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RANS – Modeling of MHD Flow Over Infinite Vertical Plate in Rotating System Under the Effect of Viscous Dissipation, Joule Heating and Radiation

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Abstract: ***In this paper Reynolds Averaged Navier-Stokes (RANS) approach have been employed so as to come up with a model capturing an MHD turbulent flow over a vertical plate provided the effect of viscous dissipation, Radiation and Joule Heating. The model equation for mean velocity and temperature in turbulent are not time dependent consequently, the Model turns out to be coupled highly non linear ODE***

Keywords: RANS, MHD, Turbulent, Radiation, viscous dissipation, Joule Heating, Modeling

1. Introduction

Magnetic fields influence in many natural and man-made flows. H.Alfvén had discoveries in *magnetohydrodynamics* with fruitful applications in different parts of plasma physics. The simplest form of MHD, Ideal MHD, assumes that the fluid has so little resistivity that it can be treated as a perfect conductor. This is the limit of infinite magnetic Reynolds number. In Ideal MHD, Lenz's law dictates that the fluid is sense tied to the magnetic field lines. The theoretical study of MHD flows has been a subject of great interest due to it's widely spread application on designing of cooling systems with liquid metals, petroleum industry, purification of crude oil, separation of matter from fluids and many other applications. Rotation viz MHD phenomena plays an important role in various phenomena like meteorology, geophysical fluids dynamics, gaseous and nuclear reactions. Formally, MHD is concerned with the mutual interaction of flow of an electrically conducting fluid and magnetic fields. The fluids in question must be electrically conducting and non-magnetic, which limits us to liquid metals, hot ionized gases (plasmas) and strong electrolyte. Model Study of effects of Hall currents on fluid flows have been discussed by various people, Recently Ayube *et al* [2], Seth *et al* [15].Beside, following Smolentsey and Morean [14] A two eddy-viscosity based models for MHD flows in a strong magnetic field when turbulence becomes Q2D have developed. Diaz *et al*[5] Consider a mathematical model related to the stationary regime of a plasma of fusion nuclear, magnetically confined in a stellarator device.Emad M.*et al*[3] presented a Model on MHD-free convection laminar flow with viscous incompressible fluid with power-law variation in surface temperature and during analysis the effects for viscous dissipation and Joule heating taken into account. And Mohammed A.*et al*[10] also came up with a mathematical modeling which dealt with MHD natural convection flow along a vertical flat plate in the presence of Joule heating. Kafousias and Daskakis [11] modeled and investigates the influence of both viscous and Joules dissipation on the magnetohydrodynamic convection flow in the stoke problem. Ola W.[12] considered one-point turbulence closure have been extended with an additional transported scalar for modeling MHD turbulence. If turbulence is entirely and chaotic, it would be inaccessible to any kind of mathematical treatment; almost all the situation of turbulence fluid motion can be mathematically modeled. Large Eddy simulation (LES)[13,8] is an approach used to model turbulent flows. In the present work we propose to carry out the modeling of Hydromagnetic flow over infinite vertical plate in Rotating System Under The influence of Viscous Dissipation, Joule Heating and Radiation together so as to extend and consolidate the aforementioned work of others and by doing this so we shall be able to broaden the formulation one step ahead than it was before along the area modeling such problems. Instead of the using LES approach we are proposed to use Reynolds averaged Navierstokes (or RANS) methods-the today's workhorse and research for industrial and research turbulence modeling application Joel[7]; this approach helped us in order to capture the turbulence characterized by random fluctuation and eddies.

2. Governing Equations Laminar Flow Scnario

With spatial coordinate's z and time, the Navier-stokes and continuity equations for the instantaneous velocity field velocity filed $u_i(z,t)$ of an incompressible fluid are, following Ola W. [12]:

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial z_i}=-\frac{1}{\rho}\frac{\partial p}{\partial z_i}+\nu\frac{\partial^2 u_i}{\partial z_i \partial z_j}+f_i \qquad (1)$$

$$\frac{\partial u_i}{\partial z_i}=0 \qquad (2)$$

And, the thermal energy equation as given by

$$\rho C_p\frac{DT}{Dt}=\kappa\nabla^2 T+\mu\phi+\frac{1}{\delta}J^2-\frac{\partial q_r}{\partial z_i} \qquad (3)$$

