Nonlinear Dynamic Study of Switched Reluctance Machine

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Abstract: ***The Switched Reluctance Machine (SRM) has recently been adopted in industrial and domestic applications due to its simple structure and low cost. But it's highly nonlinear nature makes it difficult to control. This article presents the modeling, command and control of SRM 8/6, the linear modeling is done by the "small AC signal" technique. Its control is done in closed-loop current with PID controller, voltage inverter and PWM block. The performance of the control structure are simulated on Matlab / Simulink***

Keywords: Switched Reluctance Machine, PID, AC small signal, PWM.

1. Introduction

Compared with AC and DC machines, the Switched Reluctance Machine is very reliable, easy to manufacture and low cost. Since each phase is largely independent physically, electrically and magnetically from other phases of the other machines. The SRM can reach very high speeds [1] (up to 50,000 rpm). The non-linearity of the SRM makes it difficult to control; the technique "AC small signal" allows us to have a linear model of the machine. In this article we will first present the non-linear model of the machine, the power converter and PWM block of the SRM, then the principle of the technique "AC SMALL SIGNAL" we apply it afterwards on the SRM and we end up with simulations control of the SRM with corrector.

2. Structure and model of SRM

2.1 Structure of the SRM

The SRM machine includes a toothed stator and rotor, the stator is constituted by a fixed frame comprising notches in which the conductors are housed, and the rotor is constituted by a single metal mass of soft machined iron, or stacked sheets to form teeth.

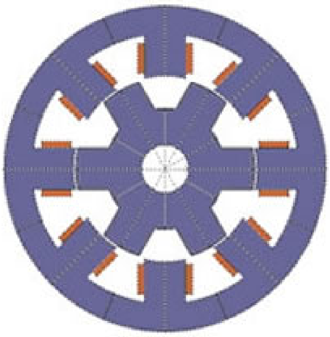

Figure 1: Structure of SRM 8/6

The operating principle of an SRM is that of an electromagnet. When a phase is energized, the rotor turns to get into the position where the flux created by the stator is maximum. This position is called conjunction position (Fig. 2 (a)). The opposite position where the flow is minimum is called the opposition position (Fig.2 (b)) [2].

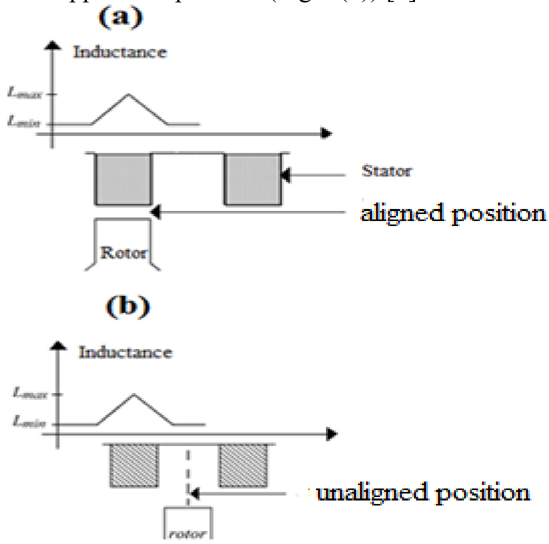

Figure 2: (a) aligned position, (b) unaligned position

2.2 Model of the SRM

NOTATIONS
V_k: The voltage at the phase terminal
I_k: The current flowing through the phase
R: Resistance of the phase
T_e: Electromagnetic torque
Φ_k: The flow in the stator pole
ω_r: Rotational speed
θ: Angular position of the rotor relative to the stator
L_k: Inductance of the phase
T_L: Load torque
J: The inertia
B: Coefficient of friction
The voltages at the windings terminals can be calculated by:

$$V_k = RI_K + \frac{d\Phi_k}{dt} \qquad k = 1,2,3,4. \qquad (1)$$

The flow in the stator pole is defined as the product of the inductance of the phase and the current flowing there through [3].

$$\Phi_k(\theta) = L_k(\theta, i) I_k(\theta) \qquad k = 1,2,3,4 . \qquad (2)$$

The rotational speed of the SRM is given by:

$$\omega_r = \frac{d\theta}{dt} \qquad (3)$$

By replacing the flow, and the speed with the expressions (2) and (3), the equation (1) will be described by the following formula:

$$V_k = RI_k(t) + I_k(t)\frac{dL_k(t)}{dt} + L_k(t)\frac{dI_k(t)}{dt} \qquad (4)$$

