On Some Modified Class of Ratio Estimators in Simple Random Sampling

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Abstract: ***This paper suggested an efficient class of modified ratio and product estimators for estimating population mean using auxiliary information. The proposed estimator provides a significant improvement over previous families of estimators in theory and leads to a better prospective of application in various applied areas. The numerical illustration shows that the proposed estimator is more efficient than the existing ones.***

Keywords: mean square error, product estimator, ratio estimator, percent relative efficiency.

1. Introduction

The use of auxiliary information in survey sampling has its own eminent role. The supplementary (auxiliary) information in survey sampling is used to enhance the efficiency of population parameters. Let $(x_i, y_i), i = 1,2,......, n$ be n pair of observations of auxiliary and study variables, respectively for the population of size N using SRSWOR(Simple Random Sampling Without Replacement).Let $\bar{X}$ and $\bar{Y}$ be the population means of auxiliary and study variables, respectively and $\bar{x}$ and $\bar{y}$ be the respective sample means. Ratio estimators are used when the line of regression of y on x passes through origin and the variables X and Y are positively correlated to each other, while product estimators are used when X and Y are negatively correlated to each other.

Khoshnevisan et al. (2007) defined the general family of estimators for estimating the population mean as

$$t = \bar{y}\left[\frac{a\bar{X} + b}{\alpha(a\bar{x} + b) + (1-\alpha)(a\bar{X} + b)}\right]^g$$

Where a and b are population parameters of the auxiliary variable such as coefficient of variation (C_x), correlation coefficient (ρ), standard deviation (σ_x), coefficient of skewness $\beta_1(x)$, coefficient of kurtosis $\beta_2(x)$ etc. g and α are suitably chosen constants which make the MSE (mean square error) of t minimum. Koyuncu and Kadilar (2009), defined a new family of estimators for the population mean as

$$\eta = k\bar{y}\left[\frac{a\bar{X} + b}{\alpha(a\bar{x} + b) + (1-\alpha)(a\bar{X} + b)}\right]^g$$

Where k is suitable chosen constant for which the mean square error (MSE) is minimum. Prasad (1989) proposed the ratio estimator as

$$\eta_1 = k\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$$

The expression for mean square error for η_1 is given as

$$MSE(\eta_1) = \bar{Y}^2\left\{k_1^2\lambda C_y^2 + \left(3k_1^2 - 2k_1\right)\lambda C_x^2 - 2\left(2k_1^2 - k_1\right)\lambda C_{yx} + (k_1 - 1)^2\right\}$$

The $MSE(\eta_1)$ is minimized for optimum values of k given as

$$k_1^* = \frac{1 + \lambda C_x^2 - \lambda C_{yx}}{1 + 3\lambda C_x^2 - 4\lambda C_{yx} + \lambda C_y^2} = \frac{A^*}{B^*}$$

Replacing k_1 with the optimum value of k_1^* we get the minimum value of MSE as

$$MSE_{\min}(\eta_1) = \left\{\bar{Y}^2\left(1 - \frac{A^*}{B^*}\right)\right\}$$

2. Proposed Estimator

Motivated by Khoshnevisan et. al (2007), Koyuncu and kadilar (2009) and Prasad(1989), we propose modified ratio estimator for estimating population mean of the study variable as

$$\eta_{r1}^* = k\bar{y}\left(\frac{\bar{X} + Md}{\bar{x}^*}\right)$$

Where $\bar{x}^*$ is the sample mean of the auxiliary variable having the relationship given as

$$\bar{X} = f\bar{x} + (1 - f)\bar{x}^*$$

To obtain the bias and MSE, let us define $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and

$e_1 = \frac{\bar{x} - (\bar{X} + Md)}{\bar{X} + Md}$. Using these notations,

$$E(e_0) = E(e_1) = 0,\ E(e_0^2) = \lambda C_y^2,\ E(e_1^2) = \lambda C_x^2,\ E(e_0 e_1) = \lambda C_{yx} = \lambda\rho C_y C_x$$

The MSE of η_{r1} is given as,

$$MSE(\eta_{r1}) = \bar{Y}^2\left\{h^2\left[k_1^2\lambda C_y^2 + \left(3k_1^2 - 2k_1\right)\lambda C_x^2 - 2\left(2k_1^2 - k_1\right)\lambda C_{yx}\right] + \left(k_1 - 1\right)^2\right\}$$

This is minimized f $MSE(\eta_{r1})$ or optimum values of k given as

$$k_{r1}^* = \frac{h^2\left(\lambda C_x^2 - \lambda C_{yx}\right)+1}{h^2\left(3\lambda C_x^2 - 4\lambda C_{yx} + \lambda C_y^2\right)+1} = \frac{A_{r1}^*}{B_{r1}^*}$$

Replacing k_{r1} with the optimum value of k_{r1}^* we get the minimum value of MSE as

$$MSE_{\min}(\eta_{r1}) = \left\{\bar{Y}^2\left(1 - \frac{A_{r1}^*}{B_{r1}^*}\right)\right\}$$

Where $\lambda = \frac{N-n}{Nn}, C_y = \frac{S_y^2}{\bar{Y}^2}, C_x = \frac{S_x^2}{\left(\bar{X}+Md\right)},$

$$C_{yx} = \frac{S_{yx}}{\bar{Y}\left(\bar{X}+Md\right)}, S_y^2 = \frac{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)}{N-1},$$

$$S_x^2 = \frac{\sum_{i=1}^{N}\left(x_i - \bar{X}\right)}{N-1},$$

$$S_{yx} = \frac{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)}{N-1}, \rho = \frac{S_{yx}}{S_x S_y}$$

and $h = \frac{n}{N-n}$

The mean square error of the estimator η_{r1} is given by

$$MSE(\eta_{r1}) = \left(\frac{n}{N-n}\right)^2 MSE(\eta_1)$$

3. Efficiency Comparison

The proposed estimator η_{r1} is more efficient than η_1 if,

$$MSE_{\min}(\eta_{r1}) < MSE(\eta_1)$$

4. Empirical Study

For numerical illustration we have taken the data of Singh and Chaudhary (1986) where,

$$N = 22, n = 5, \bar{Y} = 22.5209, \bar{X} = 1457.54,$$
$$\rho = 0.9022, S_y = 33.0459, S_x = 2552.14,$$
$$C_y = 1.4509, C_x = 1.7459$$

After simplifications, Mean square error of the proposed estimator comes as **3.19**, where as for the exixting estimator MSE=**4.45**.

Percent relative efficiency of the proposed estimator= **139.18.**

From the above results, we conclude that the proposed estimator suggested by the authors performed well in all situations and when information regarding population mean is available , the estimator is recommended for use in practice.

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