The electromagnetic equation that help in model and formulate MHD problem are *Maxwell's* equation and *Ohm's law*. Generally they are well discussed in any book on electromagnetic theory. here they are adopted as presented by Neff. [4]

$$\left\{\begin{array}{l} \nabla\times\vec{B}=\mu_e\vec{J} \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{B}=0 \\ \nabla\cdot\vec{B}=0 \end{array}\right\} \qquad (4)$$

and Ohm's law for a moving conductor taking hall current into account :

$$\vec{J}+\frac{\omega_e\tau_e}{B_0}(\vec{J}\times\vec{B})=\sigma[\vec{E}+\vec{u}\times\vec{B}] \qquad (5)$$

3. Description of the Flow

In the present study MHD flow past infinite vertical plate in rotating system in presence of a strong magnetic field is considered. The magnetic field is applied transversely along the z-axis and perpendicular to vertical plate. The plate in non-conducting and the fluid is electrically conducting. At $t=0$, the vertical plate is set into impulsive motion in its own plane(x-axis) at a constant velocity U_0. The transverse inhomogeneous magnetic field is in the z −direction. The vertical plate is kept at a higher temperature than the fluid is in $T_W > T_\infty$. Fluid flow is assumed incompressible, Newtonian electrically conducting. The flow being studied is free convectional and takes place along the x-axis is under the action of transverse variable magnetic field. The boundary layer thickness is along the z-axis hence the velocity components will changes along it. In **Figure 1**: The Geometry of the problem has depicted. It is assumed that the plate and the fluid are in a rigid body rotation with constant angular velocity, Ω, about the $z-axis$. B.L. and F.S. are designated to be a Boundary layer and Free stream. g is the gravitation force. B is applied magnetic field.

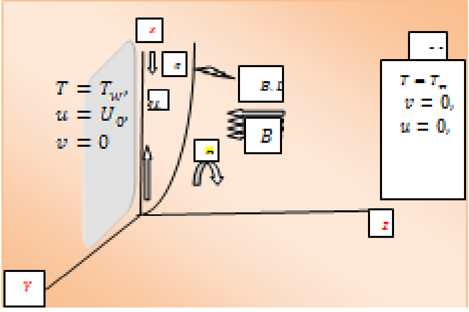

Figure 1: The Geometry of the Model

The flow of is takes place in two dimensions in general; as a result we anticipate the fluid variables to be depend on two coordinate space and that was why the geometry of the flow depicted in 3D coordinate space. The velocity u and v are depends on x, z, t . the velocity v does not change along y. For the magnetic field, for it to have an effect on the flow it must act in the direction perpendicular to the flow region. The velocity vector $\vec{q}=(u,v,0)$, $u=u(x,z,t)$ and $v=v(x,z,t)$; in fact before the flow become turbulent the third velocity component, w along was there. the magnetic field is $B=(0,0,B_0)$. The situation over the surface of the plate, $z=0$, the boundary condition, the plate is moving and as same time rotating hence we set $u=U_0$, $v=0$ and the temperature of the plate maintained constant at fixed temperature then we shall have $T=T_W$. The faraway from the plate $i.e\ z\to\infty\ u=v=0\ T=T_\infty$ and because the geometry the effect of gravity that when the bouncy force effect come into consideration.

It's been seen that the velocity and temperature over the surface of the plate and of the free stream has already captured earlier and our very concern is to know what is actually going to happen in between. The Model which can only work for laminar scenario is as :

$$\left.\begin{array}{l} \dfrac{du}{dt}+u\dfrac{du}{dx}+v\dfrac{du}{dy}+w\dfrac{du}{dz}-2v\Omega_z=\nu\nabla^2 u+\beta g(T-T_\infty)- \\ \dfrac{\sigma B_0^2(u-mv)}{\rho(1+m^2)}\ equ.(6.1) \\ \dfrac{dv}{dt}+u\dfrac{dv}{dx}+v\dfrac{dv}{dy}+w\dfrac{dv}{dz}+2u\Omega_z=\nu\nabla^2 v- \\ \dfrac{\sigma B_0^2(v+mu)}{\rho(1+m^2)}\ equ.(6.2) \\ \dfrac{dT}{dt}+u\dfrac{dT}{dx}+v\dfrac{dT}{dy}+w\dfrac{dT}{dz_z}=\alpha\nabla^2 T+\dfrac{\mu}{\rho C_p}\phi-\dfrac{1}{\rho C_p}\dfrac{dq_r}{dz_i}- \\ \dfrac{\sigma B_0^2(v+mu)^2+(u-mv)^2}{\rho C_p(1+m^2)^2}\ equ.(6.3) \end{array}\right\}$$

