Derivation of Einstein Generalized Special Relativity Using Lorentz Transformation

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Abstract: ***Lorentz transformation is utilized to derive a useful expression for space, time in the presence of any field. This expression reduces to that of special relativity and compete with that of the Einstein generalized special relativity and Savickas. These expressions indicate that the space and time are affected by the field as well as by velocity. Unlike the old Einstein generalized special relativity model. This new model is not restricted to weak fields only, but holds for all fields including strong fields.***

Keywords: Lorentz transformation; Einstein generalized special relativity; weak fields

1. Introduction

Einstein theory of special relativity (SR) is one of the biggest achievements in modern. It changes radically the classical concept of space and time. The SR theory is based on two postulates, one of them is the homogeneity of space and the other one is the constancy of the speed of light in vacuum **[1]**. By considering Lorenz transformation and the above postulates, the ordinary expressions for time, length, mass and energy were obtained **[2]**. The SR theory succeeded in explaining a large number of physical phenomena, like meson decay, pair production and nuclear binding energy. Despite of these successes SR suffers from noticeable setbacks. First of all, at low particle speed its expression of energy consists only two terms, one representing mass energy and the other one stands for the kinetic energy.

There is no term representing the potential energy **[3]**. Thus, this energy expression does not satisfy correspondence principle, for it does inconformity with Newton expression for energy which consists of kinetic energy expressions. Moreover, SR cannot explain the gravitational red shift, which states that the gravitational field changes the photon frequency **[4]**. The mass of photon is directly proportional to its frequency and the periodic time is affected by the gravitational field as well. The basic shortcomings of special theory of relativity and general theory of relativity are presented elsewhere **[5].** However, Einstein's general relativity is a generally covariant theory of gravity. The laws of Nature are relativistic, and one of the main motivations to develop quantum field theory is to reconcile quantum mechanics with special relativity. The purpose of this paper is to derive Einstein generalized special relativity using Lorentz transformation.

2. Special Relativity Theory and its Disadvantages

The rigid rod is shorter when in motion in the direction of its length than when at rest, and the more quickly it is moving, the shorter is the rod. As a consequence of its motion the clock goes more slowly than when at rest. The most famous formula in theory of relativity, $E = mc^2$. Where, m is $\gamma m'$ .Where $\gamma = \sqrt{1 - v^2 / c^2}$. Suppose the system S is absolutely at rest, then we have $t_x = t_y = t_z = t_r = t$.

According to time formula of special theory of relativity, the time t, spatial coordinate x, mass m and energy E for a particle moving with speed v is given by **[5]**;

$$t = \gamma\left[t' + \frac{vx'}{c^2}\right],\; x = \gamma(x' + vt') \qquad (1)$$

For the theory of relativity, suppose system S' is transmitted along x-axis, then the time transformation formula in Lorentz transformation reads, $t' = \gamma\left[t - \frac{vx'}{c^2}\right]$.

Conservation of energy yields the fact that the sum of the kinetic and potential energies is a constant. However, the kinetic energy of a photon is given by $h\,f$, where h is Planck's constant and f is the frequency of the photon. If the mass-energy relation $E = mc^2$, which relates the kinetic energy to the product of mass and the square of the speed of light, is introduced, then an 'effective mass' for the photon may be deduced and is given by m= h f/c^2. The equation expressing conservation of energy then becomes hf - GMm/r = hf - GMhf/rc2 = constant. This equation immediately allows the well established expression for the gravitational red-shift to be deduced. The derivation of the expression for the gravitational red-shift, no appeal has been made to any aspect of the theory of general relativity, not even the principle of equivalence. The gravitational red shift of photon of frequency f' leads to change its frequency to f according to the relation;

$$hf = hf' - V \tag{2}$$

Where V is potential. But, since; $hf = mc^2$, $hf' = m'c^2$, $f = T^{-1}$ and $f' = T'^{-1}$.

It follows that equation (2) reads;

$$(m - m')c^2 = \Delta mc^2 = -V \,,\; \left(\frac{h}{T} - \frac{h}{T'}\right) = -V$$

$$\Rightarrow h\left(\frac{T' - T}{TT'}\right) = -V \Rightarrow \left(\frac{T' - T}{TT'}\right) = -\frac{V}{h}$$

$$\text{Thus; } \Delta T = T - T' = \frac{V}{h} TT' \tag{3}$$

Moreover, the definition of force in terms of potential V requires that;

$W = K.E = \int \vec{F}.d\vec{r} = -\int \nabla V.d\vec{r} = -V$. But,

$K.E = mc^2 - m'c^2$. Hence;

$$V = (m' - m)c^2 = \Delta mc^2 \tag{4}$$

Expressions (2), (3) and (4) indicates that the potential changes mass and time which is in direct conflict with equation (1) in which potential effect is absent.

