Optimization Design and Calculation of the Variable Stiffness Coil Spring Applied to Vehicles

Qu Xian[1], Zhao Shu-en[2]

[1]School of Mechanics & Automobile Engineering, Chongqing JiaoTong University, Chongqing, 400074, China

[2]School of Mechanics & Automobile Engineering, Chongqing JiaoTong University, Chongqing, 400074, China

Abstract: ***On the basis of action and requirements of variable stiffness coil spring, constrained nonlinear optimization mathematics model of variable stiffness coil spring is established. Using a certain car front suspension stiffness of helical springs as an example, spring wire diameter d, lap number n and spring index C are design variables, spring wire shear stress is less than the allowable shear stress for the constraint, spring KP is as large as possible and spring-mass is minimum for objective function. Then, using Optimization Toolbox to optimize design and calculate. Finally, the optimal design is calculated which use the front suspension of vehicle as example. The results show that the multi-objective mathematical optimization model is simple and accurate. Meanwhile, it overcome the shortcomings of the traditional error method and graphical method.***

Keywords: multi-objective, optimal design, variable stiffness spring, vehicle

1. Introduction

The transportation is developed rapidly. Besides, the requirements of comfort are improved. Spring is parts elastic element which is used in industrial machinery commonly [2]. When it is light-load, it has small deformation. While, when it is overload**, it** has large. Meanwhile, changes of the effective spring stiffness can avoid the phenomenon of resonance cars. Now, commonly used variable stiffness helical springs are mainly contained variable pitch, variable diameter, variable spring wire diameter or several simultaneously change. In this study, the variable pitch helical springs of variable stiffness is studied.

Many researchers have done about variable stiffness spring in domestic and foreign and fruitful results have been achieved. Yao Wei learn from discrete ideas and spring design theory to propose a reverse design method of variable stiffness helical springs, the stiffness characteristics of variable stiffness coil spring are obtained by finite element simulation analysis[2]. For Shang Yue-jin, all springs in a variable-stiffness spring group are integrated as a whole system, then using optimization software of finite element analysis software to optimize [3]. However, the process of variable stiffness spring is difficult, the design is not mature. Therefore, the spring stiffness kp is large as soon as possible and spring-mass minimum are the objective function. The design variables include spring wire diameter d, the number of turns n and spring index C**.** The optimization model is established which spring wire shear force is less than the allowable. Then, using MATLAB optimization toolbox to optimize the multi-objective model**,** the stiffness of the spring is improved**.**

2. The process of optimization and analysis

2.1 Variable stiffness coil spring

Variable stiffness coil spring consist of two or more different pitch spring[4]. The three-dimensional map of variable stiffness coil spring is shown Fig.1.

Figure 1: The three-dimensional map of variable stiffness coil spring

2.2 The process of optimization

The flow chart of optimization process is shown as Fig.2.

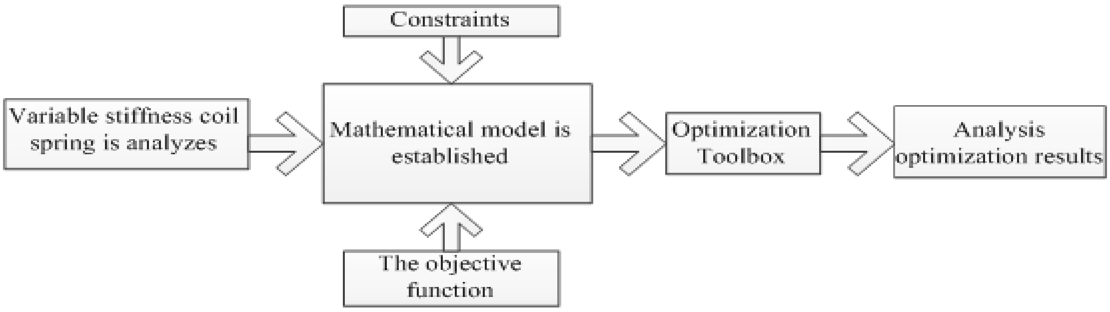

Figure 2: The flow chart of optimization process

From the Fig.2 we can see, multi-objective optimization design include the following steps generally:

1) The physical problems of design turn into mathematical models. Mathematical model describe the nature of engineering problems, reasonable and effective mathematical model is established to guarantee optimize design. The design variables are selected, the constraints are listed, the objective function is given when a mathematical model is created.

$$MinF(x)$$
$$S.tR_i(x)(i=1,2\ldots n)$$

2) Where, F(x) is object function; $R_i(x)$ is contains. The appropriate method is selected**. T**he following principles are followed when it selects optimization methods: suiting mathematical model, solving high efficiency , high precision, less downtime.
3) Computer Solution , optimized design;
4) Compare and analysis the optimization results.

