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A New Class of Homeomorphisms in Soft Topological Spaces

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Abstract: ***In this paper, we define and study the concepts of soft πgb-homeomorphism, soft πgb-regular, soft πgb-normal, soft πgb-compact and soft πgb-connectedness in soft topological spaces. Further its characterizations are established.***

Keywords: soft πgb-regular, soft πgb-normal, soft πgb-compact , soft πgb-homeomorphism, soft πgb-connected.

1. Introduction

Molodtsov[12,13] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Soft systems provide a general framework with the involvement of parameters. Soft set theory has a wider application and its progress is very rapid in different fields. Levine[10] introduced g-closed sets in general topology. Kannan [8] introduced soft g-closed sets in soft topological spaces. Muhammad Shabir and Munazza Naz [16] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. Soft semi-open sets and its properties were introduced and studied by Bin Chen[3]. Kharal et al.[9] introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [4] in 2013 studied and discussed the properties of Soft continuous mappings which are defined over an initial universe set with a fixed set of parameters. Mahanta and Das [16] introduced and characterized various forms of soft functions like semi continuous, semi irresolute, semi open soft functions.

In the paper, the concepts soft πgb-homeomorphism soft πgb-regular, soft πgb-normal, soft πgb-compact and soft πgb-connectedness in soft topological spaces are discussed and some of its characterizations are obtained.

2. Preliminaries

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P(U) denote the power set of U, and let A ⊆ E.

Definition 2.1 ([12]). A pair (F,A) is called a soft set over U, where F is a mapping given by F : A → P(U). In other words, a soft set over U is a parametrized family of subsets of the universe U. For a particular e ∈ A. F(e) may be considered the set of e-approximate elements of the soft set (F,A).

Definition 2.2. ([5]). For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if
(i) A ⊆ B, and
(ii)∀e ∈ A, F(e) $\tilde{\subseteq}$ G(e).
We write (F,A) $\tilde{\subseteq}$ (G,B). (F,A) is said to be a soft super set of (G,B), if (G,B) is a soft subset of (F,A). We denote it by (F,A) $\tilde{\supseteq}$ (G,B).

Definition 2.3. ([11]). A soft set (F,A) over U is said to be
(i) null soft set denoted by ϕ if ∀e ∈ A, F(e) = ϕ.
(ii) absolute soft set denoted by A, if ∀e ∈ A, F(e) = U.

Definition 2.4. For two soft sets (F,A) and (G,B) over a common universe U,
(i) ([11]) union of two soft sets of (F,A) and (G,B) is the soft set (H,C),where C = A ∪ B, and ∀e ∈ C,

$$H(e) = \begin{cases} F(e) & ,if\ e \in A - B \\ G(e) & ,if\ e \in B - A \\ F(e) \cup G(e) & ,if\ e \in A \cap B \end{cases}$$

We write (F,A) ∪(G,B)=(H,C).

Definition :2.5 ([5])
The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U denoted (F,A) ∩ (G,B) is defined as C= A∩ B and H(e) = F(e) ∩ G(e) for all e∈C.

Definition:2.6 ([16])
For a soft set (F,A) over the universe U, the relative complement of (F,A) is denoted by (F,A)′ and is defined by (F,A)′=(F′,A), where F′: A →P(U) is a mapping defined by F′(e)= U-F(e) for all e∈A.

Definition: 2.7 ([16])
Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:
1) ϕ, $\tilde{X}$ belong to τ
2) The union of any number of soft sets in τ belongs to τ.
3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X, τ, E) as X throughout the work.

Definition:2.8 ([16])
Let (X, τ, E) be soft space over X. A soft set (F,E) over X is said to be soft closed in X, if its relative complement (F,E)′ belongs to τ. The relative complement is a mapping F′:E→P(X) defined by F′(e)= X-F(e) for all e ∈A.

Definition:2.9 ([16])
Let X be an initial universe set , E be the set of parameters and τ={ ϕ, $\tilde{X}$ }. Then τ is called the soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete space over X. If τis the collection of all soft sets which can be defined over X, then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X.

