An Unsteady Mathematical Model of Human Dermal Region Due to Temperature Distribution of Artirel Blood Flow in Cold Atmosphere

Yogesh Shukla[1], Sonia Shivhare[2]

[1]Department of Mathematics, Amity University Madhya Pradesh, India

[2]Department of Mathematics, Amity University Madhya Pradesh, India

Abstract: ***This paper deals with the change in ambient temperature which can adversely disturb the thermostat in human body. Acute thermal disorders above and below the normal range of body temperature are termed as hyperthermia and hypothermia thermo-regulation in human dermal region. A one dimensional temperature distribution model in human dermal region involving bio-heat equation has been solved using finite element method the natural three layers of dermal part – epidermis, dermis, and subcutaneous tissues are considered for the study. The important parameters like blood mass flow rate, metabolic heat generation rate and thermal conductivity are taken in each layer. The loss of heat from the skin surface to the environment is taken due to convection, radiation, and insensible perspiration***.

Keywords: Rate of metabolism, blood mass flow rate, thermal conductivity, heat generation, finite element method

1. Introduction

Human body maintains its body core temperature constant within small range between (37 - 0.6)0C and (37 + 0.6)0C. The skin and subcutaneous tissue (SST) is the major organ that controls heat and moisture flow to and from the surrounding environment. SST plays a key role for heat transfer within human body, and hence there is the variation of temperatures in the region in accordance with surrounding temperatures which is composed of three layers: epidermis, dermis, and subcutaneous tissue. The layer's geometry is irregular at the junction of the dermis where the epidermis projects into the dermis like cone called epidermal ridges. Temperature changes can affect tissues in several ways; it can kill cells, denature proteins, slow down or speed up metabolism, involved in pathological changes etc. Heat regulation in human body is characterized by conduction, convection, radiation as well as blood perfusion, metabolism and evaporation. Metabolic heat generation and blood perfusion play an important role in heat regulation in several parts of human body. Blood perfusion is a physiological term that refers to the process of nutritive delivery of arterial blood to a capillary bed in the biological tissue. Metabolism is the sum of total of all chemical reactions involved in maintaining the living state of the cells, and thus the organism. Evaporation is the conversion of a liquid to a vapor and is always accompanied by cooling.

2. Mathematical Model

Perl's Bio heat partial differential equation in one dimensional unsteady state case for heat distribution in the tissues of SST region of human body can be written as:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + m_b c_b (T_b - T) + S = \rho c\frac{\partial T}{\partial t} \qquad (1)$$

Here the effect of metabolic heat generation and blood mass flow are given by the terms S and $m_b c_b(T_b\text{-}T)$ respectively. T_b, K, ρ, c, m_b and c_b are body core temperature, thermal conductivity, density and specific heat of tissue; blood mass flow rate and specific heat of blood respectively. Right hand side of eq. (2.1) shows the storage of heat in tissues. The first two terms of the left hand side represents conduction of heat in the tissues, caused by the temperature gradient and third term is for heat transport between the tissues and microcirculatory blood perfusion. The last term represent heat generation due to metabolism.

At the skin surface heat loss or gain due to convection, radiation and evaporation can be written as

$$-K\frac{\partial T}{\partial \eta} = h(T - T_a) + LE \qquad (2)$$

Where

$T_a =$ Atmospheric temperature

$L =$ Latent heat of evaporation

$E =$ Rate of sweat evaporation

$\eta =$ Normal to skin surface

$h =$ Heat transfer coefficient

And we have taken

$T_V = qT$ (3) (2.3) (2) Whe

Where q is known constant its value is nearer to 1.

The partial differential equation (1) coupled with equation (2) and equation (3) , written in one dimensional unsteady state and compared with "Euler-Lagrange's equation is transformed into the following equivalent form :

$$I = \frac{1}{2}\int_0^{a_3}\left[K\left(\frac{\partial T}{\partial x}\right)^2 + mc_b(T_A - T_V)^2 - 2ST + \rho C\frac{\partial T^2}{\partial t}\right]dx + \frac{1}{2}h(T_a - T)^2 + LET$$

We assumed the values T_0, T_1, T_2, T_3 to T at the points $x=0$, $x=a_1$, $x=a_2$ and $x=a_3$ respectively. Here $'a_1'$ is the thickness of epidermis and dermis together and $'a_3'$ is the total thickness of dermal region. Let $T^r(x)$ (r = 1, 2, 3) represents the linear values of T(x) in three sub regions respectively.