3. Potential-Kinetic Energy Relation

A useful expression which relates kinetic to potential energy can be obtained by using the definition of force F for a particle of mass m moving in a field with potential V dependent on the position x only. According to the Newton's second law we can use to describe its motion **[6]**;

$$\vec{F} = m\frac{dv}{dt} = -\nabla V = -m\nabla \varphi = -m\frac{\partial \varphi}{\partial x} = -m\frac{d\varphi}{dx} \tag{5}$$

Where φ is a potential assumed to be dependent on the position x only.

Thus; $m\frac{dv}{dx}\frac{dx}{dt} = -m\frac{d\varphi}{dx}$. Then; $mv\frac{dv}{dx} = -m\frac{d\varphi}{dx}$ (6)

The dependence of φ on position is obvious as far as gravity, electromagnetic and strong nuclear force is position dependent. Integrating two sides, and assuming the initial velocity to be v_o , one gets;

$$m\int_{v_o}^{v} v dv = -\int_{0}^{\varphi} d\varphi \Rightarrow \frac{1}{2}m\left[v^2 - v_o^{\,2}\right] = -m[\varphi - 0] = \varphi$$

$$\therefore v = \sqrt{v_o^{\,2} - 2\varphi} \tag{7}$$

By replacing 0 by φ_o in equation (7) one gets;

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_o^{\,2} = -m[\varphi - \varphi_o],$$

$$K.E - K.E_o = -m[\varphi - \varphi_o]$$

$$K.E + m\varphi = K.E_o + m\varphi_o \,,\; E = E_o \tag{8}$$

Where, K.E and E stand for kinetic energy and total energy. This represents the principle of energy conservation. This relation can also be obtained by using ordinary Newton's second law relation for a linear motion, where;

$$v^2 = v_o^{\,2} - 2ax \tag{9}$$

But since; $F = ma$, $V = m\varphi = Fx = \max$, $V_o = m_o\varphi_o = Fx_o = \max_o$. It follows that; $\varphi = ax$. Hence;

$$v^2 = v_o^{\,2} - 2\varphi \tag{10}$$

Consider a particle moving with constant acceleration in a direction opposite to the field acceleration. The velocity takes form;

$v = -at + c_o \quad v = -\int adt + c_o$

Since at $t = 0$ $v = v_o$, it follows that; $v_o = c_o$. Thus;

$$v = v_o - at \tag{11}$$

Similarly x is given by

$x = \int vdt + c_1 = v_o t - \frac{1}{2}at^2 + c_1$.

At, $t = 0$, $x = x_o$ .Thus; $x_o = c_1$.

Hence;

$$x = v_o t - \frac{1}{2}at^2 + x_o \tag{12}$$

In view of (11) and (12);

$$x = x_0 + \frac{1}{2}(v_o - at) + \frac{1}{2}v_o t = x_o + \frac{1}{2}(v + v_o)t \tag{13}$$

Or one can write;

$$x = x' + v_m t \tag{14}$$

Where the mean velocity is defined to be;

$$v_m = \frac{1}{2}(v + v_o) \tag{15}$$

4. General Lorenz Transformation in the presence of Motion and Field

The Lorenz transformation which makes Maxwell's equations invariant was utilized by Einstein to construct SR. This transformation is based on the fact that the space is homogeneous, which means that;

$$x = \gamma(x' + v_m t') \text{ and } x' = \gamma(x + v_m t) \tag{16}$$

Assuming that the speed of light is constant one gets;

$$x = ct \,,\; x' = ct \tag{17}$$

Inserting equation (17) in (16) yields;