Definition:2.10 ([16])
Let (X, τ, E) be a soft topological space over X and the soft interior of (F,E) denoted by Int(F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E).The soft closure of (F,E) denoted by Cl(F,E) is the intersection of all closed sets containing (F,E). Clearly , (F,E) is smallest soft closed set containing (F,E).

Int (F,E) = ∪ { (O,E): (O,E) is soft open and (O,E) $\tilde{\subseteq}$ (F,E)}.
Cl(F,E) = ∩{ (O,E): (O,E) is soft closed and (F,E) $\tilde{\subseteq}$ (O,E)}.

Definition:2.11 ([3],[8],[11])
Let U be the common universe set and E be the set of all parameters. Let (F,A) and (G,B) be soft sets over a common universe set U and A,B $\tilde{\subseteq}$E. Then (F,A) is a subset of (G,B), denoted by (F,A) $\tilde{\subseteq}$ (G,B). (F,A) equals (G,B) , denoted by (F,A)=(G,B) if (F,A) $\tilde{\subseteq}$ (G,B) and (G,B) $\tilde{\subseteq}$ (F,A).

Definition:2.12
A soft subset (A,E) of X is called
(i) a soft generalized closed (Soft g-closed)[8] if Cl(A,E) $\tilde{\subseteq}$ U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.
(ii)) a soft b-open[7] if (A,E) $\tilde{\subseteq}$Cl(Int(A,E)) Int(Cl(A,E))
(iii) **a** soft πgb-closed in X if sbcl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π-open in X.

The complement of the soft semi open , soft regular open , soft α-open, soft b-open , soft pre-open sets are their respective soft semi closed , soft regular closed , soft α-closed , soft b-closed and soft pre -closed sets.

The finite union of soft regular open sets is called soft π-open set and its complement is soft π-closed set.
The soft regular open set of X is denoted by SRO(X) or SRO(X, τ, E).

Definition:2.13 [8]
A soft topological space X is called a soft $T_{1/2}$-space if every soft g-closed set is soft closed in X.

Definition:2.14[7]
The soft regular closure of (A,E) is the intersection of all soft regular closed sets containing (A,E). (i.e)The smallest soft regular closed set containing (A,E) and is denoted by srcl(A,E).

The soft regular interior of (A,E) is the union of all soft regular open set s contained in (A,E) and is denoted by srint(A,E).

Similarly , we define soft α-closure, soft pre-closure, soft semi closure and soft b-closure of the soft set (A,E) of a topological space X and are denoted by sαcl(A,E), spcl(A,E), sscl(A,E) and sbcl(A,E) respectively.

Definition 2.15. [12] Let (F,E) be a soft set X. The soft set (F,E) is called a soft point , denoted by (x_e,E), if for the element e ∈ E, F(e) = {x} and F(e′) = ϕ for all e′ ∈ E − {e}.

Definition 2.16. Let (X, τ,E) and (Y, τ ′,E) be two soft topological spaces.A function f : (X, τ ,E) → (Y, τ ′,E) is said to be
(i) soft semi-continuous[15] if f^{-1}((G,E)) is soft semi-open in (X, τ ,E), for every soft open set G,E) of (Y, τ ′,E).
(ii) soft pre-continuous [17] if f^{-1}((G,E)) is soft pre-open in (X, τ,E), for every soft open set (G,E) of (Y, τ ′,E).
(iii) soft πgr-continuous[7] if f^{-1}((G,E)) is soft πgr-closed in (X, τ, A) for every soft closed set (G, E) in (Y, τ ′,E).
(iv)soft πgr-irresolute [7]if f^{-1}((G, E)) is soft πgr-closed in (X, τ, A) for every soft πgr- closed set (G, E) in (Y, τ ′,E).

Definition:2.20 **[7]** Let (X, τ, A) and (Y, τ*,B) be soft topological spaces and f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ be a function. Then the function f_{pu} is called soft open mapping if f_{pu}((G,A)) ∈ τ* for all (G,A)∈ τ. Similarly, a function $f_{pu}:SS(X)_A \rightarrow SS(Y)_B$ is called a soft closed map if for a closed set (F,A) in τ, the image f_{pu}((G,B)) is soft closed in τ*.

