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Soft Computing Approach to Solve Effect of Variable Viscosity and Thermal Diffusivity on Unsteady Porous Stretching Sheet

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Abstract: The present communication explores the unsteady boundary layer flow on a porous stretching sheet. The thermal diffusivity and viscosity are assumed to vary as linear function of temperature. Using the stream function, the governing partial differential equations are converted into ordinary differential equations. The obtained boundary value problem is converted into equivalent initial value problem using continuous genetic algorithm and solved by Runge-Kutta Fehlberg method. To carry out analysis the results are depicted in the form of figures.

Keywords Unsteady flow, Variable Thermal diffusivity, Variable viscosity, Permeable stretching sheet

1. Introduction

The study of an incompressible viscous fluid flow over a stretching surface through porous media is a classical problem in fluid dynamics. It has received enormous research interest over the last few decades due to its extensive and important applications in various geophysical and industrial processes. Some of its remarkable applications include creation of polymers of fixed cross-sectional profiles, aerodynamics shaping of plastic sheet by forcing through die and cooling of metallic and glass plates. Other applications can be in movement of biological fluids and in food processing industry. Due to its applications in lots of areas it has attracted many researchers towards it in recent years.

The steady, viscous and incompressible two-dimensional flow of a Newtonian fluid was first of all studied by Crane [1] where he has applied uniform stress on an elastic flat sheet with velocity varying linearly with the distance from a fixed point in its own plane. The commendable work of Crane was later extended by various researchers to explore various aspects of the flow and heat transfer occurring in an infinite domain of the fluid surrounding the stretching sheet. The work carried out on fluid includes both at rest and moving with some velocity. Mahapatra and Gupta [2] reported that a boundary layer is formed when the free stream velocity exceeds stretching velocity while studying effect of free stream velocity on stagnation-point flow towards a stretching surface. The orthogonal and oblique flow along with porosity and radiation effect on a stretching sheet was studied by Singh et al. [3, 4] respectively.

There are many situations when due to sudden stretching of a sheet the flow and heat transfer become unsteady. Pop and Na [5] investigated the unsteady flow past a wall and found that in due course of time the unsteady flow would approach the steady flow. Elbashbeshy and Bazid [6] reported similarity solution for the heat transfer of an unsteady boundary layer flow over stretching sheet and concluded that unsteadiness parameter is inversely proportional to thermal boundary layer thickness and momentum boundary layer thickness. Ishak et al. [7] investigated boundary layer flow over a continuous stretching permeable surface and reported that unsteadiness parameter is directly proportional to the heat transfer rate at the surface.

Gary et. al [8] and Mehta AND sood [9] explained that with variation in temperature, the physical properties of fluid changes. The decrease in temperature will make a local decrease in the transport phenomena by increasing the viscosity across the momentum boundary layer and as a result the rate of heat transfer at the wall is also affected. So the viscosity variation for incompressible fluids must be necessarily taken into consideration. The variable viscosity along with the application of MHD on boundary layer was explained by Mukhopadhyay et al [10].

The present work deals with unsteady fluid flow and heat transfer over a stretching sheet in presence of wall suction. Fluid viscosity and thermal diffusivity are taken as a linear function of temperature. Using the Similarity variable and similarity solutions ordinary differential equations corresponding to momentum and energy equations are derived. These equations are further solved numerically using genetic algorithm. The effects of various parameters (viz. variable thermal diffusivity, temperature dependent fluid viscosity, unsteadiness and suction) on velocity and temperature fields are explored and analysed with the help of graphs.

2. Formulation of Problem

The mathematical model considered here consists of a viscous, incompressible, unsteady flow of a fluid flowing past a heated stretching sheet. Fluid is considered in the presence of thermal radiation effect. The fluid occupies the upper half plane i.e. $y > 0$. The sheet has uniform temperature T_∞ and moving with non-uniform velocity $U\ (x,t) = \dfrac{cx}{1-\alpha t}$, where c and α are positive constants with dimension (time)$^{-1}$, c is the initial stretching rate and

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$\frac{c}{1-\alpha t}$ is the effective stretching rate which is increasing with time. The temperature of the sheet is different from that of the ambient medium. The fluid viscosity is assumed to vary with temperature while the other fluid properties are assumed constants.

