

# A New Class of Homeomorphisms in Soft Topological Spaces

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**Abstract:** In this paper, we define and study the concepts of soft  $\pi$ gb-homeomorphism, soft  $\pi$ gb-regular, soft  $\pi$ gb-normal, soft  $\pi$ gb-compact and soft  $\pi$ gb-connectedness in soft topological spaces. Further its characterizations are established.

**Keywords:** soft  $\pi$ gb-regular, soft  $\pi$ gb-normal, soft  $\pi$ gb-compact, soft  $\pi$ gb-homeomorphism, soft  $\pi$ gb-connected.

## 1. Introduction

Molodtsov[12,13] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Soft systems provide a general framework with the involvement of parameters. Soft set theory has a wider application and its progress is very rapid in different fields. Levine[10] introduced g-closed sets in general topology. Kannan [8] introduced soft g-closed sets in soft topological spaces. Muhammad Shabir and Munazza Naz [16] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. Soft semi-open sets and its properties were introduced and studied by Bin Chen[3]. Kharal et al.[9] introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [4] in 2013 studied and discussed the properties of Soft continuous mappings which are defined over an initial universe set with a fixed set of parameters. Mahanta and Das [16] introduced and characterized various forms of soft functions like semi continuous, semi irresolute, semi open soft functions.

In the paper, the concepts soft  $\pi$ gb-homeomorphism soft  $\pi$ gb-regular, soft  $\pi$ gb-normal, soft  $\pi$ gb-compact and soft  $\pi$ gb-connectedness in soft topological spaces are discussed and some of its characterizations are obtained.

## 2. Preliminaries

Let  $U$  be an initial universe set and  $E$  be a collection of all possible parameters with respect to  $U$ , where parameters are the characteristics or properties of objects in  $U$ . Let  $P(U)$  denote the power set of  $U$ , and let  $A \subseteq E$ .

**Definition 2.1** ([12]). A pair  $(F,A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F : A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parametrized family of subsets of the universe  $U$ . For a particular  $e \in A$ ,  $F(e)$  may be considered the set of  $e$ -approximate elements of the soft set  $(F,A)$ .

**Definition 2.2.** ([5]). For two soft sets  $(F,A)$  and  $(G,B)$  over a common universe  $U$ , we say that  $(F,A)$  is a soft subset of  $(G,B)$  if

(i)  $A \subseteq B$ , and

(ii)  $\forall e \in A, F(e) \subseteq G(e)$ .

We write  $(F,A) \subseteq (G,B)$ .  $(F,A)$  is said to be a soft super set of  $(G,B)$ , if  $(G,B)$  is a soft subset of  $(F,A)$ . We denote it by  $(F,A) \supseteq (G,B)$ .

**Definition 2.3.** ([11]). A soft set  $(F,A)$  over  $U$  is said to be

(i) null soft set denoted by  $\phi$  if  $\forall e \in A, F(e) = \phi$ .

(ii) absolute soft set denoted by  $A$ , if  $\forall e \in A, F(e) = U$ .

**Definition 2.4.** For two soft sets  $(F,A)$  and  $(G,B)$  over a common universe  $U$ ,

(i) ([11]) union of two soft sets of  $(F,A)$  and  $(G,B)$  is the soft set  $(H,C)$ , where  $C = A \cup B$ , and  $\forall e \in C$ ,

$$H(e) = \begin{cases} F(e) & , \text{if } e \in A - B \\ G(e) & , \text{if } e \in B - A \\ F(e) \cup G(e) & , \text{if } e \in A \cap B \end{cases}$$

We write  $(F,A) \cup (G,B) = (H,C)$ .

**Definition :2.5** ([5])

The Intersection  $(H,C)$  of two soft sets  $(F,A)$  and  $(G,B)$  over a common universe  $U$  denoted  $(F,A) \cap (G,B)$  is defined as  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$  for all  $e \in C$ .

**Definition:2.6** ([16])

For a soft set  $(F,A)$  over the universe  $U$ , the relative complement of  $(F,A)$  is denoted by  $(F,A)'$  and is defined by  $(F,A)' = (F',A)$ , where  $F' : A \rightarrow P(U)$  is a mapping defined by  $F'(e) = U - F(e)$  for all  $e \in A$ .

**Definition: 2.7** ([16])

Let  $\tau$  be the collection of soft sets over  $X$ , then  $\tau$  is called a soft topology on  $X$  if  $\tau$  satisfies the following axioms:

- 1)  $\phi, \tilde{X}$  belong to  $\tau$
- 2) The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
- 3) The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ . For simplicity, we can take the soft topological space  $(X, \tau, E)$  as  $X$  throughout the work.