Definition 2.21 [16]
Let (X,$\tilde{\tau}$) be a soft topological space over X and Y be a nonempty subset of X. Then $\tilde{\tau}_Y$={(^{Y}F,E):(F,E)∈$\tilde{\tau}$} is said to be the soft relative topology on Y and (Y,$\tilde{\tau}_Y$) is called a soft subspace of (X,$\tilde{\tau}$). We can easily verify that $\tilde{\tau}_Y$ is, in fact, a soft topology on Y.

Theorem 2.22 [16]
Let (Y,$\tilde{\tau}_Y$) be a soft subspace of a soft topological space (X,$\tilde{\tau}$) and (F,E) be a soft set over X, then (1) (F,E) is soft open in Y if and only if (F,E)=Ỹ⊓(G,E) for some (G,E)∈$\tilde{\tau}$.
(2) (F,E) is soft closed in Y if and only if (F,E)=Ỹ⊓(G,E) for some soft closed set (G,E) in X.

Definition 2.23 [16]
Let (X,$\tilde{\tau}$) be a soft topological space over X, (G,E) be a soft closed set in X and x∈X such that x∉(G,E). If there exist soft open sets (F_1,E) and (F_2,E) such that x∈(F_1,E), (G,E)⊑(F_2,E) and (F_1,E)⊓(F_2,E)=Φ, then (X,$\tilde{\tau}$) is called a soft regular space.

Definition 2.24 [16]

[7] Let $(X,\tilde{\tau})$ be a soft topological space over X, (F,E) and (G,E) soft closed sets in X such that (F,E)⊓(G,E)=Φ. If there exist soft open sets (F_1,E) and (F_2,E) such that $(F,E)\sqsubseteq(F_1,E)$, $(G,E)\sqsubseteq(F_2,E)$ and $(F_1,E)\sqcap(F_2,E)=\Phi$, then $(X,\tilde{\tau})$ is called a soft normal space.

Definition 2.25 [1]

[8] Let $(X,\tilde{\tau})$ be a soft topological space over X, (1) A family $C=\{(F_i,E):i\in I\}$ of soft open sets in $(X,\tilde{\tau})$ is called a soft open cover of X, if satisfies $\sqcup_{i\in I}(F_i,E)=\tilde{X}$. A finite subfamily of soft open cover C of X is called a finite subcover of C, if it is also a soft open cover of X. (2) X is called soft compact if every soft open cover of X has a finite subcover.

3. Soft πgb-homeomorphism

Definition 3.1: A bijection f: (X, τ, E) → (Y, τ′, E) is called soft πgb-homeomorphism if f is both soft πgb-continuous and soft πgb-open map.

Definition 3.2: A bijection f: (X, τ, E) → (Y, τ′, E) is called soft πgbC-homeomorphism if f is both soft πgb-irresolute and f^{-1} is soft πgb-irresolute.

Proposition 3.3: For any bijection f: (X, τ, E) → (Y, τ′, E), the following statements are equivalent.

(a) $f^{-1}:Y\to X$ is soft πgb-continuous.
(b) f is a soft πgb-open map.
(c) f is a soft πgb-closed map.

Proof: (a) ⇒(b).Let (A, E) be a soft open set in X. Then X- (A, E) is soft closed in X. Since f^{-1} is soft πgb-continuous,$(f^{-1})^{-1}$(X- (A, E))=f(X- (A, E))=Y-f((A, E)) is soft πgb-closed in Y. Then f((A, E)) is soft πgb-open in Y. Hence f is a soft πgb-open map.

(b) ⇒(c).Let f be a soft πgb-open map. Let (A, E) be a soft closed set in X. Then X- (A, E) is soft open in X. Since f is soft πgb-open, f(X- (A, E))=Y-f((A, E)) is soft πgb-open in Y. Then f((A, E)) is soft πgb-closed in Y. Hence f is soft πgb-closed.

(c)⇒ (a).Let (A, E) be soft closed set in X. Then f((A, E)) is soft πgb-closed in Y. That is $(f^{-1})^{-1}$((A, E)) is soft πgb-closed in Y. Hence f^{-1} is soft πgb-continuous.

