

# Expression of Mass in Einstein Generalized Special Relativity

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**Abstract:** In this work the expression of time in Einstein generalized special relativity together with the principle of conservation of momentum are utilized to find a useful expression for mass and energy of a moving particle in a field generating a constant acceleration. The energy and mass expression shows that they both depend on potential as well as velocity. The two expressions are in conformity with Savickas and Einstein generalized special relativity modes.

**Keywords:** Mass energy, Potential, Generalized, Velocity, EGSR modes, General Relativity, Special Relativity

## 1. Introduction

Einstein special theory of relativity (SR) is one of the big achievement in physics. It changes the notion of space and time radically [1-5]. Before SR scientists treat space and time as absolute concepts. But SR shows that space and time scales changes for observers moving with respect to each others with uniform velocity. Special relativity theory succeeded in explaining the Michelson–Morely experimental result which shows that the speed of light in vacuum is constant and independent of the motion of the observer or the source. It also explains the time dilation for fast moving decaying meson. It also explains successfully pair production and annihilation phenomena. Despite these successes SR suffers from noticeable setbacks. First of all its expression for energy does not reduce to its corresponding Newtonian expression at low speeds. This is since it does not recognize the potential term. In other words the Einstein energy expression at low speeds gives only two terms. One stands for the kinetic energy and the other represents the mass energy. There is no term standing for the potential energy. Moreover SR cannot explain the red shift phenomena in which the photon frequency mass and periodic time gravitational field. This bizarre situation nessesitates searching for a more generalized version of SR to cure these defects. Thus one need to show that Einstein SR is still capable to explain a wide variety of physical phenomena if it is generalized.

Such generalization was made in the Einstein generalized special relativity (EGSR) model [6]. In this model general relativity (GR) was used to find a useful expressions for time, length mass and energy to cure the a fore noted defects [7]. This present work is devoted to derive the expression of mass and energy by using momentum conservation together with the time dilation relation of EGSR.

## 2. Einstein Generalized Special Relativity

The expression of the energy momentum tensor of the gravitation field, together with the space time interval in a curved space, were used by some authors to derive expressions for time, length mass and energy in the presence of weak fields in the form [7-10];

$$t = \frac{t_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} = \gamma t_0$$

$$L = L_0 \sqrt{g_{00} - \frac{v^2}{c^2}} = \gamma^{-1} L_0$$

$$m = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} = g_{00} \gamma m_0 \quad (1)$$

$$E = m c^2 \quad (2)$$

Where

$$g_{00} = 1 + \frac{2\phi^2}{c^2}, \gamma = \frac{1}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \quad (3)$$

With  $t_0, L_0, m_0$ , standing for the mass at rest in free space, while  $t, l, m$ , represents the corresponding values in the presence of field having a potential  $\phi$  per unit for particles moving with speed  $v$ . The speed of light in vacuum is designated here by  $c$ .

## 3. The Relativistic Expression of Mass in the presence of fields by using momentum conservation

To find an expression of the mass  $m$  of particle moving with speed  $v$  in potential per unit mass  $\phi$  consider two particles colliding elastically. Let frame  $S'$  move with constant acceleration  $a$ . In a field with respect  $S$  which is in free space.

Before collision let particle 1 having a mass  $m_1$  is at rest in  $S$ , while particle 2 with mass  $m_2$  in frame  $S'$ . At the same

time  $m_1$  was thrown in the  $+y$  direction at speed  $v_1$  while  $m_2$  was thrown in the  $-y$  direction at speed  $v_2$  such that;

$$v_1 = v_2 \quad (4)$$

After collision,  $m_1$  rebounds in the  $-y$  direction at the speed  $v_1$  while:  $m_2$  rebounds in the  $+y'$  direction at the speed  $v_2$ .

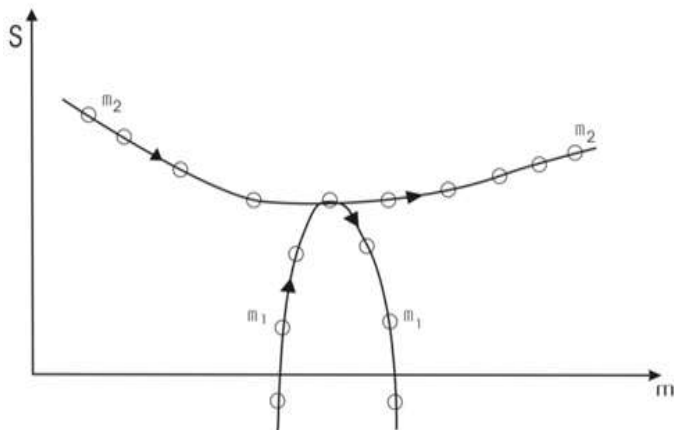


Figure 1: The two masses as observed in S

If the particles are thrown from positions  $L$  apart, the observer find collision occurs at;

