

Load Flow Analysis for Radial and Mesh Connected Distribution Systems

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Abstract: *Distribution System load flow studies play a vital role in present day scenario. Distribution Load Flow(DLF) are required for demand side management, distribution management system, studies to include dispersed generation, load scheduling, etc., for the modern growing distribution systems. DLF's are different from the load flows we use for Power Systems, because of the radial nature of the distribution systems. Also some of the methods need to be modified for weakly meshed distribution system. This paper presents ideas of three distribution load flows, namely Primitive Impedance based Distribution Load Flow (PIDLF), Current Injections based Distribution Load Flow (CIM) and Fast Decoupled Single Matrix Model Distribution Load Flow (SMM). These methods are briefed and tested on IEEE standard 15 bus, 33bus and 69 bus systems and comparative study is carried out based on the converged voltages, phase angles and the number of iterations taken for convergence.*

Keywords: Distribution Load Flow, Primitive Impedance based load flow, Current Injection ,FDC SMM

1. Introduction

Distribution power flow is an important tool for the analysis of distribution system and it is used in the operational as well as in planning stages. Distribution systems [1] are characterized by less power handling capacities, radial or weakly meshed structure, low x/r ratios ($x/r \leq 1$), unbalanced loads, large number of branches. Distribution system with their radial structure and wide ranging resistance and reactance values are inherently ill-conditioned [2] and conventional load flow methods like gauss-seidel, newton-raphson and fast decoupled techniques are inefficient insolving such networks. Transmission and distribution losses are as high as 20 to 30 percent of total power generation. Therefore, the challenge is more pronounced in case of distribution systems. Basic reason behind these huge power losses is resistive loss, as well as distribution systems are operated at much lower voltages as compared to transmission systems. Therefore, there is need to develop a load flow solutions to meet the properties of distribution system with less computational time. Several load flow algorithms specially designed for distribution systems have been proposed and published so far time to time. The recent tendency towards the Distribution Automation (DA) has led the researchers to focus on robust and efficient load flow methods [4] and distribution load flow studies are the base case study required for distribution system studies like load scheduling, distributed generation incorporation, demand side management, etc., these studies are essential for efficient and reliable operation of the distribution systems.

There are several load flow [3] methods like Direct Distribution Load Flow (DDLDF), Backward Forward Sweep Distribution Load Flow (BFS) and Vector based Distribution Load Flow (VDLF), which needs special attention while working for weakly mesh connected distribution systems, as in metropolitan cities. In this paper, three different methods are presented which works for Radial as well as weakly mesh connected systems.

The paper gives insights to three different load flow studies to study about the distribution systems and each method has its own specialty. For an instance the Primitive Impedance based Distribution Load Flow does not need matrix related operations (when compared to direct distribution load flow), Current Injection Method is simple and works for all sizes of radial distribution systems and Single Matrix Model method is very powerful for it is fast decoupled load flow for distribution systems. These methods are tested on IEEE standard distribution 15 bus, 33 bus and 69 bus systems [5] and the results are presented in the upcoming sections and comparison study with respect to converged voltages, phase angles and computational time is carried out. And the results are validated against standard load flow results using backward forward sweep (which is mostly used). This paper is organized of six sections and they are: Section I: Introduction, Section II: Primitive Impedance Based Load Flow, Section III: Current Injection Method, Section IV: Fast Decoupled Single Matrix Model, Section V: Results and Comparison, Section VI: Conclusion And References.

2. Primitive Impedance Base Distribution Load Flow (PIDLF)

The proposed method is developed based on two derived matrices, the Bus-Injection to Branch-Current (BIBC) matrix and the Branch-Current to Bus-Voltage (BCBV) matrix [6][7]. In this section, the development procedure will be described in detail. The corresponding equivalent current injection at the k^{th} iteration of solution is given by,

$$I_i^k = I_i^r(V_i^k) + jI_i^i(V_i^k) = \frac{P-jQ}{V_i^k} \quad (1)$$

Where I_i^k and V_i^k are current injections and voltage at i^{th} bus and at k^{th} iteration. And I_i^r and I_i^i are real and imaginary part of current.