3. Solution

If I_1, I_2 and I_3 are the values of I in three sub-regions then $I = \sum_{i=1}^{3} I_i$.

For solving equation (3.1) we used same conditions which are given in Table 1. The following additional conditions initial and boundary are also used :

$$T(x,0) = 22.87 + px \quad , p > 0 \ , 0 \le x \le a_3$$

$$T(a_3, t) = T_b$$

$$T(a_3, 0) = T_b$$

Where p is unknown constant.

We get,

$$I_1 = \frac{K_1}{a_1}(T_1 - T_0)^2 + \frac{h}{2}(T_0 - T_a)^2 + LET_0 + \frac{1}{6}\rho C a_1 \frac{\partial}{\partial t}\left(T_0^2 + T_1^2 + T_0 T_1\right)$$

$$I_2 = (T_2 - T_1)^2 A_1 + (T_2 a_1 - T_1 a_2)^2 A_2 + (T_2 a_1 - T_1 a_2)(T_2 - T_1) A_3 + (T_2 a_1 - T_1 a_2) A_4 + (T_2 - T_1) A_5 + A_6 + \frac{1}{6}\rho C (a_2 - a_1)\frac{\partial}{\partial t}\left(T_1^2 + T_2^2 + T_1 T_2\right)$$

$$I_3 = (T_3 - T_2)^2 B_1 + (a_3 T_2 - a_2 T_3)^2 B_2 + (T_3 - T_2) B_3 + (a_3 T_2 - a_2 T_3) B_4 + (a_3 T_2 - a_2 T_3)(T_3 - T_2) B_5 + B_6 + \frac{1}{6}\rho C (a_3 - a_2)\frac{\partial}{\partial t}\left(T_2^2 + T_3^2 + T_2 T_3\right)$$

Where,

$$A_1 = \frac{a_2 K_1 - a_1 K_3}{2(a_2 - a_1)^2} + \frac{(K_3 - K_1)(a_2 + a_1)}{4(a_2 - a_1)} + \frac{q^2 M (3a_2^2 + 2a_2 a_1 + a_1^2}{24(a_2 - a_1)},$$

$$A_2 = Mq^2 \frac{1}{4(a_2 - a_1)},$$

$$A_3 = -\frac{MqT_b (a_2 - a_1)(a_1 + 3a_3)}{12(a_3 - a_1)},$$

$$A_4 = -Mq \frac{T_b (a_2 - a_1)}{3(a_3 - a_1)} - \frac{S}{2},$$

$$A_5 = Mq^2 \frac{(a_1 + 2a_2)}{6(a_2 - a_1)},$$

$$A_6 = M\left(\frac{T_b}{a_3 - a_1}\right)^2 \frac{(a_2 - a_1)^3}{8},$$

$$B_1 = \frac{q^2 M}{6}\frac{(a_3^2 + a_2^2 + a_3 a_2)}{(a_3 - a_2)} + \frac{K_3}{2(a_3 - a_2)},$$

$$B_2 = \frac{Mq^2}{2(a_3 - a_2)},$$

$$B_3 = \frac{-T_b q M \left(2a_2^2 + 3a_3^2 + 2a_2 a_3 - 3a_1 a_3 - 3a_1 a_2\right)}{6(a_3 - a_1)} - \frac{S(a_3 + a_2)}{2}$$

$$B_4 = \frac{-T_b qM}{3}\frac{(a_3 + a_2 - 2a_1)}{(a_3 - a_1)} - S,$$

$$B_5 = \frac{Mq^2 (a_3 + a_2)}{2(a_3 - a_2)},$$

$$B_6 = M\left(\frac{T_b}{a_3 - a_2}\right)^2 \frac{\left(a_3^2 + a_2^2 + a_3 a_2 - 3a_1 a_2 - 3a_1 a_3 + 3a_1^2\right)(a_3 - a_2)}{6}$$