The mechanical equation of the SRM is:

$$j\frac{d\omega}{dt} = T_e - T_L - B\omega \qquad (5)$$

When the magnetic circuit is not saturated, the electromagnetic torque is given by:

$$T_{ek} = \frac{1}{2}\frac{dL_k}{d\theta} I_k^2 \qquad k = 1,2,3,4 . \qquad (6)$$

Note that the sign of the torque does not depend on the current direction. For a motor torque, the phase must be supplied when the inductance is increasing and for a braking torque, the phase must be supplied when the inductance decreases [4].
The average torque can be written as the superposition of torques of each motor phase:

$$T_e = \sum_{Phase=1}^{4} T_{Phase} \qquad (7)$$

3. Technique "ac small signal modeling"

3.1 Principle

The "AC small signal" technique has been widely used in power converters for many years. It is commonly used as a tool to analyze the nonlinear dynamic systems. The linear model can be obtained by disturbing and linearizing the signal about an operating point. When it is a linear model, it can be solved using conventional analysis techniques to obtain the transfer function, the output impedance and other properties.

3.2 Application of the technique "ac small signal modeling" on SRM 8/6

The mechanical equations of SRM can be written:

$$T_e - T_L = J\frac{d\omega}{dt} + B\omega \qquad (8)$$

$$T(\theta, I) = \frac{1}{2} I^2 \frac{dL(\theta, I)}{d\theta} \qquad (9)$$

(8) and (9) we have (10):

$$\frac{1}{2} I^2 \frac{dL(\theta, I)}{d\theta} - T_L = J\frac{d\omega}{dt} + B\omega \quad (10)$$

$$V = RI + L(\theta, I)\frac{dI}{dt} + \omega I \frac{dL(\theta, I)}{d\theta} \quad (11)$$

We set:

$$\left.\begin{aligned} I &= I_0 + \hat{I} \\ \omega &= \omega_0 + \hat{\omega} \\ V &= V_0 + \hat{V} \\ T_L &= T_{L0} + \hat{T}_L \end{aligned}\right\} \quad (i)$$

$\hat{I}, \hat{\omega}, \hat{V}$ *and* $\hat{T}_L$: *perturbing signal*
I_0, ω_0, V_0 *and* T_{L0} : quiescent value

We replace (i) in (11):

$$V_0 + \hat{V} = R(I_0 + \hat{I}) + L\frac{d(I_0 + \hat{I})}{dt} + \frac{dL}{d\theta}(\omega_0 I_0 + \omega_0 \hat{I} + \hat{\omega} I_0 + \hat{\omega}\hat{I}$$

From the technique « AC SMALL SIGNAL MODELING » we eliminate: $V_0, I_0 R, \frac{dI_0}{dt}, \omega_0 I_0, \hat{\omega}\hat{I}$.

$$\hat{V} = R\hat{I} + \frac{d\hat{I}}{dt} + \frac{dL}{d\theta}\omega_0 \hat{I} + \frac{dL}{d\theta} I_0 \hat{\omega}$$

$$\hat{V} = \hat{I}\left(R + \frac{dL}{d\theta}\omega_0\right) + \frac{d\hat{I}}{dt} + \frac{dL}{d\theta}\hat{\omega} I_0$$

$$L\frac{d\hat{I}}{dt} = \hat{V} - \hat{I}\left(R + \frac{dL}{d\theta}\omega_0\right) - \frac{dL}{d\theta}\hat{\omega} I_0$$

$$\frac{d\hat{I}}{dt} = \left(-\frac{R}{L} - \frac{1}{L}\frac{dL}{d\theta}\omega_0\right) - \frac{1}{L}\frac{dL}{d\theta}\hat{\omega} I_0 + \frac{\hat{V}}{L} \quad (12)$$

We replace (i) in (10) we find:

$$\frac{1}{2}(I_0{}^2 + 2I_0\hat{I} + \hat{I}^2)\frac{dL}{d\theta} - (T_{L0} + \hat{T}_L) = J\frac{d(\omega_0 + \hat{\omega})}{d\theta} I_0 + B(\omega_0 + \hat{\omega})$$

$$I_0 \hat{I}\frac{dL}{d\theta} - \hat{T}_L = J\frac{d\hat{\omega}}{dt} + B\hat{\omega}$$

$$\frac{d\hat{\omega}}{dt} = \hat{I}\frac{I_0}{J}\frac{dL}{d\theta} - \frac{B}{J}\hat{\omega} - \frac{1}{J}\hat{T}_L \qquad (13)$$