A simple distribution system shown in Fig.1 is used as an example. The power injection can be converted to the equivalent current injections the relationship between the bus

current injections and branch current can be obtained by applying Kirchoff's Current Law (KCL) to the distribution net-work. The branch currents can then be formulated as functions of equivalent current injections. For example, the branch currents are B1, B2, B3, B4, and B5 can be expressed by equivalent current injections as

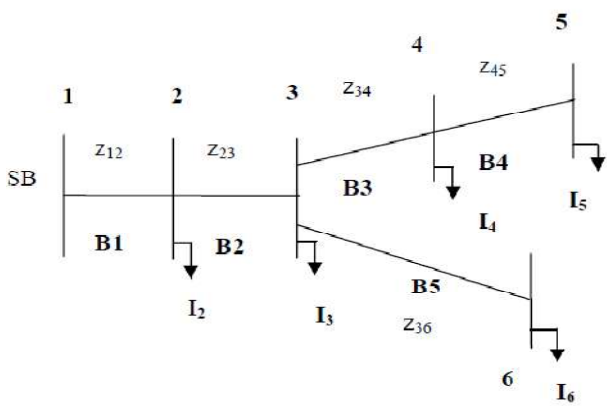


Figure 1: Simple 6 bus distribution system

Formation of BIBC matrix:-

$$\begin{aligned} B1 &= I2 + I3 + I4 + I5 + I6 \\ B2 &= I3 + I4 + I5 + I6 \\ B3 &= I4 + I5 \\ B4 &= I5 \\ B5 &= I6 \end{aligned}$$

Therefore, the relationship between the bus current injections and branch currents can be expressed as

$$\begin{bmatrix} B1 \\ B2 \\ B3 \\ B4 \\ B5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} I2 \\ I3 \\ I4 \\ I5 \\ I6 \end{bmatrix} \quad (2)$$

Equation 2 can be written as

$$[B] = [BIBC] * [I] \quad (3)$$

The constant BIBC matrix is an upper triangular matrix and contains values of 0 and +1 only. The relationship between branch currents and bus voltages are given by

$$V4 = V1 - B1 * Z12 - B2 * Z23 - B3 * Z34 \quad (4)$$

Similarly voltage at each bus can be expressed in terms of branch currents.

$$\begin{bmatrix} V1 \\ V1 \\ V1 \\ V1 \\ V1 \end{bmatrix} - \begin{bmatrix} V2 \\ V3 \\ V4 \\ V5 \\ V6 \end{bmatrix} = \begin{bmatrix} Z12 & 0 & 0 & 0 & 0 \\ Z12 & Z23 & 0 & 0 & 0 \\ Z12 & Z23 & Z34 & 0 & 0 \\ Z12 & Z23 & Z34 & Z45 & 0 \\ Z12 & Z23 & 0 & 0 & Z36 \end{bmatrix} * \begin{bmatrix} B1 \\ B2 \\ B3 \\ B4 \\ B5 \end{bmatrix} \quad (5)$$

Equation (5) can be written as,

$$[\Delta V] = [BCBV] * [B] \quad (6)$$

Sub., equation (3) in equation (6), we get

$$[\Delta V] = [BCBV] * [BIBC] * [I] = [DLF] * [I] \quad (7)$$

[DLF] is the Distribution Load Flow matrix which will be:

$$\begin{bmatrix} Z12 & Z12 & Z12 & Z12 & Z12 \\ Z12 & Z12 + Z23 & Z12 + Z23 & Z12 + Z23 & Z12 + Z23 \\ Z12 & Z12 + Z23 & Z12 + Z23 + Z34 & Z12 + Z23 + Z34 & Z12 + Z23 \\ Z12 & Z12 + Z23 & Z12 + Z23 + Z34 & Z12 + Z23 + Z34 + Z45 & Z12 + Z23 \\ Z12 & Z12 + Z23 & Z12 + Z23 & Z12 + Z23 & Z12 + Z23 + Z36 \end{bmatrix}$$

Table 1: Path Line Vector for Forming Ipathf And Ipathto

Bus no	1	2	3	4	5				6					
Location count =j	1	2	3	4	5	6	7	8	9	10	11	12	13	14
pathline (j)	0	1	1	2	1	2	3	1	2	3	4	1	2	5

A. Algorithm for Direct Distribution Load flow

1. Read the system data-n, nline, nsubstation, epsilon, itermax.
2. Form sparsity vectors
3. Read the Line data and the load data.
4. Set/Read the initial guess voltages.
5. Calculate the injections at each bus, using equation (1).
6. For all buses form ipathf and ipathto vectors as following,
 - I. Form path line vector which gives information of the total number of buses connected from substation bus to each bus
 - II. Form path size as: number of total paths for each bus.
 - III. Form ipathf and ipathto as:source and destination buses of each path within the path size for all the buses with the help of adjacent buses information (sparsity)
7. Calculate the [DLF] matrix.
8. Calculate the $[\Delta V]$ and update the bus voltages.
9. Check for convergence and if not satisfied repeat step 4-step 7 till convergence is achieved.
10. Print out the converged voltages.