The governing equations of continuity, momentum and energy under above assumptions are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

and

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}(\kappa\frac{\partial T}{\partial y}) \tag{3}$$

where u and v are velocity components along x and y axis respectively, T is the temperature, κ is the coefficient of thermal diffusivity (dependent on temperature), c_p is the specific heat, ρ is the fluid density (assumed constant), μ is the coefficient of fluid viscosity (dependent on temperature), k is the permeability of the porous medium.

Boundary conditions for the given model are:

$$u=U(x,t),\ \ v=v_w(t) \text{ and } T=T_w(x,t) \quad at\ y=0 \tag{4}$$

$$u\to 0 \text{ and } T\to T_\infty \quad as\ y\to\infty \tag{5}$$

where $v_w(t)=-v_o\frac{1}{\sqrt{1-\alpha t}}$ is the velocity of suction $(v_o>0)$ at the wall, of the fluid, $T_w(x,t)=T_\infty+\frac{T_o \operatorname{Re}_x(1-\alpha t)^{1/2}}{2}$ is the wall temperature, where $\operatorname{Re}_x=\frac{ux}{\nu}$ is the local Reynolds number based on the stretching velocity U, T_o is a reference temperature such that $0\le T_o\le T_w$ and ν is the kinematic viscosity of the ambient fluid. The expression for $v_w(t)$, $T_w(x,t), U(x,t)$ and $v_o(t)$ are valid only for time $t<\alpha^{-1}$ unless α become zero. Introducing the stream function $\psi(x,y)$ as defined by $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$, the dimensionless temperature $\theta=\frac{T-T_\infty}{T_w-T_\infty}$ and the similarity variable $\eta=\sqrt{\frac{c}{(1-\alpha t)\nu}}\,y$, $\psi=\sqrt{\frac{\nu c}{(1-\alpha t)}}\,xf(\eta)$ and

$$T=T_\infty+T_o\left[\frac{c\,x^2}{2\,\nu}\right](1-\alpha t)^{-3/2}\theta(\eta).$$

The temperature dependant fluid viscosity is given by-

$$\mu=\mu^*[a+b(T_w-T)] \tag{6}$$

where μ^* is the constant value of the coefficient of viscosity far away from the sheet and a, b are constants with $b>0$. We have used viscosity temperature relation $\mu=a-bT$ which is in perfect harmony with the relation $\mu=e^{-aT}$ when second and higher order terms neglected in the expansions. The variation of thermal diffusivity with the dimensionless temperature is written as

$$\kappa=\kappa^*[1+\beta\theta] \tag{7}$$

where β is a thermal diffusivity parameter which depends on the nature of the fluid, κ^* is the value of thermal diffusivity at the temperature T_w.

With the help of above relations the governing equations (2) and (3) finally reduces to

$$M(\frac{\eta}{2}f''+f')+f'^2-ff''=-A\theta' f''+(a+A)f'''-A\theta f''' \tag{8}$$

$$\frac{M}{2}\eta\theta'+\frac{3}{2}M\theta+2f'\theta-f\theta'=\frac{1}{P_r}(\beta\theta'^2+\theta''+\beta\theta\theta'') \tag{9}$$

where $M=(\frac{\alpha}{c})$ is the unsteadiness parameter, $A=b(T_w-T_\infty)$] is the temperature dependent viscosity parameter.

The corresponding boundary conditions then reduces to:

$$f(0)=s,\quad f'(0)=1 \text{ and } \theta(0)=1\ \ at\ \eta=0$$
$$f'(\infty)\to 0, \text{ and } \theta(\infty)\to 0 \quad as\ \eta\to\infty \tag{10}$$

where is P_r Prandtl number = $(\mu^*\frac{c_p}{\kappa})$ and S corresponds to the suction.

3. Results and Discussion

The governing boundary layer equations (8) and (9) subjected to the boundary conditions (10) were solved numerically by Runge-Kutta Fehlberg with the help of genetic algorithm. Different values of thermal diffusivity parameter β, unsteadiness parameter M, viscosity variation parameter A, were taken for numerical simulation. Numerical results were depicted graphically.