We set:

$$R_1 = R + \frac{dL}{d\theta}\omega_0$$

$$K_1 = \frac{dL}{d\theta} I_0$$

$$\hat{f} = K_1 \hat{\omega} \qquad (14)$$

Where R_1, K_1 et $\hat{f}$: the equivalent résistance, emf constant and induced emf.
Then the following equations (12) and (13) become:

$$\frac{d\hat{I}}{dt} = -\frac{1}{L}R_1 - \frac{B}{J}\hat{\omega} - \frac{1}{L}\frac{dL}{d\theta}\hat{\omega} I_0 + \frac{\hat{V}}{L}$$

$$\frac{d\hat{\omega}}{dt} = \hat{I}\frac{K_1}{J} - \frac{B}{J}\hat{\omega} - \frac{\hat{T}_L}{J}$$

The Laplace transform of equations (3), (4) and (5) can be expressed in block diagram as follows:

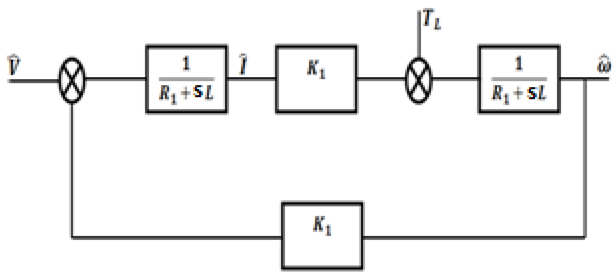

Figure 3: Small signal block diagram of SRM

The transfer function of SRM

$$H(P) = \frac{I(P)}{V(P)} = K\frac{1+ST}{a_1S^2 + a_1S + 1}$$

With

$$K = \frac{B}{R_1B+K_b^2}\ ,\ T = \frac{J}{B} \quad a_1 = \frac{JL}{R_1B+K_b^2} \text{ et } a_2 = \frac{LB+R_1J}{R_1B+K_b^2}$$

4. Control structure

The SRM is controlled in closed loop current. The control scheme contains a converter block (four asymmetric bridges), a PWM block, a block position sensor and a corrector that we will determine later.

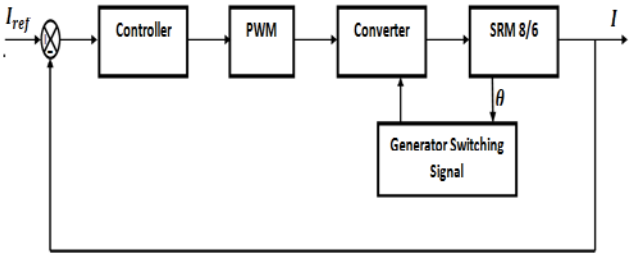

Figure 4: Control structure

4.1 Technique PWM

For the four phases, we have associated a current loop which each contains a pulse width modulator to compare the output of the corrector with the PWM carrier. When the carrier exceeds the output of the correction, the switch is turned off; this is shown in Figure 5 with the input and the wave form.

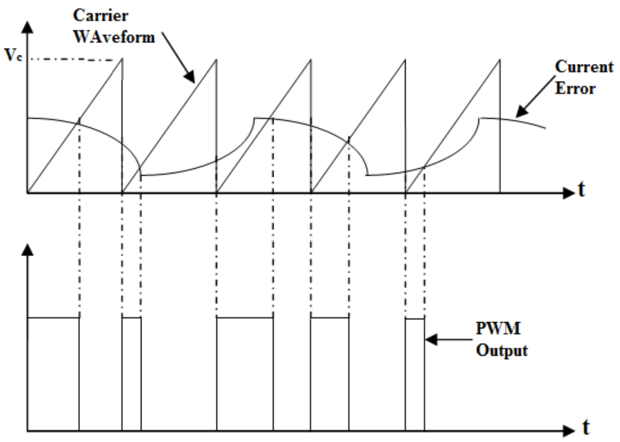

Figure 5: Output signal PWM

4.2 Block Position Sensor

The function of this block is to select the winding to be supplied, through the angle of the rotor angular position relative to the reference angle zero in an electric cycle. For a switched reluctance machine 8/6, each inductance has a periodicity of 60° (2π/6=60°), therefore, we must transform the rotor position angle, calculated from the mechanical equation so it is modulo 60°. Modulo 60° is achieved through the real function in Matlab/Simulink as shown in Figure 6 [5]:

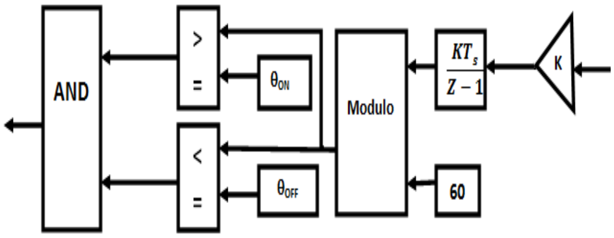

Figure 6: Block Position Sensor

4.3 Selection of the corrector

The current error is corrected with a corrector to generate a control signal for the power converter; the latter is modeled as a gain and a delay which will be determined later.
The gain of the power converter is:

$$K_{PC} = \frac{V_{DC}}{V_C}$$

With VC: the maximum control voltage of the PWM; VDC: the nominal voltage of the converter.
Assuming that the carrier frequency of the PWM is fc, the time constant of the TPC converter is:

$$T_{PC} = \frac{T_C}{2} = \frac{1}{2f_C}$$

The delay time is negligible because the converter chopping frequency of the PWM signal is high, as shown in Fig 7

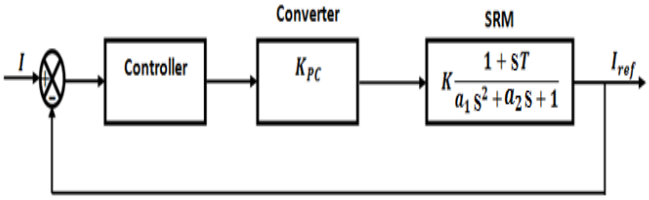

Figure 7: block diagram of the control loop

From Figure 8: Unit response. The calculated response time is important (on the order of Tr = 27.6ms above PWM period of 0.1ms) the static error is 80% and the order of overshoot is 80% which leads us to choose a PID associated with a first order filter as corrector to improve the speed, stability and accuracy

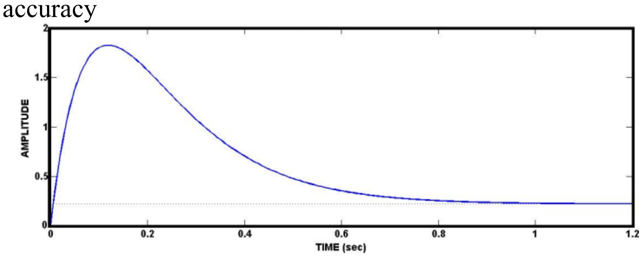

Figure 8: response to a step

The transfer function of the corrector is:

$$G(p)_c = K_c\frac{1+T_iS+T_iT_dS^2}{T_iS} \times \frac{1}{1+\tau S}$$

After simplification, the parameters of the corrector are given by:

$\tau = T$

$T_i = a_2$

$T_d = \frac{a_1}{a_2}$

The transfer function in closed loop becomes:

$$H(p) = \frac{1}{1+\frac{T_iS}{K_cK_{PC}}}$$

The improved response time is set to:

$$T_r = \frac{T_{PWM}}{1000}$$

T_{PWM} is the Pulse Width Modulation period.
According to the transfer function in closed loop, the term of the response time is:

$$T_r = 3\frac{T_i}{K_c K_{pc} K}$$

Hence the expressions of the corrector gain:

$$K_c = 3000\frac{T_i}{T_{PWM} K_{pc} K}$$

5. Simulation and interpretation of results

The model simulation is performed on the Matlab / Simulink software using blocks of SimpowerSys and Simulink. The simulation scheme is as follows:

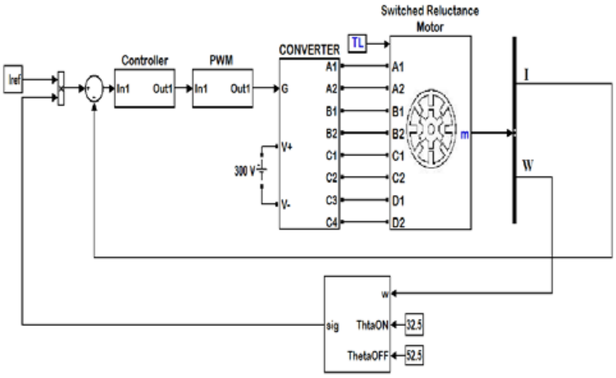

Figure 9: Control Scheme in Matlab/Simulink

The parameters used in the simulation are as follows:
The SRM parameters

Table 1. Parameters of SRM

Stator resistance (Ω)	0.05
Inertia (Kg.m²)	0.05
Friction (N.m.s)	0.02
Unaligned inductance (mH)	0.67
aligned inductance (mH)	23.6
Maximum current (A)	450
Maximum flux (V.s)	0.486
Rated current (A)	250
Rated speed (rpm)	3000
Power(KW)	40
Torque Load	125