**Definition:2.8** ([16])

Let  $(X, \tau, E)$  be soft space over  $X$ . A soft set  $(F, E)$  over  $X$  is said to be soft closed in  $X$ , if its relative complement  $(F, E)'$  belongs to  $\tau$ . The relative complement is a mapping  $F': E \rightarrow P(X)$  defined by  $F'(e) = X - F(e)$  for all  $e \in E$ .

**Definition:2.9** ([16])

Let  $X$  be an initial universe set,  $E$  be the set of parameters and  $\tau = \{ \phi, \tilde{X} \}$ . Then  $\tau$  is called the soft indiscrete topology on  $X$  and  $(X, \tau, E)$  is said to be a soft indiscrete space over  $X$ . If  $\tau$  is the collection of all soft sets which can be defined over  $X$ , then  $\tau$  is called the soft discrete topology on  $X$  and  $(X, \tau, E)$  is said to be a soft discrete space over  $X$ .

**Definition:2.10** ([16])

Let  $(X, \tau, E)$  be a soft topological space over  $X$  and the soft interior of  $(F, E)$  denoted by  $Int(F, E)$  is the union of all soft open subsets of  $(F, E)$ . Clearly,  $(F, E)$  is the largest soft open set over  $X$  which is contained in  $(F, E)$ . The soft closure of  $(F, E)$  denoted by  $Cl(F, E)$  is the intersection of all closed sets containing  $(F, E)$ . Clearly,  $(F, E)$  is smallest soft closed set containing  $(F, E)$ .

$$Int(F, E) = \cup \{ (O, E) : (O, E) \text{ is soft open and } (O, E) \tilde{\subset} (F, E) \}.$$

$$Cl(F, E) = \cap \{ (O, E) : (O, E) \text{ is soft closed and } (F, E) \tilde{\subset} (O, E) \}.$$

**Definition:2.11** ([3],[8],[11])

Let  $U$  be the common universe set and  $E$  be the set of all parameters. Let  $(F, A)$  and  $(G, B)$  be soft sets over a common universe set  $U$  and  $A, B \tilde{\subset} E$ . Then  $(F, A)$  is a subset of  $(G, B)$ , denoted by  $(F, A) \tilde{\subset} (G, B)$ .  $(F, A)$  equals  $(G, B)$ , denoted by  $(F, A) = (G, B)$  if  $(F, A) \tilde{\subset} (G, B)$  and  $(G, B) \tilde{\subset} (F, A)$ .

**Definition:2.12**

A soft subset  $(A, E)$  of  $X$  is called

- (i) a soft generalized closed (Soft g-closed)[8] if  $Cl(A, E) \tilde{\subset} U, E$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$ .
- (ii) a soft b-open[7] if  $(A, E) \tilde{\subset} Cl(Int(A, E)) \cap Int(Cl(A, E))$
- (iii) a soft  $\pi$ gb-closed in  $X$  if  $sbcl(A, E) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $X$ .

The complement of the soft semi open, soft regular open, soft  $\alpha$ -open, soft b-open, soft pre-open sets are their respective soft semi closed, soft regular closed, soft  $\alpha$ -closed, soft b-closed and soft pre-closed sets.

The finite union of soft regular open sets is called soft  $\pi$ -open set and its complement is soft  $\pi$ -closed set. The soft regular open set of  $X$  is denoted by  $SRO(X)$  or  $SRO(X, \tau, E)$ .

**Definition:2.13** [8]

A soft topological space  $X$  is called a soft  $T_{1/2}$ -space if every soft g-closed set is soft closed in  $X$ .

**Definition:2.14**[7]

The soft regular closure of  $(A, E)$  is the intersection of all soft regular closed sets containing  $(A, E)$ . (i.e) The smallest soft regular closed set containing  $(A, E)$  and is denoted by  $srlcl(A, E)$ .

The soft regular interior of  $(A, E)$  is the union of all soft regular open sets contained in  $(A, E)$  and is denoted by  $srint(A, E)$ .

Similarly, we define soft  $\alpha$ -closure, soft pre-closure, soft semi closure and soft b-closure of the soft set  $(A, E)$  of a topological space  $X$  and are denoted by  $sacl(A, E)$ ,  $splcl(A, E)$ ,  $sscl(A, E)$  and  $sbcl(A, E)$  respectively.

**Definition 2.15.** [12] Let  $(F, E)$  be a soft set  $X$ . The soft set  $(F, E)$  is called a soft point, denoted by  $(x_e, E)$ , if for the element  $e \in E$ ,  $F(e) = \{x\}$  and  $F(e') = \phi$  for all  $e' \in E - \{e\}$ .