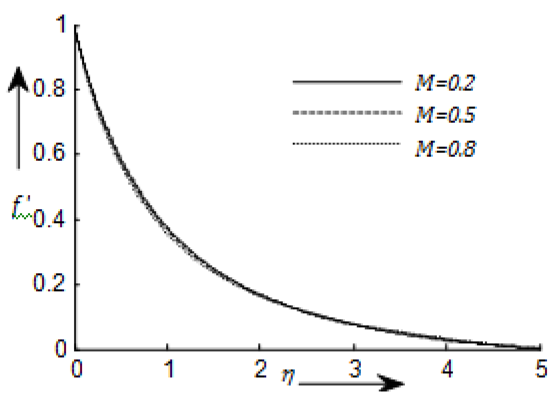

Figure 1: Variation of unsteadiness parameter with η on fluid velocity.

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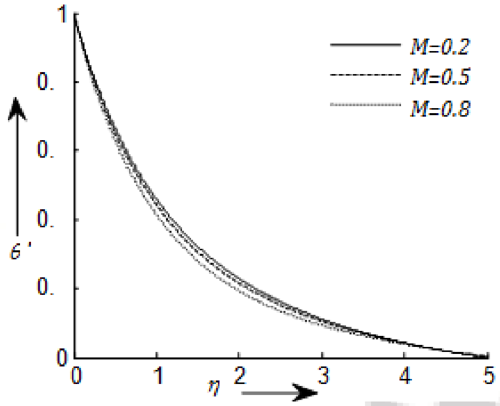

Figure 2: Variation of unsteadiness parameter with η on dimensionless temperature.

Figure 1 represents variation of velocity profile of the fluid with unsteadiness parameter. It is observed that with the increase in unsteadiness parameter M, the fluid velocity first decreases and then after certain value of η it starts increasing. We notice that a crossing over point appears in the figure. This is a special point, where all the velocity curves cross each other i.e., velocity profile exhibit different behavior before and after this point. This is contrary to the result from the paper El-Aziz (2009), where the flow is without any such point for all values of M considered. Temperature is found to decrease with increasing unsteadiness parameter as shown in Figure 2. We also notice that impact of unsteadiness parameter on temperature profile is more pronounced than on the velocity profile.

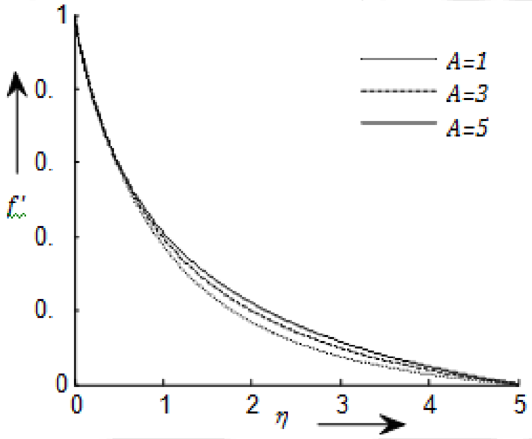

Figure 3: Variation of viscosity variation parameter with η on fluid velocity.

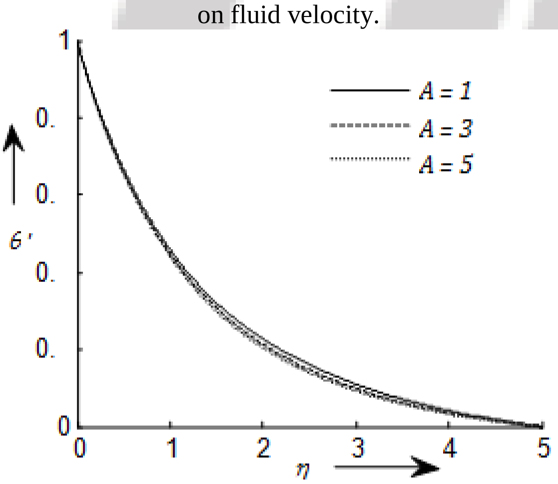

Figure 4: Variation of viscosity variation parameter with η on dimesionless temperature.

Figure 3 and 4 shows variation of viscosity variation parameter. They show that as the fluid viscosity variation parameter increases, the fluid velocity goes on increasing. This happens at all the places except near the wall. As the value of A increases the temperature decreases so as a result the thermal boundary layer decreases as well as the boundary layer thickness increases.

From the above discussion we see that both unsteadiness parameter and viscosity variation parameter affects the velocity and temperature profiles. The fluid velocity is inversely proportional to unsteadiness parameter and directly proportional to viscosity variation parameter. These two opposing effects will shows that as M increases the fluid velocity decreases and as A increases the fluid velocity decreases. From the graphs of velocity and temperature profile one can conclude that up to the crossing over point the unsteadiness parameter dominates and after crossing over point viscosity variation parameter dominates.