$ct = \gamma(ct' + v_m t') = c\gamma\left[1 + \frac{v_m}{c}\right]t'$. Therefore,

$$t = \gamma\left[1 + \frac{v_m}{c}\right]t' \tag{18}$$

$$ct' = \gamma(ct - v_m t) = c\gamma\left[1 - \frac{v_m}{c}\right]t$$

$$t' = \gamma\left[1 - \frac{v_m}{c}\right]t \tag{19}$$

Incorporating relation (18) in relation (19) yields;

$$t=\gamma^2\left[1+\frac{v_m}{c}\right]\left[1-\frac{v_m}{c}\right]t$$

Thus;

$$1=\gamma^2\left[1-\frac{v_m{}^2}{c^2}\right]$$

Hence;

$$\gamma=\frac{1}{\sqrt{1-{v_m{}^2}/{c^2}}} \tag{20}$$

If one consider the relation for a particle moving in a field which increases its velocity;

$$v^2=v_o{}^2+2\varphi$$

And by assuming v and v_o to represent the average values which are related to the maximum values v_x and v_{ox} according to the relations; $v=\frac{v_x}{\sqrt{2}}$, $v_o=\frac{v_{ox}}{\sqrt{2}}$. It follows that;

$v_x{}^2=v_{ox}{}^2+4\varphi$. Hence;

$$v_m{}^2=\left(\frac{v_{ox}+v_x}{2}\right)^2=\left(\frac{v_{ox}{}^2+v_x{}^2+2v_{ox}v_x}{4}\right)=\frac{2v_x{}^2-4\varphi+2v_x{}^2\sqrt{1-\frac{4\varphi}{v_x{}^2}}}{4} \tag{21}$$

For weak field; $\left(1-\frac{4\varphi}{v_x{}^2}\right)^{\frac{1}{2}}\approx 1-\frac{2\varphi}{v_x{}^2}$

Thus;

$$v_m{}^2=\frac{2v_x{}^2-4\varphi+2v_x{}^2-4\varphi}{4}=\frac{4v_x{}^2-8\varphi}{4}=v_x{}^2-2\varphi$$

. Hence; $\gamma=\frac{1}{\sqrt{1+\frac{2\varphi}{c^2}-\frac{v_x{}^2}{c^2}}}$

This reduces to Einstein Generalized Special Relativity for a weak field. In view of equation (21) and (21) it follows that;

$$\gamma=\frac{1}{\sqrt{1-\frac{\left(2v_x^2-4\varphi+2v_x\sqrt{v_x^2-4\varphi}\right)}{2c^2}}} \tag{22}$$

It is very striking to find that when no field exists $\varphi=0$ then;

$$\gamma=\frac{1}{\sqrt{1-{v_x^2}/{c^2}}} \tag{23}$$

Thus the model reduces to ordinary SR. Inserting (22) in (16) it follows that;

$$x=\frac{x'+\left(\frac{v_x{}^2-2\varphi+v_x\sqrt{v_x{}^2-4\varphi}}{2}\right)t'}{\sqrt{1-\left(\frac{2v_x^2-4\varphi+2v_x\sqrt{v_x^2-4\varphi}}{4c^2}\right)}} \tag{24}$$

It is clear that the position is dependent on potential as well as on speed v. The expression for t' can be obtained by eliminating x' from the two relations in (16) to get;

$$t'=\frac{t-\left(\sqrt{\frac{v_x{}^2-2\varphi+v_x\sqrt{v_x{}^2-4\varphi}}{2}}\right)x}{\sqrt{1-\frac{\left(2v_x{}^2-4\varphi+2v_x\sqrt{v_x{}^2-4\varphi}\right)}{4c^2}}} \tag{25}$$

Again time is affected by the field potential.

5. Discussion

The derivation of space-time relations according to Lorenz transformation for a particle moving with speed v in a field of potential φ in equation (11), indicates that such transformation reflects the pace-time homogeneity under these conditions. Equation (12) shows also that the speed of light in the presence of any field is assumed to be that of free space. The expressions for space and time obtained in equations (17) and (18) indicate that the space time are affected by field potential per unit mass φ as well as velocity v. These expressions reduce to that of SR and compete with M. Dirar **[7]** and Savickas models **[8]**. The advantage of this derivation compared to the two models reflects in the fact that this relation is not restricted to gravitational field only, but applies to any field. This fact is true as far as equation (7) is a general relation. Moreover, this model is not restricted to weak field since it shows that the expression for effects of field and speed on space and time holds for strong as well as weak field. Thus it is unlike M. Dirar Model which holds only for weak field.

6. Conclusion

The new derivation of EGSR is more general than other models in many respects. First of all it holds for any field including gravity, electromagnetic, weak and strong nuclear fields. The derivation is not restricted to weak fields but holds for all fields including strong fields. The derivation is based on homogeneity of space, constancy of light speed, and a general relation between speed and potential. Thus, it is more general than the other derivations of EGSR.

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