried out for MIMO OFDM also.

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