Corrector parameters

$T_i = 0.2321, T_d = 0.0562, K_c = 104810^3, \tau = 2.5$

Parameters of the converter and PWM block:
Command signal level: 10V
PWM chopping frequency: 10 KHz
DC link voltage: 300V

Thanks to the simulation framework developed in Matlab/Simulink, we were able to trace the evolution of the various electrical and mechanical quantities. The Figures 11, 12 and 13 represent the evolution of the current in one phase, the speed evolution and the evolution of the average value of the torque. This average value is calculated for a period ripple 0.001sec.

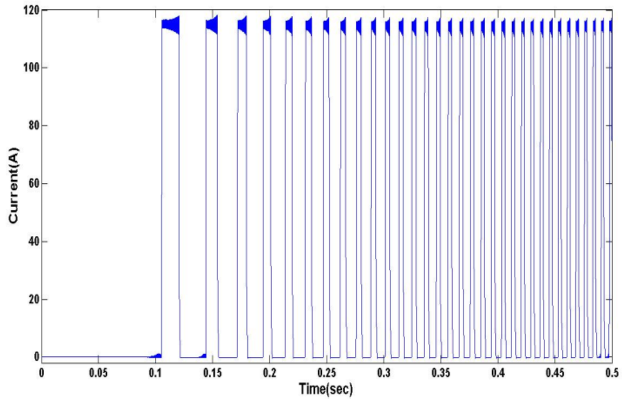

Figure 10: Current Evolution in one phase

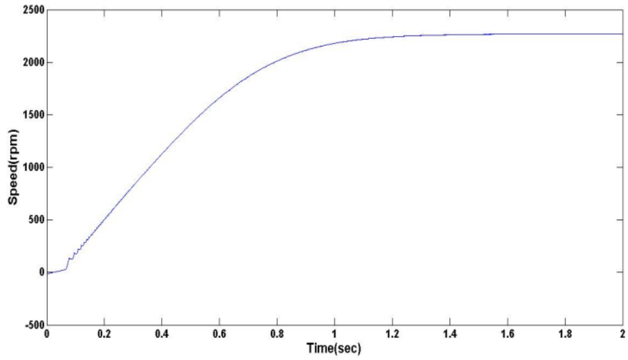

Figure 11: Speed Evolution

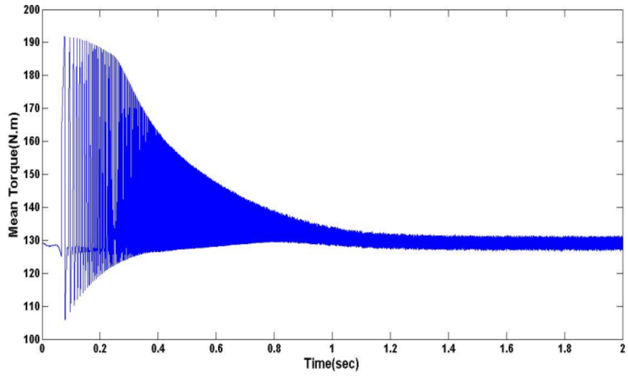

Figure 12: Evolution of the Average torque

6. Interpretation of Results

We note through the results presented:

- The average starting torque is 150N.m
- The average value of the torque in steady state is 130N.m
- The torque ripple in steady state is 30%
- The starting time of the system is 1.4sec

The torque ripple is due in consequence of the sudden switching of the switching angles θ_{ON} and θ_{OFF}. The Power during the interval $[\theta_{ON},\theta_{OFF}]$ where the inductance increases, achieves the desired maximum phase current from the ignition θ_{ON} angle. The current increases linearly from zero. Using the PID corrector allows better speed and accuracy of the current.

7. Conclusion

In this article an SRM is modeled by the "AC small signal" method, and it is controlled with current by a PID. This study was carried out on a SRM 8/6, powered by a voltage inverter. The proposed corrector for the "AC small signal" model is designed for a sufficient margin of stability, a design procedure of PID controller is suggested and the results were verified by simulation on Matlab / Simulink. The torque ripple prevents the excellent performance of the SRM. Our future work is to improve the performance of this command; we minimize the torque ripples with the DTC method.

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Author Profile

Khalid GRARI was born in Morocco in 1988. He completed B.Tech with first class division in Electrical Engineering; he received engineering degree from National School of Applied sciences Oujda. His research interest control of switched reluctance motor