3. Current Injection Method Based Load Flow (CIM)

This paper describes a sparse Newton Raphson formulation for the solution of the power flow problem, comprising 2n current injection equations written in rectangular coordinates [8]. The Jacobian matrix has the same structure as the (2n x 2n) nodal admittance matrix, in which each network branch is represented by a (2 x 2) block. Except for PV buses, the off-diagonal (2 x 2) blocks of the proposed Jacobian equations are equal to those of the nodal admittance matrix.

$$[\Delta J] = [Y] * [\Delta V] \quad (8)$$

$$\begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \\ I_{m4} \\ I_{m5} \\ I_{m6} \\ I_{r1} \\ I_{r2} \\ I_{r3} \\ I_{r4} \\ I_{r5} \\ I_{r6} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{16} & G_{11} & G_{12} & \dots & G_{16} \\ B_{21} & B_{22} & \dots & B_{26} & G_{21} & G_{22} & \dots & G_{26} \\ B_{31} & B_{32} & \dots & B_{36} & G_{31} & G_{32} & \dots & G_{36} \\ B_{41} & B_{42} & \dots & B_{46} & G_{41} & G_{42} & \dots & G_{46} \\ B_{51} & B_{52} & \dots & B_{56} & G_{51} & G_{52} & \dots & G_{56} \\ B_{61} & B_{62} & \dots & B_{66} & G_{61} & G_{62} & \dots & G_{66} \\ G_{11} & G_{12} & \dots & G_{16} & -B_{11} & -B_{12} & \dots & -B_{16} \\ G_{21} & G_{22} & \dots & G_{26} & -B_{21} & -B_{22} & \dots & -B_{26} \\ G_{31} & G_{32} & \dots & G_{36} & -B_{31} & -B_{32} & \dots & -B_{36} \\ G_{41} & G_{42} & \dots & G_{46} & -B_{41} & -B_{42} & \dots & -B_{46} \\ G_{51} & G_{52} & \dots & G_{56} & -B_{51} & -B_{52} & \dots & -B_{56} \\ G_{61} & G_{62} & \dots & G_{66} & -B_{61} & -B_{62} & \dots & -B_{66} \end{bmatrix} * \begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \\ V_{m4} \\ V_{m5} \\ V_{m6} \\ V_{r1} \\ V_{r2} \\ V_{r3} \\ V_{r4} \\ V_{r5} \\ V_{r6} \end{bmatrix}$$

The imaginary components of the current mismatches are ordered first, so that matrix Y* becomes diagonal dominant (a consequence of the fact that susceptances B_{km} are much larger than conductances G_{km}).

4. Calculation of Current Mismatches

The active and reactive power mismatches for bus k is given by:

$$\Delta P_k = P_k^{sp} - P_k^{calc} \tag{8}$$

$$\Delta Q_k = Q_k^{sp} - Q_k^{calc} \tag{9}$$

Where:

$$P_k^{calc} = V_{rk} I_{rk}^{calc} + V_{mk} I_{mk}^{calc}$$

$$Q_k^{calc} = V_{mk} I_{rk}^{calc} - V_{rk} I_{mk}^{calc}$$

By simple manipulation of the above equations, the current mismatches can be expressed only in terms of power mismatches and voltages at bus k:

$$\Delta I_{rk} = \frac{V_{rk} \Delta P_k + V_{mk} \Delta Q_k}{V_k^2} \tag{10}$$

$$\Delta I_{mk} = \frac{V_{mk} \Delta P_k - V_{rk} \Delta Q_k}{V_k^2} \tag{11}$$

Where:

$$V_k^2 = V_{rk}^2 + V_{mk}^2$$

By applying Newton Raphson method we can get power flow solutions of Distribution Systems.