**Definition 2.16.** Let  $(X, \tau, E)$  and  $(Y, \tau', E)$  be two soft topological spaces. A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be

- (i) soft semi-continuous[15] if  $f^{-1}((G, E))$  is soft semi-open in  $(X, \tau, E)$ , for every soft open set  $(G, E)$  of  $(Y, \tau', E)$ .
- (ii) soft pre-continuous [17] if  $f^{-1}((G, E))$  is soft pre-open in  $(X, \tau, E)$ , for every soft open set  $(G, E)$  of  $(Y, \tau', E)$ .
- (iii) soft  $\pi$ gr-continuous[7] if  $f^{-1}((G, E))$  is soft  $\pi$ gr-closed in  $(X, \tau, A)$  for every soft closed set  $(G, E)$  in  $(Y, \tau', E)$ .
- (iv) soft  $\pi$ gr-irresolute [7] if  $f^{-1}((G, E))$  is soft  $\pi$ gr-closed in  $(X, \tau, A)$  for every soft  $\pi$ gr-closed set  $(G, E)$  in  $(Y, \tau', E)$ .

**Definition:2.20** [7] Let  $(X, \tau, A)$  and  $(Y, \tau^*, B)$  be soft topological spaces and  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a function. Then the function  $f_{pu}$  is called soft open mapping if  $f_{pu}((G, A)) \in \tau^*$  for all  $(G, A) \in \tau$ . Similarly, a function  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  is called a soft closed map if for a closed set  $(F, A)$  in  $\tau$ , the image  $f_{pu}((G, B))$  is soft closed in  $\tau^*$ .

**Definition 2.21** [16]

Let  $(X, \tilde{\tau})$  be a soft topological space over  $X$  and  $Y$  be a nonempty subset of  $X$ . Then  $\tilde{\tau}_Y = \{ (F, E) : (F, E) \in \tilde{\tau} \}$  is said to be the soft relative topology on  $Y$  and  $(Y, \tilde{\tau}_Y)$  is called a soft subspace of  $(X, \tilde{\tau})$ . We can easily verify that  $\tilde{\tau}_Y$  is, in fact, a soft topology on  $Y$ .

**Theorem 2.22** [16]

Let  $(Y, \tilde{\tau}_Y)$  be a soft subspace of a soft topological space  $(X, \tilde{\tau})$  and  $(F, E)$  be a soft set over  $X$ , then (1)  $(F, E)$  is soft open in  $Y$  if and only if  $(F, E) = \tilde{Y} \cap (G, E)$  for some  $(G, E) \in \tilde{\tau}$ . (2)  $(F, E)$  is soft closed in  $Y$  if and only if  $(F, E) = \tilde{Y} \cap (G, E)$  for some soft closed set  $(G, E)$  in  $X$ .

**Definition 2.23** [16]

Let  $(X, \tilde{\tau})$  be a soft topological space over  $X$ ,  $(G, E)$  be a soft closed set in  $X$  and  $x \in X$  such that  $x \notin (G, E)$ . If there exist soft open sets  $(F_1, E)$  and  $(F_2, E)$  such that  $x \in (F_1, E)$ ,  $(G, E) \sqsubseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \phi$ , then  $(X, \tilde{\tau})$  is called a soft regular space.

**Definition 2.24 [16]**

[7] Let  $(X, \tilde{\tau})$  be a soft topological space over  $X$ ,  $(F, E)$  and  $(G, E)$  soft closed sets in  $X$  such that  $(F, E) \cap (G, E) = \Phi$ . If there exist soft open sets  $(F_1, E)$  and  $(F_2, E)$  such that  $(F, E) \sqsubseteq (F_1, E)$ ,  $(G, E) \sqsubseteq (F_2, E)$  and  $(F_1, E) \cap (F_2, E) = \Phi$ , then  $(X, \tilde{\tau})$  is called a soft normal space.

**Definition 2.25 [1]**

[8] Let  $(X, \tilde{\tau})$  be a soft topological space over  $X$ , (1) A family  $C = \{(F, E) : i \in I\}$  of soft open sets in  $(X, \tilde{\tau})$  is called a soft open cover of  $X$ , if satisfies  $\sqcup_{i \in I} (F, E) = \tilde{X}$ . A finite subfamily of soft open cover  $C$  of  $X$  is called a finite subcover of  $C$ , if it is also a soft open cover of  $X$ . (2)  $X$  is called soft compact if every soft open cover of  $X$  has a finite subcover.

**3. Soft  $\pi$ gb-homeomorphism**

**Definition 3.1:** A bijection  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is called soft  $\pi$ gb-homeomorphism if  $f$  is both soft  $\pi$ gb-continuous and soft  $\pi$ gb-open map.

**Definition 3.2:** A bijection  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is called soft  $\pi$ gbC-homeomorphism if  $f$  is both soft  $\pi$ gb-irresolute and  $f^{-1}$  is soft  $\pi$ gb-irresolute.