4. Turbulence Modeling

All flows encountered in engineering applications, from simple ones to complex three dimensional ones, become unstable above a certain Reynolds number. In turbulent flow the hydrodynamic and thermodynamic characteristics undergo chaotic fluctuation and hence, vary highly irregularly in space and time.(from the smallest turbulent eddies characterized by Kolmogorov micro-scales, to the flow features comparable with the size of the geometry). A turbulent (outwardly disordered) regime of fluid motion arises as a laminar flow loses its stability when the dimensionless Reynolds number $R = {UL}/{v}$ (where U and L are the characteristics velocity and Linear length scale of the flow, respectively, v is the molecular kinematic viscosity) exceeds some critical value Re_{cr} or turbulence arises either from the growth with small perturbation in a laminar flow or from the convective instability of motion. Re is the most general characteristics of a turbulized fluid.

Joel [7] There are several possible approaches for the numerical simulation of turbulent flows. The first and most intuitive one, is by directly numerically solving the governing equations over the whole range of the turbulent scales (temporal and spatial). This deterministic approach is referred as Direct Numerical Simulation (DNS). In DNS, a fine enough mesh and small enough time-step size must be used so that all of the turbulent scales are resolved. Although some simple problems have been solved using DNS, it is not possible to tackle industrial problems due to that prohibitive computer cost imposed by the mesh and time-step requirements. Hence, this approach is mainly used for benchmarking, research and academic applications.Another approach used to model turbulent model flows is Large Eddy Simulation (LES). Here, large scale turbulent structures are directly simulated whereas the small turbulent scales are filtered out and modeled by turbulence models called subgrid scale models. According to turbulent theory, small scale eddies are more uniform and have more or less common characteristics; therefore, modeling small scale turbulence appears more appropriate, rather than resolving it. the computational cost of LES is less than that of DNS.

In the RANS, equations are derived by decomposing the flow variables of the governing equation into time-mean (obtained over an appropriate time interval) and fluctuating part, and then time averaging the entire equations. Time averaging the governing equations gives rise to new terms, these new quantities must be related to the mean flow variables through turbulence models. This process introduces further assumption and approximations. The turbulences models are primarily developed based on experiment data obtained from relatively simple flows under controlled conditions. This in turn limits the range of applicability of the turbulence models. That is, no single RANS turbulence model is capable of providing accurate solution over a wide range of flow condition and geometries. Hereafter, we limit our discussion to Reynolds averaging.

4.1 Reynolds Averaging of the Model

In turbulent flow, the transport phenomena variables $(i.e., u, v, w, T, p, etc.)$ always vary with time. The instantaneous velocity value for a general flow variable say velocity u for a turbulent flow of moving fluid, provided for any location (x, y, z) can be expressed as summation of its *Mean* and its *Fluctuation* due to the small perturbation:

$u = \bar{u}(x, y, z, t) + u'(x, y, z, t)$ where

$$\bar{u}(x, y, z) = \frac{1}{\Delta t}\left[\int_t^{t+\Delta t} u(x, y, z, t)\,dt\right] \tag{7}$$

is the *Time-averaged* velocity at point (x, y, z). The time interval for the *Time-averaged,* Δt, must be very long compared with the duration of fluctuation. The mean value of the fluctuation must be zero, $i.e$

$$\overline{u'} = \frac{1}{\Delta t}\left[\int_t^{t+\Delta t} u'(x, y, z, t)\,dt\right] = 0 \tag{8}$$

Similarly, the velocity components in the y- and z- direction can be expressed as:

$v = \bar{v}(x, y, z, t) + v'(x, y, z, t)$

$w = \bar{w}(x, y, z, t) + w'(x, y, z, t)$ and

$T = \bar{T}(x, y, z, t) + T'(x, y, z, t)$

The model equations discussed in the previous section can be transformed into Reynolds averaging equations to govern turbulent flow. The Reynolds's averaging rules shall be used to transform equations governing laminar flow to turbulent flow. Time averaging of transport phenomena equations should provide the net effect of the turbulent perturbation.