3. Mathematical model of variable stiffness coil spring

a. Determining the design variables

The design variables which affect spring stiffness and spring -mass are spring wire diameter *d*, the number of turns n and spring index *C*.

$$x=[d \quad C \quad n_1 \quad \cdots \quad n_j]^T=[x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_{j+2}]^T$$

Where, *d* is the diameter of spring wire; n_1, n_2 ... n_j are the spring segments of different pitches, it is 2 or 3.

b. Objective function is determined

According to the characteristics of the spring work and its specific requirements, the objective function is established. In the process of spring design, the life, mass and stiffness are considered, economy is also considered [5]. In this study, spring stiffness as large as possible and spring-mass minimum are the objective function, as follow.

$$Maxf(x)_1=k_p=Gd^4/8D_2^3(n_1+n_2+n_3) \quad (1)$$

$$M\inf_1(x)=\sum_{j=1}^{3}\left(n_j\pi^2 D_2 d^2\rho\right)/4 \quad (2)$$

Where , n_i(i=1,2,3......j) are the number of segments with different pitches; *n* are number of coils; D_2 is spring diameter; *ρ*is spring material density; *d* is spring wire diameter of spring; *G* is shearing modulus of elasticity, GPa. For the multi-objective design problems of variable stiffness helical springs, one goal is to make the spring-mass minimum, the other is the spring stiffness as large as possible. According to the ideas of the same objective function, the original Multi-objective function is constructed a new one by a method, then using new one to evaluate[7]. Therefore, f2 (x) / f1 (x) is the evaluation function to solve design variables $x=[d \quad C \quad n_1 \quad n_2 \quad n_3]^T$.

$$\min f(x)=f_2(x)/f_1(x)=\frac{\sum_{j=1}^{3}\left(n_j\pi^2 D_2 d^2\rho\right)/4}{Gd^4/8D_2^3 d^4(n_1+n_2+n_3)}$$
$$=\frac{\sum_{i=3}^{5}x_i\pi^2 D_2 x_1\rho/4}{Gx_1^4/8D_2^3(x_3+x_4+x_5)} \quad (3)$$

The objective function is settled, as follows.

$$\min f(x)=2000D_2^4(x_3+x_4+x_5)^2\pi^2\rho/Gx_1^3 \quad (4)$$

3.3 Constraint is determined

In this study, the variable stiffness helical springs of vehicle front suspension are studied. The constraints mainly include spring strength, spring wire diameter, spring index, the fatigue strength of spring and stability constraints, as follow.

[2] Strength conditions is

$$\tau=8KCP/\pi d^2\le[\tau] \quad (5)$$

Where, τ, [τ] are shear force of spring and allowable shear force respectively; K is compensation factor, which compensates for the effects of the spring wire helix angle and curvature. For circular cross-section of the spring , K can be calculated, as follows

$$K\approx\frac{4C-1}{4C+1}+\frac{0.615}{C}$$

[3] Maximum deformation conditions[4]

Free height of the spring is

$$H_0=\sum_{i=1}^{3}n_i t_i+d \quad (6)$$

after tight, the height is

$$H_a=1.01d\left(\sum_{i=1}^{3}n_i-1\right)+2d/3 \quad (7)$$

the maximum deformation of spring is

$$\delta_{\max}=F_{\max}/k=8F_{\max}c^3(n_1+n_2+n_3)/Gd \quad (8)$$

There

$$\delta_{\max}\le H_0-H_a \quad (9)$$

[4] spring index conditions

$$4\le C\le 16 \quad (10)$$

[5] Spring fatigue strength conditions[6]

$$S=\tau_0+0.75\tau_{\min}/\tau_{\max}\ge[S] \quad (11)$$

Where,[S]is allowable safety factor; τ0 is pulse fatigue limit of spring material.