**Proposition 3.3:** For any bijection  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ , the following statements are equivalent.

- $f^{-1}: Y \rightarrow X$  is soft  $\pi$ gb-continuous.
- $f$  is a soft  $\pi$ gb-open map.
- $f$  is a soft  $\pi$ gb-closed map.

**Proof:** (a)  $\Rightarrow$  (b). Let  $(A, E)$  be a soft open set in  $X$ . Then  $X-$   $(A, E)$  is soft closed in  $X$ . Since  $f^{-1}$  is soft  $\pi$ gb-continuous,  $(f^{-1})^{-1}(X- (A, E)) = f(X- (A, E)) = Y-f((A, E))$  is soft  $\pi$ gb-closed in  $Y$ . Then  $f((A, E))$  is soft  $\pi$ gb-open in  $Y$ . Hence  $f$  is a soft  $\pi$ gb-open map.

(b)  $\Rightarrow$  (c). Let  $f$  be a soft  $\pi$ gb-open map. Let  $(A, E)$  be a soft closed set in  $X$ . Then  $X-$   $(A, E)$  is soft open in  $X$ . Since  $f$  is soft  $\pi$ gb-open,  $f(X- (A, E)) = Y-f((A, E))$  is soft  $\pi$ gb-open in  $Y$ . Then  $f((A, E))$  is soft  $\pi$ gb-closed in  $Y$ . Hence  $f$  is soft  $\pi$ gb-closed.

(c)  $\Rightarrow$  (a). Let  $(A, E)$  be soft closed set in  $X$ . Then  $f((A, E))$  is soft  $\pi$ gb-closed in  $Y$ . That is  $(f^{-1})^{-1}(f((A, E)))$  is soft  $\pi$ gb-closed in  $Y$ . Hence  $f^{-1}$  is soft  $\pi$ gb-continuous.

**Proposition 3.4:** Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be a bijective and soft  $\pi$ gb-continuous map. Then the following statements are equivalent.

- $f$  is a soft  $\pi$ gb-open map.
- $f$  is a soft  $\pi$ gb-homeomorphism.
- $f$  is a soft  $\pi$ gb-closed map.

**Proof:** (a)  $\Rightarrow$  (b) Follows from the definition.

(b)  $\Rightarrow$  (c) Let  $(A, E)$  be a soft closed set in  $X$ . Then  $X-$   $(A, E)$  is soft open in  $X$ . Since  $f$  is a soft  $\pi$ gb-homeomorphism,  $f(X- (A, E)) = Y-f((A, E))$  is soft  $\pi$ gb-open in  $Y$ . Then  $f((A, E))$  is soft  $\pi$ gb-closed in  $Y$ . Hence  $f$  is a soft  $\pi$ gb-closed map.

(c)  $\Rightarrow$  (a) Let  $(A, E)$  be a soft open set in  $X$ . Then  $X-$   $(A, E)$  is soft closed in  $X$ . Since  $f$  is a soft  $\pi$ gb-closed map,  $f(X- (A, E)) = Y-f((A, E))$  is soft  $\pi$ gb-closed in  $Y$ . Then  $f((A, E))$  is soft  $\pi$ gb-open in  $Y$ . Hence  $f$  is a soft  $\pi$ gb-open map.

**Proposition 3.5:** If  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  and  $g: (Y, \tau', E) \rightarrow (Z, \tau'', E)$  are soft  $\pi$ gbC-homeomorphisms, then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is also a soft  $\pi$ gbC-homeomorphism.

**Proof:** Let  $(A, E)$  be a soft  $\pi$ gb-open set in  $(Z, \tau'', E)$  Now  $(g \circ f)^{-1}((A, E)) = f^{-1}(g^{-1}((A, E))) = f^{-1}((A, E))$ , where  $(A, E) = g^{-1}((A, E))$ . By hypothesis,  $(A, E)$  is soft  $\pi$ gb-open in  $(Y, \tau', E)$  and again by hypothesis,  $f^{-1}((A, E))$  is soft  $\pi$ gb-open in  $(X, \tau, E)$ . Therefore  $(g \circ f)$  is soft  $\pi$ gb-irresolute. Also for a soft  $\pi$ gb-open set  $(G, E)$  in  $(X, \tau, E)$ , we have  $(g \circ f)((G, E)) = g(f((G, E))) = g((W, E))$ , where  $(W, E) = f((G, E))$ . By hypothesis,  $f((G, E))$  is soft  $\pi$ gb-open in  $(Y, \tau', E)$  and again by hypothesis,  $g((W, E))$  is soft  $\pi$ gb-open in  $(Z, \tau'', E)$ . Therefore  $(g \circ f)^{-1}$  is soft  $\pi$ gb-irresolute. Hence  $g \circ f$  is soft  $\pi$ gbC-homeomorphism.