Note that for turbulent flow, the model equations are not time dependent - since after time averaging of the momentum and energy equations, the term involving $d\bar{u}/dt$, $d\bar{v}/dt$ and $d\bar{T}/dt$ vanish i.e. become zero automatically; The model equations for the mean velocity and temperature in turbulent flows are not time dependent. For instance following Scott [16]

$$\frac{d\bar{u}_i}{dt} = 0 \tag{9}$$

4.1.1Time Averaged Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \;(\textit{incompressible flow})$$

$u = \bar{u} + u'(t), v = \bar{v} + v'(t)$ and $w = \bar{w} + w'(t)$ hence

$$\frac{\partial(\bar{u} + u')}{\partial x} + \frac{\partial(\bar{v} + v')}{\partial y} + \frac{d(\bar{w} + w')}{\partial z} = 0$$

Integrating over period $0 \to \Delta t$

$$\frac{\partial(\bar{u})}{\partial x} + \frac{d(\bar{v})}{\partial y} + \frac{d(\bar{w})}{\partial z} = 0 \tag{10}$$

4.1.2 Time-Averaged Momentum Equation

x-direction momentum equation is :

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{dy} + w\frac{\partial u}{\partial z} - 2v\Omega_z = v\nabla^2 u + \beta g(T - T_\infty) - \frac{\sigma B_0^2 (u - mv)}{\rho(1 + m^2)}$$

Multiplying continuity equation by u as:

$u \cdot \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\right)$ and adding to the above equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u^2}{\partial x}+\frac{\partial(uv)}{\partial y}+\frac{\partial(uw)}{\partial z}-2v\Omega_z=v\nabla^2u+\beta g(T-T_\infty)-\frac{\sigma B_0^2(u-mv)}{\rho(1+m^2)}$$

Averaging over period $0\to\Delta t$

$$\frac{\partial\bar{u}}{\partial t}+\frac{\partial\overline{u^2}}{\partial x}+\frac{\partial\overline{(uv)}}{\partial y}+\frac{\partial\overline{(uw)}}{\partial z}-2\bar{v}\Omega_z=v\nabla^2\bar{u}+\beta g(\bar{T}-T_\infty)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}\quad equ.(10)$$

Then, Consequently from **(9)**

$$\frac{\partial\overline{u^2}}{\partial x}+\frac{\partial\overline{(uv)}}{\partial y}+\frac{\partial\overline{(uw)}}{\partial z}-2\bar{v}\Omega_z=v\nabla^2\bar{u}+\beta g(\bar{T}-T_\infty)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}$$

Substituting for $u=\bar{u}+u'(t), v=\bar{v}+v'(t)$, $w=\bar{w}+w'(t)$ and $T=\bar{T}+T'(t)$

$$\frac{\partial\overline{(\bar{u}+u')^2}}{\partial x}+\frac{\partial\overline{((\bar{u}+u')(\bar{v}+v'))}}{\partial y}+\frac{\partial\overline{((\bar{u}+u')(\bar{w}+w'))}}{\partial z}-2\left(\overline{\bar{v}+v'}\right)\Omega_z=v\nabla^2(\overline{\bar{u}+u'})+\beta g((\overline{\bar{T}+T'})-T_\infty)-\frac{\sigma B_0^2((\overline{\bar{u}+u'})-m(\overline{\bar{v}+v'}))}{\rho(1+m^2)}$$

$$\left[\frac{\partial\overline{u^2}}{\partial x}+\frac{\partial\overline{(u')^2}}{\partial x}\right]+\left[\frac{\partial\overline{(uv)}}{\partial y}+\frac{\partial\overline{(u'v')}}{\partial y}\right]+\left[\frac{\partial\overline{(uw)}}{\partial z}+\frac{d\overline{(u'w')}}{dz}\right]-2\bar{v}\Omega_z=v\nabla^2\bar{u}+\beta g(\bar{T}-T_\infty)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}$$

from chain law: $\frac{\partial\overline{u^2}}{\partial x}=2\bar{u}\frac{\partial\bar{u}}{\partial x}$,

