Weakly b-Continuous Functions

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Abstract: In 1961, Levine [7] introduced weakly continuous functions and in 1987, Noiri [12] introduced and studied weakly \( \theta \)-continuous functions. Later on Ekici [5], in 2008, introduced and studied BR-continuous and hence weakly BR-continuous functions in a similar fashion, by means of b-regular and b-open [4] sets. This prompted us to introduce and study weakly b-continuous by making use of b-open sets. We studied several characterizations of weakly b-continuous functions. Some basic properties including restrictions and compositions of such functions have also been studied.

Keywords: Weakly continuous functions, weakly \( \theta \)-continuous functions, Weakly BR-continuous functions, Weakly b-continuous and b-open sets.

1. Introduction

Levine [7] introduced the concept of a weakly continuous function. In 2008 Ekici [5] has introduced and studied the class of functions namely BR-continuous functions and weakly BR-continuous functions by making use of b-regular sets. He obtained some characterizations of weakly BR-continuous functions and established relationships among such functions and several other existing functions. In a similar manner here our purpose is to introduce and study generalizations in form of new classes of functions namely weakly b-continuous. The author [6] has already introduced and studied b-continuous functions.

Let \((X, \tau)\) and \((Y, \sigma)\) (or \(X\) and \(Y\)) denote topological spaces. For a subset \(A\) of \(X\), the closure \(A\) and the interior of \(A\) are denoted by \(cl(A)\) and \(int(A)\) respectively. A subset \(A\) is said to be regular open (resp. regular closed) if \(A=\text{int}(cl(A))\) (resp. \(A=\text{cl}(\text{int}(A))\)). A subset \(A\) is said to be preopen [9] (resp. semi open [8], b-open [4], \(\alpha\)-open [11], semi preopen [3] or \(\beta\)-open [1]) if \(G\subset cl(int(cl(A)))\). A subset \(G\) of \(X\) is called \(b\)-neighbourhood of \(x\) if there exists a \(b\)-open set \(B\) containing \(x\). The set \(cl(U)\cap int(cl(V))\) for each \(b\)-open set \(U\) containing \(x\) such that \(B\subset cl(V)\).

A point \(x\in X\) is said to be a \(\theta\)-cluster point of \(A\) [14] if \(A\cap cl(U)\neq\emptyset\) for every open set \(U\) containing \(x\). The set of all \(\theta\)-cluster points of \(A\) is called \(\theta\)-closure of \(A\) and is denoted by \(cl(A)\). A subset \(A\) is said to be \(\theta\)-closed if \(A=cl(A)\). The complement of a \(\theta\)-closed set is \(\\emptyset\).

2. Definitions and Characterizations

Definition 2.1:- A function \(f: X \rightarrow Y\) is said to be weakly continuous [6] (resp. strongly \(\theta\)-continuous [14]) if for each \(x\in X\) and each open set \(V\) of \(Y\) containing \(f(x)\), there exists a \(b\)-open set \(U\) containing \(x\) such that \(f(U)\subset V\).

Definition 2.2:- A function \(f: X \rightarrow Y\) is said to be weakly continuous [7] (resp. weakly \(\alpha\)-continuous [12]) if for each \(x\in X\) and each open set \(V\) of \(Y\) containing \(f(x)\), there exists an open (resp. \(\alpha\)-open) set \(U\) containing \(x\) such that \(f(U)\subset V\).

Definition 2.3:- A function \(f: X \rightarrow Y\) is said to be weakly b-continuous if for each \(x\in X\) and each open set \(V\) of \(Y\) containing \(f(x)\), there exists a \(b\)-open set \(U\) containing \(x\) such that \(f(U)\subset V\).