$$y = \frac{1}{2} L \quad (5)$$

and that in  $S'$  find it occurs at;

$$y' = \frac{1}{2} L \quad (6)$$

The round trip for  $m_1$  as measured in  $SI$  is;

$$t_0 = \frac{L}{v_1} \quad (7)$$

Which is the same for  $m_2$  in  $S'$ ;

$$t_0 = \frac{L}{v_2} \quad (8)$$

The momentum conservation in  $S$  requires;

$$m_1 v_1 = m_2 v_2 \quad (9)$$

Where  $m_1, m_2, v_1, v_2$  are the masses and velocities as measured in  $S$ . In the  $S$

frame the speed  $v_2$  is given by;

$$v_2 = \frac{L}{t} \quad (10)$$

but

$$t = \gamma t_0 \quad (11)$$

thus

$$v_2 = \frac{L}{\gamma t_0} \quad (12)$$

Substitute Eqs.(7) , (10) in Eq.(9) yields;

$$m_1 \left( \frac{L}{t_0} \right) = m_2 \left( \frac{L}{\gamma t_0} \right)$$

$$m_1 = \frac{m_2}{\gamma}$$

Since for  $S$   $m_1$  is at rest while  $m_2$  moves , thus;

$$m_1 = m_0, m_2 = m$$

hence

$$m = \gamma m_0 = \frac{m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \quad (13)$$

$$m = \frac{m_0}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \quad (14)$$

This expression indicates that the mass is affected by field potential as well as on velocity  $v$ . To find energy expression consider a particle moving with initial velocity  $v$  with constant acceleration  $a$ . The velocity  $v_f$  at any time  $t$  is given by;

$$v_f^2 = v^2 - 2ax = v^2 - 2\phi \quad (15)$$

Where the potential  $V$  is given by;

$$V = m\phi = ma \quad (16)$$

Thus Eq.(14) can be rewritten as;

$$m = \frac{m_0}{\sqrt{1 - \frac{v_f^2}{c^2}}} \quad (17)$$

Thus the kinetic energy  $T$  is given by;

$$\begin{aligned} T &= \int F dx = \int \frac{d(mv_f)}{dt} dx = \int \frac{dx}{dt} d(mv_f) \\ &= \int v_f d(mv_f) = [mv_f^2] - \int m v_f dv_f \\ T &= mc^2 - m_0 c^2 \end{aligned} \quad (18)$$

Thus the energy is given by;

$$\begin{aligned} E &= mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v_f^2}{c^2}}} \\ &= \frac{m_0 c^2}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \end{aligned} \quad (19)$$

For low velocity;

$$E = m_0 c^2 \left( 1 - \frac{v_f^2}{c^2} \right)^{-1/2} = m_0 c^2 \left( 1 + \frac{1}{2} \frac{v_f^2}{c^2} \right)$$

$$E = m_0 c^2 + \frac{m_0 v_f^2}{2} = m_0 c^2 + \frac{m_0}{2} (v^2 - 2\phi)$$

$$= m_0 c^2 + \frac{1}{2} m_0 v^2 - m_0 \phi$$

$$E = m_0 c^2 + T + V \quad (20)$$

Thus it reduces to the ordinary Newtonian energy expression, as shown also by same authors [11].

#### 4. Discussion

The expression for the energy in Eq.(19) is obtained by using momentum conservation together with time dilation relation of the EGSR theory. This expression shows that the mass and energy are affected by the kinetic energy per unit mass  $\frac{1}{2} v^2$  as well as the potential  $\phi$  per unit mass. This expression is inconformity with common sence and with EGSR [12] and Savickas model [11].

The expression of energy satisfies correspondence principle as it reduces to Newtonian energy relation as for as it consists of potential term beside kinetic term. A cording to Eq.(20) for low speed of the observer  $v$  and as far as the rest mass of photon is extremely small, thus;

$$T = \frac{1}{2} m_0 v^2 \rightarrow 0$$

but since photon energy in vacuum and inside a field are given by;

$$hf_0 = m_0 c^2, hf = m c^2$$

thus equation (20) given by;

$$hf = hf_0 + V$$

thus the energy expression as shown also by the same authors.

#### 5. Conclusions

The fact that the mass and energy expression resembles that of savickas and the EGSR indicates that this expression rests on a solid ground. Such expressions can cure some of SR setbacks like Newtonian limit and red shift phenomena reference

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## References

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