Proposition 3.4: Let f: (X, τ, E) → (Y, τ′, E) be a bijective and soft πgb-continuous map. Then the following statements are equivalent.

(a) f is a soft πgb-open map.
(b) f is a soft πgb-homeomorphism.
(c) f is a soft πgb-closed map.

ProoF: (a)⇒(b) Follows from the definition.

(b)⇒(c) Let (A, E) be a soft closed set in X. Then X- (A, E) is soft open in X. Since f is a soft πgb-homeomorphism, f(X- (A, E))= Y-f((A, E)) is soft πgb- open in Y. Then f((A, E)) is soft πgb-closed in Y. Hence f is a soft πgb-closed map.

(c)⇒(a) Let (A, E) be a soft open set in X. Then X- (A, E) is soft closed in X. Since f is a soft πgb-closed map, f (X-(A, E)) =Y-f((A, E)) is soft πgb-closed in Y. Then f((A, E)) is soft πgb-open in Y. Hence f is a soft πgb-open map.

Proposition 3.5: If f: (X, τ, E) → (Y, τ′, E) and g: (Y, τ′, E) → (Z, τ″, E) are soft πgbC-homeomorphisms, then g∘f: (X, τ, E)→ (Z, τ″, E) is also a soft πgbC-homeomorphism.

Proof: Let (A, E) be a soft πgb-open set in (Z, τ″, E) Now $(g\circ f)^{-1}$((A, E))=f^{-1}(g^{-1}((A, E)))=f^{-1}((A, E)),where (A, E)=g^{-1}((A, E)).By hypothesis, (A, E) is soft πgb-open in (Y, τ′, E) and again by hypothesis,f^{-1}((A, E)) is soft πgb-open in (X, τ, E).Therefore (g∘f) is soft πgb-irresolute. Also for a soft πgb-open set (G,E) in (X, τ, E) ,we have (g∘f)((G,E))=g(f((G,E)))=g((W,E)),where (W,E)=f(G,E).By hypothesis, f((G,E)) is soft πgb-open in (Y, τ′, E) and again by hypothesis, g(W,E) is soft πgb-open in (Z, τ″, E) .Therefore $(g\circ f)^{-1}$is soft πgb-irresolute. Hence g∘f is soft πgbC-homeomorphism.

Proposition 3.6: For a soft topological space (X, τ, E), the collection SπgbCh (X, τ, E) forms a group under the composition of functions.

Proof: Define Ψ: SπgbCh (X, τ, E) × $(1, 2)^*$- SπgbCh (X, τ, E) →SπgbCh (X, τ, E) by Ψ (f, g)= (g ∘ f) for every f, g ∈ SπgbCh(X, τ, E). Then by proposition 3.5, (g∘f)∈ SπgbCh(X, τ, E) Hence SπgbCh (X, τ, E) is soft closed. We know that the composition of maps is associative. The identity map i: (X, τ, E) →(X, τ, E) is a SπgbC-homeomorphism and i∈SπgbCh(X, τ, E) Also i∘f=f∘i=f for every f∈SπgbCh(X, τ, E).For any f∈SπgbCh(X, τ, E), $f\circ f^{-1}=f^{-1}\circ f=i$. Hence inverse exists for each element of SπgbCh(X, τ_1, τ_2).Thus SπgbCh(X, τ, E) is a group under composition of maps.

Proposition 3.7: Every soft πgb-homeomorphism from a soft πgb-space into another soft πgb-space is a soft homeomorphism.

Proof: Let f:(X, τ, E) → (Y, τ′, E), be a soft πgb-homeomorphism. Then f is bijective, soft πgb-continuous and soft πgb-open. Let (A, E) be an soft open set in (X, τ, E).Since f is soft πgb-open and since (Y, τ′, E) is soft πgb-space, f((A, E)) is soft open in (Y, τ′, E).This implies f is soft open map. Let (A, E) be soft closed in (Y, τ′, E).Since f is soft πgb-continuous and since (X, τ, E) is soft πgb-space, f^{-1}((A, E)) is soft closed in((X, τ, E).Therefore f is soft continuous. Hence f is a soft homeomorphism.