The bus voltage corrections in polar coordinates, at a generic iteration (h+1) are given by:

$$\Delta V_k = \frac{V_{rk}}{V_k} \Delta V_{rk} + \frac{V_{mk}}{V_k} \Delta V_{mk} \tag{12}$$

$$\Delta \theta_k = \frac{V_{rk}}{V_k} \Delta V_{mk} - \frac{V_{mk}}{V_k} \Delta V_{rk} \tag{13}$$

$$\left. \begin{aligned} V_k^{(h+1)} &= V_k^{(h)} + \Delta V_k^{(h)} \\ \theta_k^{(h+1)} &= \theta_k^{(h)} + \Delta \theta_k^{(h)} \end{aligned} \right\} \tag{14}$$

B. Algorithm for Current Injection Method

1. Read the system data- n, nline, nsubstation, epsilon, itermax.
2. Read the Line data and the load data.
3. Form Sparsity vectors..
4. Initialize the Bus Voltages to 1+j0 p. u.
5. Calculate the Power Injections and Current Injections [i] at all the buses.

6. Initialize the iteration count k=1.
7. set ΔP=0.0 & ΔQ=0.0;
8. Calculate Pcal&Qcal and calculate power mismatches at every bus.
9. Calculate the ΔP&ΔQ and check for convergence. If so goto step 100.
10. If not, calculate the Jacobian Matrix, [J] using the real and reactive parts of Y_{bus}.
11. Take $[\Delta I] = \begin{Bmatrix} \Delta I_m \\ \Delta I_r \end{Bmatrix}$.
12. Calculate Voltage mismatches using Gauss Elimination and update the Voltages.
13. If iter<=itermax, advance iter and go to step 7. Else goto step 90.

Step 100: Calculate the line flows and losses and print the converged voltages and phase angles.

Step 90: Display the Problem has not converged in itermax iterations.

Single Matrix Model (SMM)

Single Matrix Model is like Fast Decoupled load flow, with a simple modification that there is only one matrix for our calculations; though it takes more iterations, this method is having less computational time.

C. Algorithm for SMM

- Step1: Read the system, line data and bus data.
- Step 2: Form Sparsity (itagf, itagto, adjq and adjl) vectors and then form Ybus using sparsity.
- Step 3: Initialize all elements of B' to zero.
- Step 4: Form B' matrix: diagonal elements as sum of admittance of all connected lines and off diagonal elements as the admittance of that particular line.
- Step 5: And then modify it at B'(nlsack, nslack).
- Step 6: Triangularise B' matrix using Cholesky method.
- Step 7: setdelPmax=0; delQmax=0- the convergence criteria to be checked.
- Step 8: Start the iterative process.
- Step 9: Calculate Pcal and Qcal at each bus, using the voltages and admittance data.
- Step 10: Calculate the power mismatches- delP and delQ at each bus. Find out the maximum mismatch- delPmax and delQmax.
- Step 11: if delPmax <= epsilon & delQmax <= epsilon goto step 100; if iter >= itermax go to step 99.
- Step 12: Modify delQ as a*delP + b*delQ; where a, b are random constants ranging from 0.35 to 0.7.
- Solve [delQ] = [B'] * [delV] using Cholesky, to get delV.
- Step 13: Update voltage magnitudes using the obtained delV.
- Step 14: iter = iter + 0.5.
- Step 15: Set delPmax = 0. delQmax = 0.
- Step 16: Repeat the calculations like in steps 6, 7, 8.
- Step 17: Now solve [delP] = [B'] * [deldel] to get deldel;
- Step 18: update del for all buses using the obtained deldel.
- Step 19: Calculate delPmax and delQmax.
- Step 20: if delPmax <= epsilon & delQmax <= epsilon

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goto step 100
Step 21:iter=iter+0.5
ifiter>=itermax
goto step 99
elsegoto step 7.
Step 99:Print 'Problem didn't converge in Itermax iterations'.
Step 100:Print, 'Problem converged in iter iterations' and
printout the converged voltages,phase angles and lineflows.
    