From figure 5 and 6 show one can see that as the value of Prandtl number increases the fluid velocity goes on increasing and the temperature decreases.

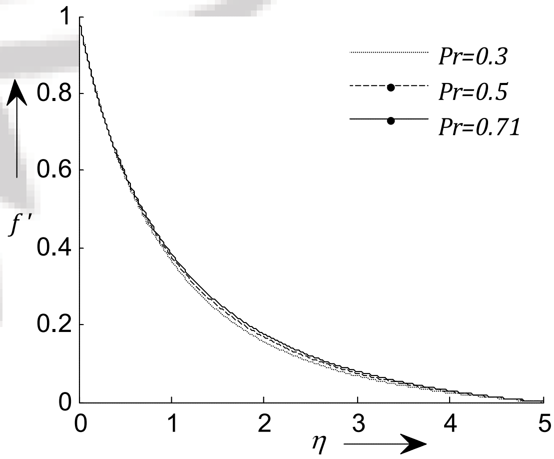

Figure 5: Variation of Prandtl number parameter with η on fluid velocity.

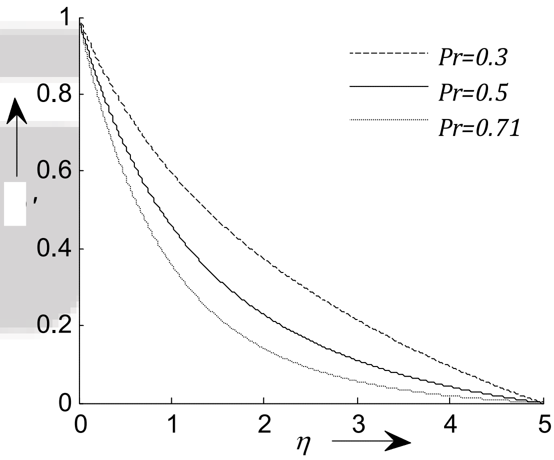

Figure 6: Variation of Prandtl number parameter with η on dimesionless temperature.

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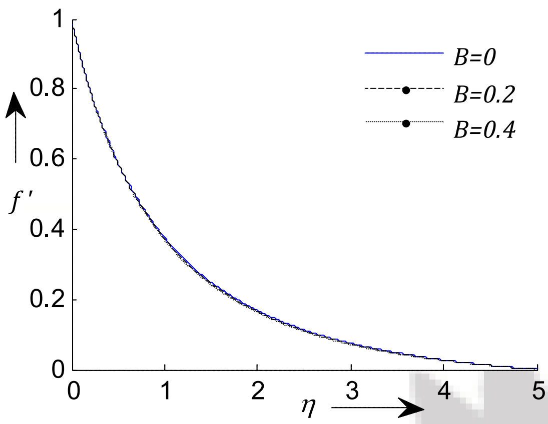

Figure 7: Variation of thermal diffusivity parameter with η on fluid velocity.

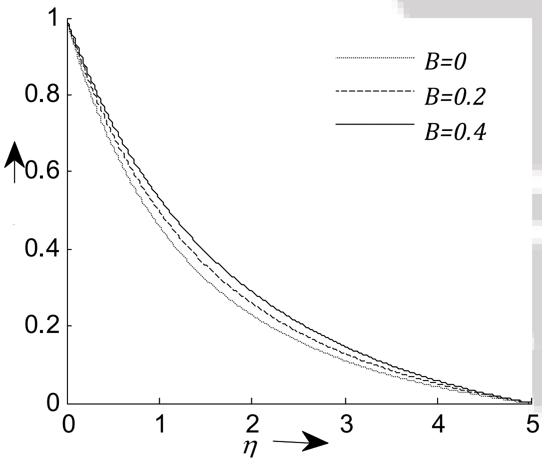

Figure 8: Variation of thermal diffusivity parameter with η on dimesionless temperature.

Figure 7 and 8 shows the velocity and temperature profiles with the variation of thermal diffusivity parameter. Figure 1 shows that as the value of thermal diffusivity parameter increases, the fluid velocity goes on increasing and also an increase is shown in the temperature of temperature profile as shown in figure 2. The reason behind that is that due to increase in thermal diffusivity parameter, the thermal boundary gets thickened.

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