Proposition 3.8: Every soft πgb-homeomorphism from a soft πgb-space into another soft πgb-space is a soft πgbC-homeomorphism.

Proof: Let f :(X, τ, E) → (Y, τ′, E) be a soft πgb-homeomorphism. Then f is bijective, soft πgb-continuous and soft πgb-open. Let (A, E) be an soft πgb-closed set in (Y, τ′, E) Then (A, E) is soft closed in (Y, τ′, E) Since f is soft πgb-continuous f^{-1}((A, E)) is soft πgb-closed in (X, τ, E).Hence f is a soft πgb-irresolute map. Let (V,E) be soft πgb-open in (X, τ, E).Then (V,E) is soft open in (X, τ, E).Since f is soft πgb-open, f((V,E)) is soft πgb-open set in (Y, τ′, E).That is $(f^{-1})^{-1}$((V,E)) is soft πgb-open in (Y, τ′, E) and hence f^{-1} is soft πgb-irresolute. Thus f is soft πgbC-homeomorphism.

4. Soft πgb-regular and Soft πgb –normal spaces

Definition 4.1: A soft topological space (X, τ, E) is said to be soft πgb-regular if for every soft closed set (G,E) and each point x∉(G,E), there exist disjoint soft πgb-open sets (F_1,E) and (F_2,E) such that (G,E) $\tilde{\subseteq}$ (F_1,E), x∈(F_2,E), $(F_1,E) \cap (F_2,E)$ =ϕ.

Theorem 4.2: Let (X, τ, E) be a soft topological space. If X is a soft πgb-regular space then for every point x ∈X and each soft open set (G,E) containing x, there exists a soft open set (F,E) in X such that x ∈ (F,E) $\tilde{\subseteq}$sπgb-cl((F,E)) $\tilde{\subseteq}$ (G,E).

Proof: Let x ∈ X and (G,E) be any soft open set in X such that x ∈ (G,E). Then X −(G,E) is a soft closed set in X such that x ∉X −(G,E). Since X is sπgb-regular space, there exist soft πgb-open sets (F,E) , (H,E) in X such that x ∈ (F,E), X –((G,E)) $\tilde{\subseteq}$ (H,E) and (F,E) ∩(H,E) = ϕ. Now we have (F,E) ∩(H,E) =ϕ implies sπgb-cl((F,E))∩((H,E))=ϕ. Also X-((G,E)) $\tilde{\subseteq}$ (H,E). Hence sπgb-cl((F,E)) $\tilde{\subseteq}$ (G,E). Therefore x ∈(F,E) $\tilde{\subseteq}$ πgb-cl((F,E)) $\tilde{\subseteq}$ (G,E).

Theorem 4.3: If f: (X, τ, E) → (Y, τ′, E), is a bijection, soft πgb- irresolute, soft closed map and Y is sπgb-regular space then X is also sπgb-regular space.

Proof: Let x ∈ X and (F,E) be any soft closed set in X such that x ∉(F,E). Since f is a bijection, there exists a point y ∈ Y such that f(x) = y ⇒ x = f^{-1}(y). Also since f is soft closed map, f((F,E)) is a soft closed set in Y such that x ∉ (F,E) ⇒ f(x) ∉ f((F,E)) ⇒ y ∉ f((F,E)).Since Y is soft πgb-regular space, there exist πgb-open sets (A,E), (B,E) in Y such that y ∈ (A,E), f((F,E)) $\tilde{\subseteq}$ (B,E) and (A,E) ∩(B,E) = ϕ. Since f is soft πgb-irresolute, f^{-1}((A,E)), f^{-1}((B,E)) are soft πgb- open sets in X. Now we have y ∈ (A,E) ⇒ f^{-1}(y) ∈ f^{-1}((A,E)) ⇒ x ∈ f^{-1}((A,E)); f((F,E)) ⊂ (B,E) ⇒ f^{-1}[f((F,E))] ⊂ f^{-1}((B,E))⇒ (F,E) ⊂ f^{-1}((B,E)) and f^{-1}((A,E) ∩ (B,E)) = f^{-1}(ϕ) ⇒ f^{-1}((A,E)) ∩ f^{-1}((B,E)) = ϕ, since f is a bijection. Thus, for every point x ∈ X and each soft closed set (F,E) in X such that x ∉ (F,E), there exist soft πgb- open sets f^{-1}((A,E)), f^{-1}((B,E)) in X such that x∈f^{-1}((A,E)), (F,E) ⊂f^{-1}((F,E)) and f^{-1}((A,E)) ∩ f^{-1}((B,E)) = ϕ. Hence X is a soft πgb-regular space.