```

D. Results and Comparison

Above three algorithms have been tested on IEEE standard 15 bus, 33 bus, 69 bus system and results are presented and comparison study has been carried out.

E. IEEE 15 Bus System Results

Graphs for the converged voltages and phase angles are presented in fig 5. and fig 6.

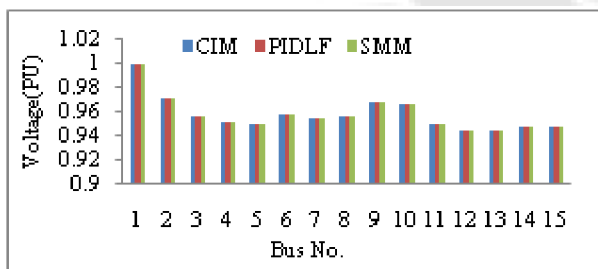


Figure 5: Converged voltage magnitudes

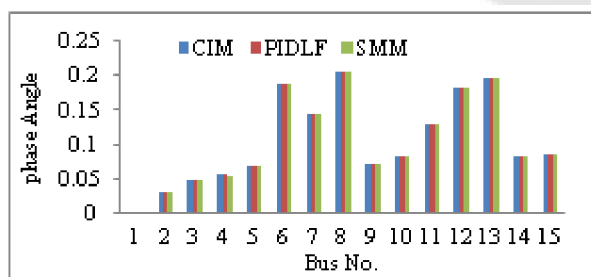


Figure 6: Converged phase angles

Table 1: IEEE 15 bus distribution system load flow results

NUMBER OF BUSES	15 BUS RESULTS					
	VOLTAGE(PU)			PHASE ANGLE(degree)		
	CIM	PIDLF	SMM	CIM	PIDLF	SMM
1	1	1	1	0	0	0
2	0.97128	0.96885	0.97031	0.0322	0.03212	0.032169
3	0.95667	0.95427	0.95571	0.04958	0.04945	0.049528
4	0.9509	0.94852	0.94995	0.05674	0.0566	0.056687
5	0.94991	0.94754	0.94896	0.06892	0.06875	0.06885
6	0.95822	0.95583	0.95726	0.19009	0.18962	0.189903
7	0.95476	0.95237	0.9538	0.14489	0.14453	0.144746
8	0.95694	0.95455	0.95599	0.20573	0.20522	0.205525
9	0.96797	0.96555	0.967	0.07219	0.07201	0.072121
10	0.96689	0.96448	0.96593	0.08522	0.085	0.085131
11	0.94995	0.94757	0.949	0.13176	0.13143	0.131625
12	0.94582	0.94346	0.94488	0.18265	0.1822	0.18247
13	0.94451	0.94215	0.94357	0.19892	0.19842	0.198721
14	0.9486	0.94623	0.94766	0.08508	0.08487	0.084997
15	0.94844	0.94606	0.94749	0.08716	0.08694	0.087074

Table 2: IEEE 15 bus distribution system total losses

15 BUS RESULTS LOSS (p.u)					
CIM		PIDLF		SMM	
P _{Loss}	Q _{Loss}	P _{Loss}	Q _{Loss}	P _{Loss}	Q _{Loss}
0.618129	0.576964	0.53	0.55	0.5894	0.5655

F. IEEE 33 bus system results:

Results for IEEE 33 bus system are presented in APPENDIX-I

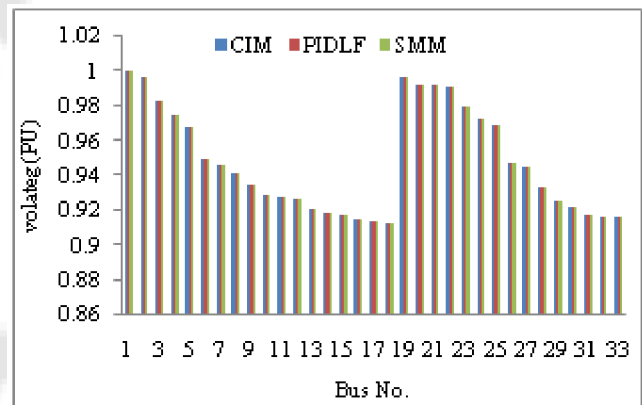


Figure 7: Converged voltage magnitudes

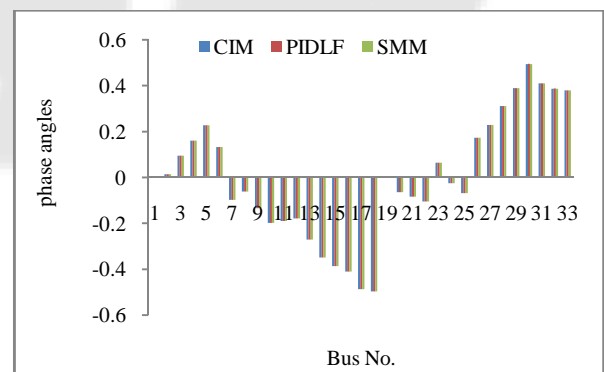


Figure 8: Converged phase angles

Table 3: IEEE 33 bus distribution system total losses

33 BUS RESULTS LOSS (p.u)					
CIM		PIDLF		SMM	
P_{loss}	Q_{loss}	P_{loss}	Q_{loss}	P_{loss}	Q_{loss}
0.07456	0.05174	0.0725549	0.050309	0.0769	0.05221

G. IEEE 69 Bus System Results

Results for IEEE 33 bus system are presented in APPENDIX-I

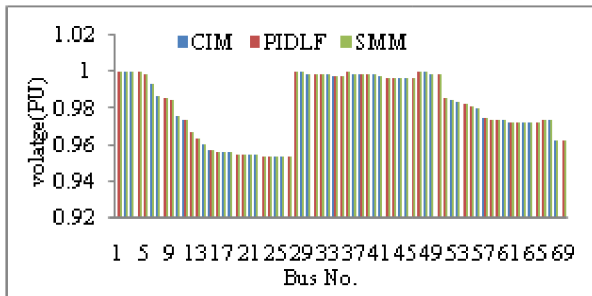


Figure 9: Converged voltage magnitudes

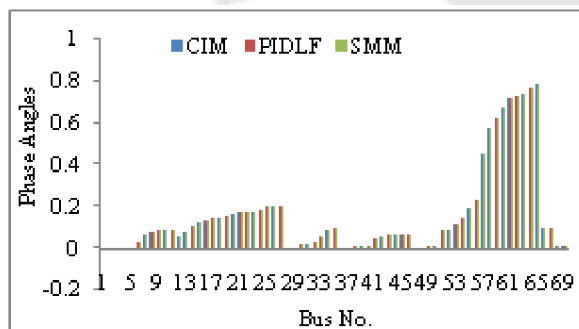


Figure 10: Converged phase angles

Table 4: IEEE 69 bus distribution system total losses

69 BUS RESULTS LOSS (p.u)					
CIM		PIDLF		SMM	
P_{loss}	Q_{loss}	P_{loss}	Q_{loss}	P_{loss}	Q_{loss}