$$\frac{\partial\overline{(uv)}}{\partial y}=\bar{u}\frac{\partial(\bar{v})}{\partial y}+\bar{v}\frac{\partial(\bar{u})}{\partial y}$$

and $\frac{\partial\overline{(uw)}}{\partial y}=\bar{u}\frac{\partial(\bar{w})}{\partial y}+\bar{w}\frac{\partial(\bar{u})}{\partial y}$ then

$$\left[2\bar{u}\frac{\partial\bar{u}}{\partial x}+\frac{\partial\overline{(u')^2}}{\partial x}\right]+\left[\bar{u}\frac{\partial(\bar{v})}{\partial y}+\bar{v}\frac{\partial(\bar{u})}{\partial y}+\frac{\partial\overline{(u'v')}}{\partial y}\right]+\left[\bar{u}\frac{\partial(\bar{w})}{\partial y}+\bar{w}\frac{\partial(\bar{u})}{\partial y}+\frac{d\overline{(u'w')}}{dz}\right]-2\bar{v}\Omega_z=v\nabla^2\bar{u}+\beta g(\bar{T}-T_\infty)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}\quad equ.(11)$$

Multiplying the time-averaged continuity equation by $\bar{u}$ as: $\bar{u}\left[\frac{d(\bar{u})}{dx}+\frac{d(\bar{v})}{dy}+\frac{d(\bar{w})}{dz}=0\right]$

Subtracting equation **(11)** with this equation, it yields:

$$\left[\bar{u}\frac{\partial\bar{u}}{\partial x}+\frac{\partial\overline{(u')^2}}{\partial x}\right]+\left[\bar{v}\frac{\partial(\bar{u})}{\partial y}+\frac{\partial\overline{(u'v')}}{\partial y}\right]+\left[\bar{w}\frac{\partial(\bar{u})}{\partial y}+\frac{d\overline{(u'w')}}{dz}\right]-2\bar{v}\Omega_z=v\nabla^2\bar{u}+\beta g(\bar{T}-T_\infty)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}$$

Then:

$$\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial(\bar{u})}{\partial y}+\bar{w}\frac{\partial(\bar{u})}{\partial y}-2\bar{v}\Omega_z=v\nabla^2\bar{u}+\beta g(\bar{T}-T_\infty)-\frac{\sigma B_0^2(\bar{u}-m\bar{v})}{\rho(1+m^2)}-\frac{\partial\overline{(u')^2}}{\partial x}-\frac{\partial\overline{(u'v')}}{\partial y}-\frac{d\overline{(u'w')}}{dz}\quad equ.(12)$$

y-direction momentum equation is :

$$\frac{dv}{dt}+u\frac{dv}{dx}+v\frac{dv}{dy}+w\frac{dv}{dz}+2u\Omega_z=v\nabla^2v-\frac{\sigma B_0^2(v+mu)}{\rho(1+m^2)}$$

with similar procedure:

$$\bar{u}\frac{\partial\bar{v}}{\partial x}+\bar{v}\frac{\partial(\bar{v})}{\partial y}+\bar{w}\frac{\partial(\bar{v})}{\partial y}+2\bar{u}\Omega_z=v\nabla^2\bar{v}-\frac{\sigma B_0^2(\bar{v}+m\bar{u})}{\rho(1+m^2)}-\frac{\partial\overline{(v')^2}}{\partial x}-\frac{\partial\overline{(u'v')}}{\partial y}-\frac{d\overline{(v'w')}}{dz}\quad equ.(13)$$

4.1.3 Time-Averaged Energy Equation: From (6.1-6.3)

$$\frac{dT}{dt}+u\frac{dT}{dx}+v\frac{dT}{dy}+w\frac{dT}{dz}=\alpha\nabla^2T+\frac{\mu}{\rho C_p}\phi-\frac{1}{\rho C_p}\frac{dq_r}{dz_i}-\frac{\sigma B_0^2(v+mu)^2+(u-mv)^2}{\rho C_p(1+m^2)^2}$$