Definition 4.4: A space X is said to be soft πgb -normal if for any pair of disjoint soft closed sets $(F_1$,E) and $(F_2$,E), there exist disjoint soft πgb -open sets (U,E) and (V,E) such that $(F_1$,E) $\tilde{\subseteq}$ (U,E) and $(F_2$,E) $\tilde{\subseteq}$ (V,E).

Definition 4.5: A function f: (X, τ, E) → (Y, τ′, E) is said to be soft M- πgb-closed if f(U) is soft πgb-open in Y for each soft πgb-closed set in X.

Lemma 4.6: A mapping f: (X, τ, E) → (Y, τ′, E) is soft M-πgb-closed if and only if for each soft subset (B,E) in Y and each soft πgb-open set (U,E) in X containing f^{-1}(B,E),there exists a soft πgb-open set (V,E) containing (B,E) such that f^{-1}((V,E)) $\tilde{\subseteq}$ (U,E).

Theorem 4.7: If f is an soft M-πgb-closed, soft continuous function from a sπgb-normal space onto a space Y, then Y is sπgb-normal.

Proof: Let (A,E) and (B,E) be two disjoint soft closed sets of Y. Then f^{-1}((A,E)) and f^{-1}((B,E)) are disjoint soft closed sets of X. Since X is **s**πgb-normal, there exists disjoint soft πgb open sets (U,E) and (V,E) such that f^{-1}((A,E)) $\tilde{\subseteq}$ (U,E) and f^{-1}((B,E)) $\tilde{\subseteq}$ (V,E).By **lemma 2.4.34**, there exists a soft πgb open sets (G,E) and (H,E) of Y such that (A,E) $\tilde{\subseteq}$ (G,E) and (B,E) ⊂(H,E),f^{-1}((G,E)) $\tilde{\subseteq}$ (U,E) and f^{-1}((H,E)) $\tilde{\subseteq}$ (V,E).Since (U,E) and (V,E) are disjoint , (G,E) and (H,E) are disjoint and hence Y is sπgb-normal.

Theorem 4.8: If X is sπgb –normal, then for every pair of soft open sets (U,E) and (V,E) whose union is X, there exist soft πgb-closed sets (A,E) and (B,E) such that (A,E) $\tilde{\subseteq}$ (U,E), (B,E) $\tilde{\subseteq}$ (V,E) and (A,E) ∪(B,E) = X.

Proof: Let (U,E) and (V,E) be a pair of soft open sets in a soft πgb-normal space X such that X =(U,E)∪(V,E). Then X-(U,E) , X-(V,E) are disjoint closed sets. Since X is sπgb -normal there exist disjoint sπgb -open sets (U_1,E) and (V_1,E) such that X-(U,E) $\tilde{\subseteq}$ (U_1,E) and X-(V,E) $\tilde{\subseteq}$ (V_1,E). Let (A,E) = X-(U_1,E), (B,E) = X – (V_1,E). Then (A,E) and (B,E) are soft πgb-closed sets such that (A,E) $\tilde{\subseteq}$ (U,E) , (B,E) $\tilde{\subseteq}$ (V,E) and (A,E)∪ (B,E) = X.

Definition 4.9:A topological space X is SπGBO-compact if every soft πgb-open cover of X has a finite sub cover.

Definition 4.10: A subset (B,E) of a topological space X is said to be SπGBO-compact if (B,E) is SπGBO-compact as a subspace of X

Theorem 4.11: Suppose **S**πGBO(X, τ) be soft closed under arbitrary unions. Let (X, τ, E) be soft compact space. If (A, E) is soft closed set in X, then (A, E) is soft πgb compact.