Since, T_3 is equal to body core temperature, so we minimize I for T_0, T_1, T_2, T_3. Accordingly, we get following system of algebraic equations:

$$L_1 T_0 + L_2 T_1 + \frac{1}{6}\rho C\left[2a_1 \frac{\partial T_1}{\partial t} + a_1 \frac{\partial T_2}{\partial t}\right] = W_0$$

$$M_1 T_0 + M_2 T_1 + M_3 T_2 + \frac{1}{6}\rho C\left[a_1 \frac{\partial T_0}{\partial t} + 2a_2 \frac{\partial T_1}{\partial t} + (a_2 - a_1)\frac{\partial T_2}{\partial t}\right] = W_1$$

$$N_1 T_1 + N_2 T_2 + \frac{1}{6}\rho C\left[(a_2 - a_1)\frac{\partial T_1}{\partial t} + 2(a_3 - a_2)\frac{\partial T_2}{\partial t}\right] = W_2$$

Where,

$$L_1 = \frac{K_1}{a_1} + h$$

$$L_2 = -\frac{K_1}{a_1}$$

$$W_0 = hT_a - LE$$

$$M_1 = -\frac{K_1}{a_1}$$

$$M_2 = \frac{K_1}{a_1} + 2A_1 + 2a_2^2 A_2 - 2a_2 A_5$$

$$W_1 = A_3 - a_2 A_4$$

$$N_1 = -2A_1 - 2a_1 a_2 A_2 + (a_1 + a_2) A_5$$

$$N_2 = 2A_1 + 2a_1^2 A_2 - 2a_1 A_5 + 2B_1 + 2a_3^2 B_2 - 2a_3 B_5$$

$$W_2 = [2B_1 + 2a_2 a_3 B_2 - (a_2 + a_3) B_5] T_3 - A_3 + a_1 A4 + B_3 - a_3 B_4$$

Equation (4), (5) and (6) are written in the following matrix form:

$$C\dot{T} = -\overline{K}T + W$$

Where

$$C = \frac{1}{6}\rho c\begin{bmatrix} 2a_1 & a_1 & 0 \\ a_1 & 2a_2 & a_{2-}a_1 \\ 0 & a_2 - a_1 & 2(a_3 - a_1) \end{bmatrix}$$

$$\dot{T} = \begin{bmatrix} \frac{\partial T_0}{\partial t} \\ \frac{\partial T_1}{\partial t} \\ \frac{\partial T_2}{\partial t} \end{bmatrix}$$

$$\overline{K} = \begin{bmatrix} L_1 & L_2 & 0 \\ M_1 & M_2 & M_3 \\ 0 & N_1 & N_2 \end{bmatrix}$$

$$T = \begin{bmatrix} T_0 \\ T_1 \\ T_2 \end{bmatrix} \; \& \; W = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}$$

This system of equation (7) is solved by Crank-Nilcoson method, therefore we get

$$\left(C + \frac{1}{2}\overline{K}\Delta t\right)T^{V+1} = \left(C - \frac{1}{2}\overline{K}\Delta t\right)T^{V} + W\Delta t$$

Where Δt is time interval and V= 0, 1, 2, ……..n (number of interval with respect to time).

4. Numerical Result

For numerical result we make use of following values of physical quantities.

$h = 0.15 \times 10^{-3}$ $Cal/cm2\ 5^0\ C$

$L = 579$ cal/gm

$K_1 = 0.5 \times 10^{-3}$ cal/cm 5^0 C

$K_3 = 1.0 \times 10^{-3}$ cal/cm 5^0 C

$T_a = 33\,^oC$

$T_b = 37\,^oC$

$a_1 = 0.10\ cm$

$a_2 = 0.35\ cm$

$a_3 = 0.50\ cm$

$M = 0.52 \times 10^{-3}$ cal/cm 5^0 C

$S = 0.3 \times 10^{-3}$ $cal/cm^3\ S$

$E = 0.008 \times 10^{-3}\ gm/cm^2\ S$

$\rho = 1.05\ g/cm^3$ $C = 0.83\ cal/gm^0c$

5. Numerical Result and Discussion

The SST region is divided into three parts namely epidermis (the outer one), dermis (under the epidermis) and sub dermis (below the dermis layer). The value of K, M, and S are assumed constant epidermis layer in dermis layer. K, M, S and other values are calculated using Lagrange's interpolation polynomial. No blood vassals present in sub dermis so values of those parameters are taken zero.