Then

$$\bar{u}\frac{d\bar{T}}{dx}+\bar{v}\frac{d\bar{T}}{dy}+\bar{w}\frac{d\bar{T}}{dz}=\alpha\nabla^2 T-\frac{\partial(\overline{u'T'})}{\partial x}-\ldots$$

$$\frac{\partial(\overline{v'T'})}{\partial y}-\frac{\partial(\overline{w'T'})}{\partial z}+\frac{\mu}{\rho C_p}\phi-\ldots$$

$$\frac{1}{\rho C_p}\frac{dq_r}{dz_i}+\frac{\sigma B_0^2(v+mu)^2+(u-mv)^2}{\rho C_p(1+m^2)^2} equ.(14)$$

$$\alpha=\frac{\kappa}{\rho C_p}$$

Now, for tow-dimensional turbulent boundary time-averaged equations (10), (12) (13) and (14), we also must take note the geometry of the problem together with the following : $\frac{\partial^2}{\partial x^2} \ll \frac{\partial^2}{\partial y^2}$, $\frac{\partial^2(\overline{u'})}{\partial x^2} \ll \frac{\partial^2(\overline{u'v'})}{\partial y^2}$ Since (y-direction fluctuation is larger than x-direction velocity fluctuation) , $\frac{\partial^2(\overline{T'u'})}{\partial x^2} \ll \frac{\partial^2(\overline{T'v'})}{\partial y^2}$ Then

$$\frac{\partial\bar{u}}{\partial x}=0 equ.(15.1)$$

$$-2\bar{v}\Omega_z=v\frac{d^2u}{dz^2}-\frac{d(\overline{u'w'})}{dz}+\beta(\bar{T}-T_\infty)-\frac{\sigma B_0(\bar{u}-m\bar{v})}{\rho(1+m^2)}$$

$$-2\bar{v}\Omega_z=\frac{1}{\rho}\frac{d}{dz}\left[\mu\frac{d\bar{u}}{dz}-\rho\overline{u'w'}\right]+\beta(\bar{T}-T_\infty)-\ldots$$

$$\frac{\sigma B_0(\bar{u}-m\bar{v})}{\rho(1+m^2)} equ.(15.2)$$

$$2\bar{u}\Omega_z=\frac{1}{\rho}\frac{d}{dz}\left[\mu\frac{d\bar{v}}{dz}-\rho\overline{v'w'}\right]-\frac{\sigma B_0(\bar{v}+m\bar{u})}{\rho(1+m^2)} equ.(15.3)$$

$$\frac{d}{dz}\left[\alpha\frac{d\bar{T}}{dz}-\rho\overline{T'w'}\right]-\frac{\mu}{\rho C_p}\left[\left(\frac{d\bar{u}}{dz}\right)^2+\left(\frac{d\bar{v}}{dz}\right)^2\right]-\ldots$$

$$\frac{1}{\rho C_p}\frac{dq_r}{dz}+\sigma\frac{B_0\left[(\bar{v}+m\bar{u})^2+(\bar{u}+m\bar{v})^2\right]}{\rho C_p(1+m^2)^2}=0 equ.(15.4)$$

The set of equation in (15) are incompressible Reynolds-Average-Stokes (RANS) equation. Notice that in equations the mean variables are independent of time. The Molecular shear stress : $\mu\frac{d\bar{u}}{dz}$ and $\mu\frac{d\bar{v}}{dz}$, The eddy shear stress $:-\rho\overline{u'w'}\equiv\rho\varepsilon_m\left(\frac{d\bar{u}}{dz}\right)$, ε_m is momentum eddy diffusivity or turbulent eddy viscosity. ε_m is a property flow field and not a physical property of the fluid.

Note that velocity fluctuations $(\overline{u'w'})$ are assumed to be induced by $\left(\frac{d\bar{u}}{dz}\right)$.

From 15.3

The molecular heat flux $-\rho\overline{T'w'}\equiv\rho\varepsilon_H\left(\frac{d\bar{T}}{dz}\right)$, ε_H is eddy diffusivity. $\varepsilon_H=\frac{\varepsilon_m}{P_{r_t}}$ where $P_{r_t}=\frac{\varepsilon_m}{\varepsilon_H}$

5. Boussinesq Approximation

The Reynolds averaged approach to turbulence modeling requires that the Reynolds stress in the above to be appropriately modeled (however, it is possible to derive its own governing equation, but it is much simpler to model this term).