**Proposition 3.6:** For a soft topological space  $(X, \tau, E)$ , the collection  $S_{\pi}gbCh(X, \tau, E)$  forms a group under the composition of functions.

**Proof:** Define  $\Psi: S_{\pi}gbCh(X, \tau, E) \times (1, 2)^* - S_{\pi}gbCh(X, \tau, E) \rightarrow S_{\pi}gbCh(X, \tau, E)$  by  $\Psi(f, g) = (g \circ f)$  for every  $f, g \in S_{\pi}gbCh(X, \tau, E)$ . Then by proposition 3.5,  $(g \circ f) \in S_{\pi}gbCh(X, \tau, E)$  Hence  $S_{\pi}gbCh(X, \tau, E)$  is soft closed. We know that the composition of maps is associative. The identity map  $i: (X, \tau, E) \rightarrow (X, \tau, E)$  is a  $S_{\pi}gbCh$ -homeomorphism and  $i \in S_{\pi}gbCh(X, \tau, E)$  Also  $i \circ f = f \circ i = f$  for every  $f \in S_{\pi}gbCh(X, \tau, E)$ . For any  $f \in S_{\pi}gbCh(X, \tau, E)$ ,  $f \circ f^{-1} = f^{-1} \circ f = i$ . Hence inverse exists for each element of  $S_{\pi}gbCh(X, \tau_1, \tau_2)$ . Thus  $S_{\pi}gbCh(X, \tau, E)$  is a group under composition of maps.

**Proposition 3.7:** Every soft  $\pi$ gb-homeomorphism from a soft  $\pi$ gb-space into another soft  $\pi$ gb-space is a soft homeomorphism.

**Proof:** Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ , be a soft  $\pi$ gb-homeomorphism. Then  $f$  is bijective, soft  $\pi$ gb-continuous and soft  $\pi$ gb-open. Let  $(A, E)$  be an soft open set in  $(X, \tau, E)$ . Since  $f$  is soft  $\pi$ gb-open and since  $(Y, \tau', E)$  is soft  $\pi$ gb-space,  $f((A, E))$  is soft open in  $(Y, \tau', E)$ . This implies  $f$  is soft open map. Let  $(A, E)$  be soft closed in  $(Y, \tau', E)$ . Since  $f$  is soft  $\pi$ gb-continuous and since  $(X, \tau, E)$  is soft  $\pi$ gb-space,  $f^{-1}((A, E))$  is soft closed in  $(X, \tau, E)$ . Therefore  $f$  is soft continuous. Hence  $f$  is a soft homeomorphism.

**Proposition 3.8:** Every soft  $\pi$ gb-homeomorphism from a soft  $\pi$ gb-space into another soft  $\pi$ gb-space is a soft  $\pi$ gbC-homeomorphism.

**Proof:** Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be a soft  $\pi$ gb-homeomorphism. Then  $f$  is bijective, soft  $\pi$ gb-continuous and soft  $\pi$ gb-open. Let  $(A, E)$  be an soft  $\pi$ gb-closed set in  $(Y, \tau', E)$  Then  $(A, E)$  is soft closed in  $(Y, \tau', E)$  Since  $f$  is soft  $\pi$ gb-continuous  $f^{-1}((A, E))$  is soft  $\pi$ gb-closed in  $(X, \tau, E)$ . Hence  $f$  is a soft  $\pi$ gb-irresolute map. Let  $(V, E)$  be soft  $\pi$ gb-open in  $(X, \tau, E)$ . Then  $(V, E)$  is soft open in  $(X, \tau, E)$ . Since  $f$  is soft  $\pi$ gb-open,  $f((V, E))$  is soft  $\pi$ gb-open set in  $(Y, \tau', E)$ . That is  $(f^{-1})^{-1}((V, E))$  is soft  $\pi$ gb-open in  $(Y, \tau', E)$  and hence  $f^{-1}$  is soft  $\pi$ gb-irresolute. Thus  $f$  is soft  $\pi$ gbC-homeomorphism.



#### 4. Soft $\pi$ gb-regular and Soft $\pi$ gb -normal spaces

**Definition 4.1:** A soft topological space  $(X, \tau, E)$  is said to be soft  $\pi$ gb-regular if for every soft closed set  $(G, E)$  and each point  $x \notin (G, E)$ , there exist disjoint soft  $\pi$ gb-open sets  $(F_1, E)$  and  $(F_2, E)$  such that  $(G, E) \tilde{\subset} (F_1, E)$ ,  $x \in (F_2, E)$ ,  $(F_1, E) \cap (F_2, E) = \phi$ .