[6] Unstable conditions

The spring is fixed at both ends so

$$b=H_0/D_2\le 5.3 \quad (12)$$

[7] Helix angle conditions is

$$\alpha=\arctan(t/\pi D)\le 9 \quad (13)$$

3.4 Solving the problem

In summary , the optimization model of variable pitch helical spring is

$$\min f(x)=2000D_2^4(x_3+x_4+x_5)^2\pi^2\rho/Gx_1^3$$

$g_1(x) = 8KCP/\pi x_1^2 - [\tau] \le 0$;

$g_2(x) = 4 - x_2 \le 0$;

$g_3(x) = x_3 - 16 \le 0$;

$g_4(x) = [S] - \tau_0 + 0.75\tau_{\min}/\tau_{\max} \le 0$;

$g_5(x) = H_0/x_2 - 5.3 \le 0$;

$$g_6(x) = 8F_{\max} x_2^3 (x_3 + x_4 + x_5)/Gx_1 \le \left(\sum_{i=3}^{5} x_i t_i + x_i\right) - (1.01 x_1 (\sum_{i=3}^{5} x_i - 1) + \frac{2x_1}{3}).$$

4. Optimization design

MATLAB optimization toolbox provides a complete solution to solve optimization problems [8]. Variable stiffness helical springs are constrained nonlinear optimization problem, using the call function fmincon to solve minimum [9].

Part programs as follows.

Using file editor to create a file M with the objective function (my fun.m:

Function f=myfun(x)

$$f = 2000 \cdot D_2 \hat{} 4(x(3)+x(4)$$

$$+x(5))\hat{}2 \cdot \pi \hat{} 2 \cdot \rho/(G \cdot x(1)\hat{}3)$$

Due to constraints in the nonlinear constrained, So M file is wrote which describe the non-linear constraints (mycoun. m) [10].

....

% Multi-objective optimization function is called.

[x, fval, exit flag, output, lambda]=........

fmincon(@ myfun,x0,A,b,[],[],lb,[],@ mycoun)

After running the program, the results are accurate approximation, find the optimal solution.

4.1 Example of optimization design

At present, variable stiffness coil springs are used in many automobile manufacturers. Taking the front suspension spring of vehicle as example in order to test the feasibility of method. According to the system design theory, the spring parameters are $k_{\min}$=50N/mm, k_{max}=80N/mm;cylinder numbers $1 \le n_1 \le 3$,$3 \le n_2 \le 5$, $1 \le n3 \le 3$; the wire diameter of spring is $12 \le d \le 22$; spring diameter is $120 \le D_2 \le 180$; The minimum spring load is 4KN, the maximum is16.39KN.According to the spring design parameters of original vehicle suspension, the materials is 50Crv, density isρ = 7900kg/m3.shear modulus is G=81Gpa; allowable shear force is [τ]=810Mpa; pulse fatigue limit is τ0=2000Mpa; safety factor is S_{min}=1.3.Then the data is substituted into program, the optimal results are shown in table 1.

Table1 Analysis results

	Original proposal	Optimization proposal
Intermediate diameterD_2	149	141
Diameter d	19	19.5
Number of segments n_1	2	2
Number of segments n_2	5	3
Number of segments n_3	2	2
Quality stiffness ratio	0.1592	0.0980

5. Conclusion

From the viewpoint of system theory, the variable stiffness coil springs of variable pitch is designed by multi- objective optimization. Taking the ratio of spring-mass and stiffness minimum as objective function**, mathematical** model is built. Then, using MATLAB optimization toolbox to optimize and analyze. Finally, taking the vehicle front suspension springs as example, the ratio of the mass and stiffness is calculated. The method is simple and convenient, the quality and efficiency are improved. Besides, This optimization is not only apply to variable pitch coil spring ,but also apply to other springs. For construction machinery manufacturing industry, it has practical significance.

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Author Profile

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Author Profile

QU Xian she studied at Chongqing Jiaotong University for Vehicle Engineering from 2012.

Zhao Shuen, male graduated Dr. of Chongqing University. Now he works in Chongqing Jiaotong University.