Let's write these terms as a function of $x, t, u, \& v$ & $i.e.$ we need a function such that:

$\overline{u'w'}=F(x,y,u,v)$

To go forward we adopt the Boussinsq approximation

$\tau_t=-\rho\overline{u'v'}=\varepsilon_m\frac{\partial\bar{u}}{\partial y}$

ε_m is not a property of the fluid like μ but depends on the mean velocity u. we use the semi empirical methods to resolve the Reynolds shear stress terms in equations; and that lead us to the study and use of the *Prandtly mixing length hypothesis* which for a long time has been an important tool in the analysis of turbulent boundary layers.

The Reynold shear $\rho\,\overline{u'v'}$ represents the flux of x-mmomentum in the direction of y. Prandtly assumed that this momentum was transported by eddies which moved in the y-direction over a distance l without interaction ($i.e$ momentum is assumed to be conserved over distance l) and then mixed with existing fluid at the new location McComb[9]

From His experiment Prandtl deduce that:

$\rho\,\overline{u'v'}=-\rho l^2\left(\frac{\partial\bar{u}}{\partial y}\right)^2$

At this stage further assuming are taken $i.e$ $l=ky$ where n is the *Von Karman* constant, $n=0.4$ McComb[9].

We thus finally have

$\rho\,\overline{uv}=-\rho n^2 y^2\left(\frac{\partial\bar{u}}{\partial y}\right)^2$

Thus, the approximation of the terms due to the turbulence effect for our model shall be:

$$\left.\begin{aligned}
&-\rho\overline{u'w'} \approx \rho n^2 z^2\left(\frac{d\bar{u}}{dz}\right)^2 \approx \rho\varepsilon_m\frac{d\bar{u}}{dz}\\
&-\rho\overline{w'v'} \approx \rho n^2 z^2\left(\frac{d\bar{v}}{dz}\right)^2 \approx \rho\varepsilon_m\frac{d\bar{v}}{dz}\\
&-\overline{w'T'} \approx \varepsilon_m\frac{d\bar{T}}{dz}\,\varepsilon_H = \frac{\varepsilon_m}{P_{r_t}} = \frac{n^2z^2}{P_{r_t}}\left(\frac{d\bar{u}}{dz}\right)\\
&\varepsilon_m = n^2z^2\frac{d\bar{u}}{dz} = n^2z^2\frac{d\bar{v}}{dz}\\
&-\overline{w'T'} \approx \frac{n^2z^2}{P_{r_t}}\frac{d\bar{T}}{dz}
\end{aligned}\right\} equ.(16)$$

Model Turbulence equation

$$-2\Omega v = \upsilon\frac{d^2u}{dz^2} + \frac{d}{dz}\left(n^2z^2\left(\frac{du}{dz}\right)^2\right) + g\beta(T - T_\infty) - \ldots$$

$$\frac{\sigma B_0^2}{\rho}\left(\frac{u - mv}{1 + m^2}\right) equ.(17.1)$$

$$2\Omega u = \upsilon\frac{d^2v}{dz^2} + \frac{d}{dz}\left(n^2z^2\left(\frac{dv}{dz}\right)^2\right) - \frac{\sigma B_0^2}{\rho}\left(\frac{v + mu}{1 + m^2}\right) equ.(17.2)$$

$$\alpha\frac{d^2T}{dz^2} + \frac{d}{dz}\left(\frac{n^2z^2}{\Pr_t}\frac{du}{dz}\frac{dT}{dz}\right) - \frac{1}{\rho c_p}\frac{dq_r}{dz} + \frac{\mu}{\rho c_p}\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right]$$

$$+\frac{\sigma B_0^2}{\rho c_p}\left[\frac{(u - mv)^2 + (v + mu)^2}{(1 + m^2)^2}\right] = 0 equ.(17.3)$$

With the Boundary condition

$u = U_0,\ v = 0,\ T = T_w$, at z = 0,

$u = 0,\ v = 0,\ T = T_\infty$, as $z \to \infty$.