Proof. Let (A, E) be soft πgb-closed subset of a SπGBO-compact space X. Then $(A, E)^c$ is soft πgb-open in X. Let (M,E) = {$(G_\alpha$,E) : α∈Λ} be a soft cover of (A,E) by soft πgb- open sets in X. (A,E) $\tilde{\subseteq}$∪{ $(G_\alpha$,E): α∈Λ}.Then (M,E)∪$(A,E)^c$ is a soft πgb-open cover of X. By definition, every soft πgb open cover has a finite sub cover. Since X is πGBO–compact, there exists a finite Λ_o of Λ of X. Say X= {$(G_\alpha$,E): α∈Λ_0}∪$(A,E)^c$. But (A, E) and $(A, E)^c$ are disjoint. Hence (A, E) $\tilde{\subseteq}$∪{$(G_\alpha$,E): α∈Λ_0}. This implies soft πgb-open cover (M, E) of (A,E) contains a finite sub cover. Therefore (A, E) is SπGBO-compact relative to X. Therefore every soft πgb-closed subset of a SπGBO-compact space X is soft πgb-compact.

Theorem 4.12: A surjective soft πgb-continuous image of a SπGBO-compact space is soft compact.

Proof. Let f: (X, τ, E) → (Y, τ′, E), be a soft πgb-continuous map from a SπGBO-compact space X into a soft topological space Y. Let {(A_i , E) ; i∈Λ } be a soft open cover of Y. Then {f^{-1}((A_i ,E)): i∈Λ} is a soft πgb open cover of X. Since X is SπGBO-compact, every soft πgb-open cover of X has a finite subcover say $\{f^{-1}((A_1, E)), f^{-1}((A_2, E)), \ldots\ldots f^{-1}((A_n, E)\}$. Since f is onto, $\{(A_1, E), (A_2,E) \ldots\ldots (A_n,E)\}$ is a soft cover of Y, which is finite. Therefore Y is soft compact.

Definition 4.13: Two non-empty subsets (F,E) and (G,E) of a soft topological space (X, τ, E) are called soft πgb-separated if and only if (F,E)∩sπgb-cl((G,E))= ϕ and sπgb-cl((F,E))∩(G,E)= ϕ.

Definition 4.14: A soft topological space (X, τ, E) is said to be soft πGB-connected if X cannot be expressed as a disjoint union of two non empty soft πgb-open sets.
A soft subset of X is soft πGB connected if it is soft πGB-connected as a soft subspace.

Theorem 4.15: A soft topological space (X, τ, E) is soft πGB-connected if and only if X and ϕ are the only soft subsets of X which are both soft πgb-open and soft πgb-closed.
Proof. Let (X, τ, E) be soft πGB-connected Let (G,E) be any soft πgb- open and soft πgb-closed subset in X. Then $(G,E)^c$ is both soft πgb-open and soft πgb-closed. Then X is a disjoint union of soft πgb -open sets (G,E) and $(G,E)^c$. This contradicts the fact that X is SπGB-connected, then either (G,E) = ϕ (or) (G,E) =X.
Conversely, assume X= (A,E)U(B,E) where (A,E) and (B,E) are disjoint non empty soft πgb-open subsets of X then (A,E) is both soft πgb-open and soft πgb-closed .By assumption (A,E)= ϕ or X which is a contradiction. Hence X is soft πGB-connected

Theorem 4.16: If f :(X, τ, E) → (Y, τ′, E) is a soft πgb-irresolute surjection and X is soft πGB-connected, then Y is soft πGB-connected.
Proof. Suppose Y is not soft πGB-connected. Then Y= (A,E)∪ (B,E) where (A,E) and (B,E) are two non empty disjoint soft πgb-open sets in Y. Since f is soft πgb-irresolute and onto,X= $f^{-1}((A,E)) \cup f^{-1}((B,E))$ where $f^{-1}(A,E)$ and $f^{-1}((B,E))$ are disjoint non empty soft πgb-open sets in X. This contradicts the fact that X is soft πGB-connected. Hence Y is soft πGB-connected.

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