0.153936	0.066577	0.1687	0.06945	0.15387	0.0683

5. Conclusion

Three different distribution load flow methodologies had been presented and the comparative study had been carried out. PIDLF can work for mesh connected systems by making some modifications while forming the [BIBC]matrix and requires very less memory. The PIDLF method does not require any Gauss Elimination or LU decomposition. This method is fast and simple. The CIM method is simple, requires Gauss Elimination and is of Newtonian approach-so quadratic convergence is observed. It can work well for mesh connected systems also. The SMM method is the most accurate method and also requires less memory due to use of sparsity. This is like FDC, so very fast computing and requires LU decomposition. SMM method shows geometric convergence-uses Cholesky Decomposition and it can work well for mesh connected systems also. Thus all the three discussed load flows can work effectively for both Radial as well as Meshed systems and that too with little or no modifications.

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APPENDIX-1: IEEE 33 Bus Load Flow Results

NUMBER OF BUSES	33 BUS RESULTS					
	VOLTAGE(PU)			PHASE ANGLE(degree)		
	CIM	PIDLF	SMM	CIM	PIDLF	SMM
1	1	1	1	0	0	0
2	0.99702	0.994529	0.99602	0.013502	0.0134682	0.013488
3	0.98293	0.98047	0.98194	0.095065	0.0948273	0.09497
4	0.97545	0.973007	0.97447	0.160677	0.1602753	0.160516
5	0.96805	0.965628	0.96708	0.227314	0.2267457	0.227087
6	0.94965	0.947273	0.9487	0.132883	0.1325508	0.13275
7	0.94616	0.943797	0.94522	-0.09745	-0.097202	-0.09735
8	0.94132	0.938964	0.94038	-0.06137	-0.06122	-0.06131
9	0.93505	0.93271	0.93411	-0.13445	-0.134116	-0.13432
10	0.92925	0.926927	0.92832	-0.19747	-0.196973	-0.19727
11	0.92839	0.926069	0.92746	-0.19021	-0.189738	-0.19002
12	0.92689	0.924574	0.92596	-0.17872	-0.178275	-0.17854
13	0.92078	0.918475	0.91986	-0.27004	-0.269363	-0.26977
14	0.91851	0.916215	0.91759	-0.34872	-0.347846	-0.34837
15	0.9171	0.914805	0.91618	-0.3864	-0.385434	-0.38601
16	0.91573	0.913442	0.91482	-0.40966	-0.408631	-0.40925
17	0.9137	0.911419	0.91279	-0.48692	-0.485705	-0.48644
18	0.9131	0.910813	0.91218	-0.49651	-0.49527	-0.49601
19	0.99649	0.994002	0.9955	0.002671	0.0026643	0.002668
20	0.99292	0.990434	0.99192	-0.06431	-0.064148	-0.06424
21	0.99221	0.98973	0.99122	-0.08367	-0.083458	-0.08358
22	0.99157	0.989095	0.99058	-0.10402	-0.103755	-0.10391
23	0.97934	0.976893	0.97836	0.064103	0.0639427	0.064039