Following Cogley et al. [1], we assume the fluid medium is optically thin with a relatively low density and the radiative heat flux q_r is given as

$$\frac{\partial q_r}{\partial z} = -4\varepsilon^2(T - T_\infty),$$

where $\varepsilon << 1$ is the radiation absorption coefficient. The model energy balance equation then become:

$$\alpha\frac{d^2T}{dz^2} + \frac{d}{dz}\left(\frac{n^2z^2}{\Pr_t}\frac{du}{dz}\frac{dT}{dz}\right) + 4\varepsilon^2(T - T_\infty) + \ldots$$

$$\frac{\mu}{\rho c_p}\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right] + \frac{\sigma B_0^2}{\rho c_p}\left[\frac{(u - mv)^2 + (v + mu)^2}{(1 + m^2)^2}\right] = 0$$

Following Marchello and Toor [6], for high turbulence intensity, the turbulent Prandtl number is given as $\Pr_t = \sqrt{\Pr}$.

Dimensionless Variables and Quantities:

$$\eta = \frac{zU_0}{\upsilon},\quad U = \frac{u}{U_0},\quad V = \frac{v}{U_0},\quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$M = \frac{\sigma B_0^2\upsilon}{\rho U_0^2},\quad m = w_e\tau_e, R = \frac{\Omega\upsilon}{U_0^2},\quad \Pr = \frac{\upsilon}{\alpha},$$

$$Ec = \frac{U_0^2}{c_p(T_w - T_\infty)},\quad Gr = \frac{g\beta\upsilon(T_w - T_\infty)}{U_0^3}, Nr = \frac{4\varepsilon^2\upsilon}{\rho c_p U_0^2},$$

Note:
R = Rotational parameter
Ec= Eckert number
Gr = Grashof number,
Nr = Radiation parameter
M = magnetic field parameter
m = Hall parameter
Pr = Prandtl number
$\Pr_t$ = Turbulent Prandtl number
n = von Karman constant (= 0.4)

Dimensionless Model Equations:

$$-2RV = \frac{d^2U}{d\eta^2} + \frac{d}{d\eta}\left(n^2\eta^2\left(\frac{dU}{d\eta}\right)^2\right) + Gr\theta - M\left(\frac{U - mV}{1 + m^2}\right) equ.(18.1)$$

$$-2RU = \frac{\partial^2V}{\partial\eta^2} + \frac{\partial}{\partial\eta}\left(n^2\eta^2\left(\frac{\partial V}{\partial\eta}\right)^2\right) - M\left(\frac{V + mU}{1 + m^2}\right) equ,(18.2) \quad \mathbf{(17.1)}$$

$$\frac{1}{\Pr}\frac{d^2\theta}{d\eta^2} + \frac{d}{d\eta}\left(\frac{n^2\eta^2}{\Pr_t}\frac{dU}{d\eta}\frac{d\theta}{d\eta}\right) + Nr\theta + Ec\left[\left(\frac{dU}{d\eta}\right)^2 + \left(\frac{dV}{d\eta}\right)^2\right] + \quad \mathbf{(17.3)}$$

$$MEc\left[\frac{(U - mV)^2 + (V + mU)^2}{(1 + m^2)^2}\right] = 0 equ.(18.3)$$

with

$U = 1,\ V = 0,\ \theta = 1$, at η = 0,

$U = 0,\ V = 0,\ \theta = 0$, as $\eta \to \infty$.

The coupled highly non -linear ordinary differential equ.(18.1) - equ.(18.3) equations are the final equations Turbulence model equations

6. Conclusion

We have developed the RANS-MHD modeling flow over a plate in rotating system given the effect of viscous dissipation, Joule heating and Radiation as a heat source ; This model enables us to analyze the momentum and the temperature dynamics provided variation of the set of values of the aforementioned parameters. Moreover along the model the turbulence effect -eddy viscosity has been properly captured into the model. Since turbulence flow has got vital application in area of engineering hence its noteworthy to realize how such model vitally useful.

7. Future Scope of the Work

Due to the tremendous need in the area of modeling of phenomena's & applications in industrial, meteorological, and oceanographical, consequently the present work could be possibly extended to further studies; As recommended approach we suggest that similar modeling can be carried out by considering a porous medium embedded into the system and applying a strong variable magnetic field in the flow region. Moreover the work could have also extend in such a way that assuming conditions leading to Q2D turbulence; such flows demonstrate the Hartmann layers at the walls perpendicular to the magnetic field and the core, where the flow is essentially two-dimensional.

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