In present paper mathematical model has been developed to analyze the temperature variation in dermal region. The total thickness is taken 0.95cm.The thickness of subcutaneous; dermis and epidermis are 0.50cm, 0.35cm and 0.10cm respectively.
The values of physical and physiological parameters have been taken from Cooper and Trazek as given below.

Table 1: Values for physical and physiological parameter

Thermal Conductivity (cal/cm min ^oC)	Heat Transfer Coefficient h (cal/cm^2 min ^oC)	Specific Heat of Tissues c (cal/gm ^oC)
K_1=0.060, K_2=0.045, K_3=0.030	0.009	0.830
Blood Density of Tissues ρ (gm/cm^3)	Latent Heat L (cal/gm)	Body Core Temperature T_b (^oC)
1.090	579.0	37

Table 2: M, S and E for different atmospheric temperature

Atmospheric Temperature T_a (^oC)	Rate of Evaporation E (gm/ cm^{2}min)	Blood Mass Flow Rate M (cal/ cm min. ^oC)	Rate of metabolism S (cal/cm^3 min^1)
23	0, 0.24x10^{-3}, 0.48x10^{-3}	0.0180	0.0180
33	0.24x10^{-3},0.48x10^{-3}	0.0315	0.0180

Numerical solutions are obtained for sub -dermis, dermis and epidermis region. Initially it is assumed that SST region is fully insulated. So temperature of each layer at time t=0 is equal to the 37^0 C.

References

[1] W.Perl *Heat and matter distribution in body tissue and determination of tissue blood flow by local clearance methods" journal of theoretical biology 2(3),1962,201-235.*
[2] Perl,W. and Hirsch, R.L. ,*Local blood flow in kidney tissue by heat clearance method, J. Theoret.Biol. 10(2), 1966, 251-280*
[3] *Cooper, T.E. and Trezek, G.J.,A Probe technique for determining the thermal conductivity of tissues, J. Heat transfer, ASME 94 , 133-140,1972,a.*
[4] *Cooper, J.E. and Trezek, G.T.,on the freezing of tissues, J. Heat transfer , ASME, 94,251-253,1972,b.*
[5] *Patterson, A.M., Measurement of temperature profiles in human Skin, S.Afr J. Sci, 72, 78-79,1976.*
[6] *Saxena, V.P. and Arya, D., Steady state heat distribution in epidermis, dermis and Subcutaneous tissues" J.Theovet, Biol. 89, (1981),423-432.*
[7] Saxena V.P. and Bindra J.S., Indian Journal of pure and Applied Mathematics,18(9), (1987), 846-855.
[8] Yadav , A.S., " Mathematical Study of heat flow in human skin with thermal injury" Ph.D. Thesis, J.U. Gwalior,1998.
[9] *Trezek G.J. and Cooper T.E. ,Analytical determination of cylindrical source temperature field and their relation*

to thermal diffusivity of brain tissue, Thermal prov. in Bio-tech ; ASME, NY, 1-15,1968.

[10] *Saxena, V.P., Application of similarity transformation to unsteady state heat migration problem in human skin and subcutaneous tissues,Proc. 6th Int. Heat Transfer Conf. 3, 65-68,1978.*

[11] Saxena V.P. and Pardasani K.R. , "Effect of dermal tumors on temperature distribution in skin with variable blood flow" Bull. Math Bio. 53(4), 525-536 (1991).

[12] D. B. Gurung," two dimensional temperature distribution model in human dermal region exposed at low ambient temperatures with air flow" Kathmandu university journal of science, engineering and technology, vol. 8, no. ii,11-24(2012).