24	0.97267	0.970238	0.9717	-0.02463	-0.024571	-0.02461
25	0.96935	0.966922	0.96838	-0.06834	-0.068164	-0.06827
26	0.94772	0.945349	0.94677	0.17234	0.1719092	0.172168
27	0.94515	0.942791	0.94421	0.228495	0.2279238	0.228267
28	0.93371	0.93138	0.93278	0.311443	0.3106644	0.311132
29	0.9255	0.923182	0.92457	0.38935	0.3883766	0.388961
30	0.92194	0.919634	0.92102	0.494624	0.4933874	0.494129
31	0.91778	0.915483	0.91686	0.410214	0.4091885	0.409804
32	0.91686	0.91457	0.91595	0.387171	0.3862031	0.386784
33	0.91658	0.914287	0.91566	0.379441	0.3784924	0.379062

APPENDIX-II: IEEE 69 Bus Load Flow Results

NUMBER OF BUSES	69 BUS RESULTS					
	VOLT AGE(PU)			PHASE ANGLE(degree)		
	CIM	PIDLF	SMM	CIM	PIDLF	SMM
1	1	1	1	0	0	0
2	0.999977	0.9974771	0.99897702	-0.000614	-0.0006125	-0.0006134
3	0.999953	0.9974531	0.99895305	-0.001226	-0.0012229	-0.0012248
4	0.9999	0.9974003	0.9989001	-0.003071	-0.0030633	-0.0030679
5	0.99933	0.9968317	0.99833067	-0.01242	-0.012389	-0.0124076
6	0.993216	0.990733	0.99222278	0.032568	0.0324866	0.0325354
7	0.986994	0.9845265	0.98600701	0.072869	0.0726868	0.0727961
8	0.98549	0.9830263	0.98450451	0.082641	0.0824344	0.0825584
9	0.984708	0.9822462	0.98372329	0.086455	0.0862389	0.0863685
10	0.976279	0.9738383	0.97530272	0.085971	0.0857561	0.085885
11	0.974354	0.9719181	0.97337965	0.085564	0.0853501	0.0854784
12	0.967557	0.9651381	0.96658944	0.056506	0.0563647	0.0564495
13	0.964086	0.9616758	0.96312191	0.081182	0.080979	0.0811008
14	0.960598	0.9581965	0.9596374	0.103781	0.1035215	0.1036772
15	0.957287	0.9548938	0.95632971	0.127637	0.1273179	0.1275094
16	0.956745	0.9543531	0.95578826	0.132762	0.1324301	0.1326292
17	0.955774	0.9533846	0.95481823	0.142135	0.1417797	0.1419929
18	0.955762	0.9533726	0.95480624	0.142238	0.1418824	0.1420958
19	0.955192	0.952804	0.95423681	0.152722	0.1523402	0.1525693
20	0.954976	0.9525886	0.95402102	0.161259	0.1608559	0.1610977
21	0.954645	0.9522584	0.95369036	0.173439	0.1730054	0.1732656
22	0.954633	0.9522464	0.95367837	0.173867	0.1734323	0.1736931
23	0.954496	0.9521098	0.9535415	0.177721	0.1772767	0.1775433
24	0.954277	0.9518913	0.95332272	0.185149	0.1846861	0.1849639
25	0.954006	0.951621	0.95305199	0.199332	0.1988337	0.1991327
26	0.953977	0.9515921	0.95302302	0.204282	0.2037713	0.2040777
27	0.953969	0.9515841	0.95301503	0.205697	0.2051828	0.2054913
28	0.999936	0.9974362	0.99893606	-0.001458	-0.0014544	-0.0014565
29	0.999733	0.9972337	0.99873327	-0.000779	-0.0007771	-0.0007782