**Theorem 4.2:** Let  $(X, \tau, E)$  be a soft topological space. If  $X$  is a soft  $\pi$ gb-regular space then for every point  $x \in X$  and each soft open set  $(G, E)$  containing  $x$ , there exists a soft open set  $(F, E)$  in  $X$  such that  $x \in (F, E) \tilde{\subset} \pi$ gb-cl $((F, E)) \tilde{\subset} (G, E)$ .

**Proof:** Let  $x \in X$  and  $(G, E)$  be any soft open set in  $X$  such that  $x \in (G, E)$ . Then  $X - (G, E)$  is a soft closed set in  $X$  such that  $x \notin X - (G, E)$ . Since  $X$  is soft  $\pi$ gb-regular space, there exist soft  $\pi$ gb-open sets  $(F, E)$ ,  $(H, E)$  in  $X$  such that  $x \in (F, E)$ ,  $X - ((G, E)) \tilde{\subset} (H, E)$  and  $(F, E) \cap (H, E) = \phi$ . Now we have  $(F, E) \cap (H, E) = \phi$  implies  $\pi$ gb-cl $((F, E)) \cap ((H, E)) = \phi$ . Also  $X - ((G, E)) \tilde{\subset} (H, E)$ . Hence  $\pi$ gb-cl $((F, E)) \tilde{\subset} (G, E)$ . Therefore  $x \in (F, E) \tilde{\subset} \pi$ gb-cl $((F, E)) \tilde{\subset} (G, E)$ .

**Theorem 4.3:** If  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ , is a bijection, soft  $\pi$ gb-irresolute, soft closed map and  $Y$  is soft  $\pi$ gb-regular space then  $X$  is also soft  $\pi$ gb-regular space.

**Proof:** Let  $x \in X$  and  $(F, E)$  be any soft closed set in  $X$  such that  $x \notin (F, E)$ . Since  $f$  is a bijection, there exists a point  $y \in Y$  such that  $f(x) = y \Rightarrow x = f^{-1}(y)$ . Also since  $f$  is soft closed map,  $f((F, E))$  is a soft closed set in  $Y$  such that  $x \notin (F, E) \Rightarrow f(x) \notin f((F, E)) \Rightarrow y \notin f((F, E))$ . Since  $Y$  is soft  $\pi$ gb-regular space, there exist soft  $\pi$ gb-open sets  $(A, E)$ ,  $(B, E)$  in  $Y$  such that  $y \in (A, E)$ ,  $f((F, E)) \tilde{\subset} (B, E)$  and  $(A, E) \cap (B, E) = \phi$ . Since  $f$  is soft  $\pi$ gb-irresolute,  $f^{-1}((A, E))$ ,  $f^{-1}((B, E))$  are soft  $\pi$ gb-open sets in  $X$ . Now we have  $y \in (A, E) \Rightarrow f^{-1}(y) \in f^{-1}((A, E)) \Rightarrow x \in f^{-1}((A, E))$ ;  $f((F, E)) \subset (B, E) \Rightarrow f^{-1}[f((F, E))] \subset f^{-1}((B, E)) \Rightarrow (F, E) \subset f^{-1}((B, E))$  and  $f^{-1}((A, E) \cap (B, E)) = f^{-1}(\phi) \Rightarrow f^{-1}((A, E)) \cap f^{-1}((B, E)) = \phi$ , since  $f$  is a bijection. Thus, for every point  $x \in X$  and each soft closed set  $(F, E)$  in  $X$  such that  $x \notin (F, E)$ , there exist soft  $\pi$ gb-open sets  $f^{-1}((A, E))$ ,  $f^{-1}((B, E))$  in  $X$  such that  $x \in f^{-1}((A, E))$ ,  $(F, E) \subset f^{-1}((B, E))$  and  $f^{-1}((A, E)) \cap f^{-1}((B, E)) = \phi$ . Hence  $X$  is a soft  $\pi$ gb-regular space.

**Definition 4.4:** A space  $X$  is said to be soft  $\pi$ gb -normal if for any pair of disjoint soft closed sets  $(F_1, E)$  and  $(F_2, E)$ , there exist disjoint soft  $\pi$ gb -open sets  $(U, E)$  and  $(V, E)$  such that  $(F_1, E) \tilde{\subset} (U, E)$  and  $(F_2, E) \tilde{\subset} (V, E)$ .

**Definition 4.5:** A function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be soft  $M$ -  $\pi$ gb-closed if  $f(U)$  is soft  $\pi$ gb-open in  $Y$  for each soft  $\pi$ gb-closed set in  $X$ .