Theorem 2.4:- For a function \(f: X \rightarrow Y\), the following are equivalent:

(a) \(f\) is weakly b-continuous at \(x\in X\).
(b) for each neighbourhood \(V\) of \(f(x)\), there exists a \(b\)-open set \(U\) containing \(x\) (or \(b\)-neighbourhood \(U\) of \(x)\) such that \(f(U)\subset V\).
(c) \(b\)-cl(\(f^{-1}(int(cl(V)))\))\(\subset f^{-1}(cl(V))\) for every subset \(V\) of \(Y\).
(d) \(b\)-cl(\(f^{-1}(int(F)))\)\(\subset f^{-1}(F)\) for every regular closed subset \(F\) of \(Y\).
(e) \(b\)-cl(\(f^{-1}(V))\)\(\subset f^{-1}(cl(V))\) for every open set \(V\) of \(Y\).
(f) \(f^{-1}(V)\subset b\)-int(\(f^{-1}(cl(V)))\) for every open set \(V\) of \(Y\).
(g) \(b\)-cl(\(f^{-1}(V))\)\(\subset f^{-1}(cl(V))\) for each preopen set of \(Y\).
(h) \(f^{-1}(V)\subset b\)-int(\(f^{-1}(cl(V)))\) for each preopen set of \(V\) of \(Y\).

Proof:- (a)\(\Rightarrow\)(b) obvious by definition.
(a)\(\Rightarrow\)(c) Let \(V\subset Y\) and \(x\in X\). Then \(f(x)\in Y\) and there exists an open \(U\) containing \(x\), such that \(f(U)\subset V\). We have \(cl(U)\cap int(cl(V))\neq\emptyset\). Since \(f\) is weakly b-
There exists an open set G containing f(x), such that f(W)∩V≠∅. We have, cl(f(G))⊂cl(V) and hence f is weakly b-continuous at x∈X.

Theorem 2.7:- For a function f : X → Y, the following are equivalent :
(a) f is weakly b-continuous.
(b) f(b-cl(G))⊂θ-cl(f(G)) for each subset G of X.
(c) b-cl(f−1(H))⊂θ-cl(H) for each subset H of Y.

Proof:- (a)⇒(b) Let G⊂X, x∈b-cl(G) and U be any open set in Y containing f(x). There exists a b-open W containing x such that f(W)⊂cl(U). Since, x∈b-cl(G), then W⊂f(G). This implies that f(W)⊂f(G)⊂cl(U)∩H(f(G)) and f(x)∈θ-cl(H). Thus, f(b-cl(G))⊂θ-cl(H).

Theorem 2.8:- If f−1(θ-cl(V)) is b-closed in X for every subset V of Y, then f is weakly b-continuous.

Proof:- Let V⊂Y. Since f−1(θ-cl(V)) is b-closed in X, then f−1(b-cl(V))⊂b-cl(f−1(V))⊂θ-cl(V). This follows directly from Theorem 2.7. Since f is weakly b-continuous, f−1(V) is b-closed in X for every θ-closed subset V of Y.

Corollary 2.10:- Let f : X → Y be a weakly b-continuous function, then f−1(V) is b-open in X for every θ-closed subset V of Y.

Theorem 2.11:- Let f : X → Y be a function. If Y is regular then following are equivalent :
(a) f is weakly b-continuous.
(b) f is b-continuous.
(c) f is strongly θ-b-continuous if and only if f is continuous [13].

Proof:- Let x∈X and V be an open subset of Y containing f(x). Since Y is regular, there exists an open set H of Y containing f(x) such that f(H)⊂V. Since f is weakly b-continuous, there exists a b-open set U of X containing x such that f(U)⊂θ-cl(H)⊂V. Thus f is b-continuous. Converse is obvious.

Lemma 2.12 [4]:- The intersection of an α-open set and a b-open set is a b-open set.
(b) the restriction f/A : A i → Y is weakly b-continuous for each i ∈ I.

**Proof:-**

(a)⇒(b) Let i ∈ I and A i be an α-open set in X. Let x ∈ A i and V be an open set in Y containing f/A i (x) = f(x).

Since f is weakly b-continuous, so, there exists a b-open set G containing x such that f(G) ⊂ cl(V). Moreover G ∩ A i is b-open in A i containing x and f/A i (G ∩ A i) = f(G) ∩ f(A i) ⊂ cl(V). Hence f/A i is weakly b-continuous.

(b)⇒(a) Let x ∈ X and V be an open set in Y containing f(x). There exists i ∈ I, such that x ∈ A i. Since f/A i : A i → Y is weakly b-continuous, there exists a b-open set G in A i containing x such that f(A i(G) ⊂ cl(V). Since each A i is α-open in X then G is b-open in X containing x and f(G) ⊂ cl(V). Hence f is weakly b-continuous.

**References**


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