30	0.999457	0.9969584	0.99845754	0.019094	0.0190463	0.0190749
31	0.999411	0.9969125	0.99841159	0.021999	0.021944	0.021977
32	0.999247	0.9967489	0.99824775	0.034217	0.0341315	0.0341828
33	0.998896	0.9963988	0.9978971	0.055565	0.0554261	0.0555094
34	0.99839	0.995894	0.99739161	0.084674	0.0844623	0.0845893
35	0.99813	0.9956347	0.99713187	0.096757	0.0965151	0.0966602
36	0.999925	0.9974252	0.99892508	-0.00101	-0.0010075	-0.001009
37	0.999544	0.9970451	0.99854446	0.002508	0.0025017	0.0025055
38	0.999241	0.9967429	0.99824176	0.011105	0.0110772	0.0110939
39	0.999161	0.9966631	0.99816184	0.013239	0.0132059	0.0132258
40	0.999156	0.9966581	0.99815684	0.013345	0.0133116	0.0133317
41	0.997629	0.9951349	0.99663137	0.050111	0.0499857	0.0500609
42	0.997086	0.9945933	0.99608891	0.063714	0.0635547	0.0636503
43	0.997026	0.9945334	0.99602897	0.065055	0.0648924	0.0649899
44	0.997015	0.9945225	0.99601799	0.065293	0.0651298	0.0652277
45	0.996936	0.9944437	0.99593906	0.067548	0.0673791	0.0674805
46	0.996935	0.9944427	0.99593807	0.067554	0.0673851	0.0674864
47	0.99989	0.9973903	0.99889011	-0.002932	-0.0029247	-0.0029291
48	0.99972	0.9972207	0.99872028	0.000599	0.0005975	0.0005984
49	0.999327	0.9968287	0.99832767	0.00853	0.0085087	0.0085215
50	0.999269	0.9967708	0.99826973	0.009515	0.0094912	0.0095055
51	0.985447	0.9829834	0.98446155	0.085292	0.0850788	0.0852067
52	0.985359	0.9828956	0.98437364	0.089834	0.0896094	0.0897442
53	0.983837	0.9813774	0.98285316	0.116641	0.1163494	0.1165244
54	0.982887	0.9804298	0.98190411	0.149397	0.1490235	0.1492476
55	0.9816	0.979146	0.9806184	0.190314	0.1898382	0.1901237
56	0.980473	0.9780218	0.97949253	0.228784	0.228212	0.2285552
57	0.97524	0.9728019	0.97426476	0.454191	0.4530555	0.4537368
58	0.974129	0.9716937	0.97315487	0.578149	0.5767036	0.5775709
59	0.973792	0.9713575	0.97281821	0.621488	0.6199343	0.6208665
60	0.973437	0.9710034	0.97246356	0.669107	0.6674342	0.6684379
61	0.972846	0.9704139	0.97187315	0.720173	0.7183726	0.7194528
62	0.972762	0.9703301	0.97178924	0.728313	0.7264922	0.7275847
63	0.972679	0.9702473	0.97170632	0.737846	0.7360014	0.7371082
64	0.972407	0.969976	0.97143459	0.769264	0.7673408	0.7684947
65	0.972207	0.9697765	0.97123479	0.792489	0.7905078	0.7916965
66	0.974214	0.9717785	0.97323979	0.093076	0.0928433	0.0929829
67	0.974214	0.9717785	0.97323979	0.093184	0.092951	0.0930908
68	0.962995	0.9605875	0.96203201	0.006544	0.0065276	0.0065375
69	0.962993	0.9605855	0.96203001	0.006644	0.0066274	0.0066374