**Lemma 4.6:** A mapping  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft  $M$ - $\pi$ gb-closed if and only if for each soft subset  $(B, E)$  in  $Y$  and each soft  $\pi$ gb-open set  $(U, E)$  in  $X$  containing  $f^{-1}(B, E)$ , there exists a soft  $\pi$ gb-open set  $(V, E)$  containing  $(B, E)$  such that  $f^{-1}((V, E)) \tilde{\subset} (U, E)$ .

**Theorem 4.7:** If  $f$  is a soft  $M$ - $\pi$ gb-closed, soft continuous function from a soft  $\pi$ gb-normal space onto a space  $Y$ , then  $Y$  is soft  $\pi$ gb-normal.

**Proof:** Let  $(A, E)$  and  $(B, E)$  be two disjoint soft closed sets of  $Y$ . Then  $f^{-1}((A, E))$  and  $f^{-1}((B, E))$  are disjoint soft closed sets of  $X$ . Since  $X$  is soft  $\pi$ gb-normal, there exist disjoint soft  $\pi$ gb open sets  $(U, E)$  and  $(V, E)$  such that  $f^{-1}((A, E)) \tilde{\subset} (U, E)$  and  $f^{-1}((B, E)) \tilde{\subset} (V, E)$ . By lemma 2.4.34, there exists a soft  $\pi$ gb open sets  $(G, E)$  and  $(H, E)$  of  $Y$  such that  $(A, E) \tilde{\subset} (G, E)$  and  $(B, E) \subset (H, E)$ ,  $f^{-1}((G, E)) \tilde{\subset} (U, E)$  and  $f^{-1}((H, E)) \tilde{\subset} (V, E)$ . Since  $(U, E)$  and  $(V, E)$  are disjoint,  $(G, E)$  and  $(H, E)$  are disjoint and hence  $Y$  is soft  $\pi$ gb-normal.

**Theorem 4.8:** If  $X$  is soft  $\pi$ gb -normal, then for every pair of soft open sets  $(U, E)$  and  $(V, E)$  whose union is  $X$ , there exist soft  $\pi$ gb-closed sets  $(A, E)$  and  $(B, E)$  such that  $(A, E) \tilde{\subset} (U, E)$ ,  $(B, E) \tilde{\subset} (V, E)$  and  $(A, E) \cup (B, E) = X$ .

**Proof:** Let  $(U, E)$  and  $(V, E)$  be a pair of soft open sets in a soft  $\pi$ gb-normal space  $X$  such that  $X = (U, E) \cup (V, E)$ . Then  $X - (U, E)$ ,  $X - (V, E)$  are disjoint closed sets. Since  $X$  is soft  $\pi$ gb -normal there exist disjoint soft  $\pi$ gb -open sets  $(U_1, E)$  and  $(V_1, E)$  such that  $X - (U, E) \tilde{\subset} (U_1, E)$  and  $X - (V, E) \tilde{\subset} (V_1, E)$ . Let  $(A, E) = X - (U_1, E)$ ,  $(B, E) = X - (V_1, E)$ . Then  $(A, E)$  and  $(B, E)$  are soft  $\pi$ gb-closed sets such that  $(A, E) \tilde{\subset} (U, E)$ ,  $(B, E) \tilde{\subset} (V, E)$  and  $(A, E) \cup (B, E) = X$ .

**Definition 4.9:** A topological space  $X$  is soft  $\pi$ GBO-compact if every soft  $\pi$ gb-open cover of  $X$  has a finite sub cover.

**Definition 4.10:** A subset  $(B, E)$  of a topological space  $X$  is said to be soft  $\pi$ GBO-compact if  $(B, E)$  is soft  $\pi$ GBO-compact as a subspace of  $X$ .

**Theorem 4.11:** Suppose soft  $\pi$ GBO $(X, \tau)$  be soft closed under arbitrary unions. Let  $(X, \tau, E)$  be soft compact space. If  $(A, E)$  is soft closed set in  $X$ , then  $(A, E)$  is soft  $\pi$ gb compact.

**Proof:** Let  $(A, E)$  be soft  $\pi$ gb-closed subset of a soft  $\pi$ GBO-compact space  $X$ . Then  $(A, E)^c$  is soft  $\pi$ gb-open in  $X$ . Let  $(M, E) = \{(G_\alpha, E) : \alpha \in \Lambda\}$  be a soft cover of  $(A, E)$  by soft  $\pi$ gb-open sets in  $X$ .  $(A, E) \tilde{\subset} \cup \{(G_\alpha, E) : \alpha \in \Lambda\}$ . Then  $(M, E) \cup (A, E)^c$  is a soft  $\pi$ gb-open cover of  $X$ . By definition, every soft  $\pi$ gb open cover has a finite sub cover. Since  $X$  is soft  $\pi$ GBO-compact, there exists a finite  $\Lambda_0$  of  $\Lambda$  of  $X$ . Say  $X = \{(G_\alpha, E) : \alpha \in \Lambda_0\} \cup (A, E)^c$ . But  $(A, E)$  and  $(A, E)^c$  are disjoint. Hence  $(A, E) \tilde{\subset} \cup \{(G_\alpha, E) : \alpha \in \Lambda_0\}$ . This implies soft  $\pi$ gb-open cover  $(M, E)$  of  $(A, E)$  contains a finite sub cover. Therefore  $(A, E)$  is soft  $\pi$ GBO-compact relative to  $X$ . Therefore every soft  $\pi$ gb-closed subset of a soft  $\pi$ GBO-compact space  $X$  is soft  $\pi$ gb-compact.

**Theorem 4.12:** A surjective soft  $\pi$ gb-continuous image of a soft  $\pi$ GBO-compact space is soft compact.

**Proof:** Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ , be a soft  $\pi$ gb-continuous map from a soft  $\pi$ GBO-compact space  $X$  into a soft topological space  $Y$ . Let  $\{(A_i, E) ; i \in \Lambda\}$  be a soft open cover of  $Y$ . Then  $\{f^{-1}((A_i, E)) ; i \in \Lambda\}$  is a soft  $\pi$ gb open cover of  $X$ . Since  $X$  is soft  $\pi$ GBO-compact, every soft

$\pi$ gb-open cover of  $X$  has a finite subcover say  $\{f^{-1}((A_1, E), f^{-1}((A_2, E)), \dots, f^{-1}((A_n, E))\}$ . Since  $f$  is onto,  $\{(A_1, E), (A_2, E), \dots, (A_n, E)\}$  is a soft cover of  $Y$ , which is finite. Therefore  $Y$  is soft compact.

**Definition 4.13:** Two non-empty subsets  $(F, E)$  and  $(G, E)$  of a soft topological space  $(X, \tau, E)$  are called soft  $\pi$ gb-separated if and only if  $(F, E) \cap \pi\text{gb-cl}((G, E)) = \phi$  and  $\pi\text{gb-cl}((F, E)) \cap (G, E) = \phi$ .

**Definition 4.14:** A soft topological space  $(X, \tau, E)$  is said to be soft  $\pi$ GB-connected if  $X$  cannot be expressed as a disjoint union of two non empty soft  $\pi$ gb-open sets. A soft subset of  $X$  is soft  $\pi$ GB connected if it is soft  $\pi$ GB-connected as a soft subspace.

**Theorem 4.15:** A soft topological space  $(X, \tau, E)$  is soft  $\pi$ GB-connected if and only if  $X$  and  $\phi$  are the only soft subsets of  $X$  which are both soft  $\pi$ gb-open and soft  $\pi$ gb-closed.

**Proof.** Let  $(X, \tau, E)$  be soft  $\pi$ GB-connected. Let  $(G, E)$  be any soft  $\pi$ gb-open and soft  $\pi$ gb-closed subset in  $X$ . Then  $(G, E)^c$  is both soft  $\pi$ gb-open and soft  $\pi$ gb-closed. Then  $X$  is a disjoint union of soft  $\pi$ gb-open sets  $(G, E)$  and  $(G, E)^c$ . This contradicts the fact that  $X$  is soft  $\pi$ GB-connected, then either  $(G, E) = \phi$  (or)  $(G, E) = X$ .

Conversely, assume  $X = (A, E) \cup (B, E)$  where  $(A, E)$  and  $(B, E)$  are disjoint non empty soft  $\pi$ gb-open subsets of  $X$  then  $(A, E)$  is both soft  $\pi$ gb-open and soft  $\pi$ gb-closed. By assumption  $(A, E) = \phi$  or  $X$  which is a contradiction. Hence  $X$  is soft  $\pi$ GB-connected.

**Theorem 4.16:** If  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is a soft  $\pi$ gb-irresolute surjection and  $X$  is soft  $\pi$ GB-connected, then  $Y$  is soft  $\pi$ GB-connected.

**Proof.** Suppose  $Y$  is not soft  $\pi$ GB-connected. Then  $Y = (A, E) \cup (B, E)$  where  $(A, E)$  and  $(B, E)$  are two non empty disjoint soft  $\pi$ gb-open sets in  $Y$ . Since  $f$  is soft  $\pi$ gb-irresolute and onto,  $X = f^{-1}((A, E)) \cup f^{-1}((B, E))$  where  $f^{-1}((A, E))$  and  $f^{-1}((B, E))$  are disjoint non empty soft  $\pi$ gb-open sets in  $X$ . This contradicts the fact that  $X$  is soft  $\pi$ GB-connected. Hence  $Y$  is soft  $\pi$ GB-connected.

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