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Against Fantasy: In n no more than n

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Abstract: ***Departing from how natural entities gain dimension including in some extent mathematical entities, it is shown that in an entity of n dimensions there can be entities of n or less dimensions but not of a greater dimension. This is made first non-mathematically and then mathematically. Clarifying that the universe is four dimensional, being space three dimensional it is concluded that in the universe or our corner of the universe there are entities of no more than four or three dimensions, including their movements. Then a hypothesis and a conjecture are stated, showing first that in nature there are not negative dimensions and second that there are no more than three dimensions. Some final relevant comments are posed as a coda.***

Keywords: Dimension, Extension, Negative Dimension, Three Dimension

1. Introduction

Practically there has not been a preoccupation for the dimensionality of nature until the appearance of general relativity where a four dimension was proposed and from this the work of Klein (Griffiths, 1987) where a fifth dimension was stated, but it is not until the rise of string and superstring theories that the possibility of greater dimensions in nature was proposed (Green, Schwarzd, and Witten, 1998; Polchinski, 1999), this was also done in quantum gravity (Smolin, 2001) leading to the creation of a world of fantasy. Here a restating of the true dimensionality of nature is intended. This is accomplished first by a non-mathematical demonstration based on the way how dimensions are obtained in nature followed by a mathematical one reinforcing the proposition. In addition, as an interestingly related topic it will be shown that some mathematical objects also fulfill the same criterion in order to gain dimension. As complementary to the above it will also be stated as a hypothesis a demonstration proving that in nature there is not any negative dimension and also a conjecture setting the dimensionality of nature as three dimensions, which is an interesting consequence of the analysis done. Finally, some relevant comments are posed as a coda.

2. How Entities Gain Dimension in Nature

In nature an entity gains dimension by getting extension in a given direction following an axe that makes some determined angle with the axe or axes of the previous dimension. Thus from a point (which has dimension zero, being this the basic dimension in nature) we have a line (dimension one) which develops by the addition of points extending in an axe which makes a zero angle respect to the point, the next dimension (dimension two) is gained by the addition of lines following an axe that makes a 90^0 angle with respect to the line to which they are added and this continues under the same process. Actually the angle could be any, here it has been used special angles to make clear the process. Interestenglyeven some mathematical entities gain dimension by gaining extension which is seen in the ideas stated by Hurewicz and Wallman (1948) where it is appreciated that the dimensionality (which is expressed as n) gained by aspace increases if it gains in extension by way of the neighborhoods of dimension n-1. The same is seen in Nagata (1983) with the ideas of an open cover U of a topological space R, the one of the strong inductive dimension, and the definition of C. W. Dowkes on local dimension. These ideas are better noted in algebraic topology (Munkres, 1984) where the n-simplices clearly get dimension extending among their vertices. Finally, this also appears in fractals (Edgar, 1991; Feder, 1989; Mandelbrot, 1982), where having fractional dimension is due to the fact of gaining extension towards a greater dimension without getting it.

3. How Many Dimensions Can Have Entities inside Another of a Determined Dimension

3.1 According to the process above mentioned by which an entity gains dimension we have, as a consequence, that in an entity of dimension n only can be entities of dimension n or less, but not of dimension greater than n because to get a dimension greater then n, say n+1, the respective entity must expand following an axe with a determined angle to the axe or axes that guide the expansion of the dimension n entity, and to do so it must extend out of the dimension n entity. For instance, see what happen in a dimension 2 entity. We have that in an entity of dimension 2 only can be entities of dimension 2 or less like lines and points, but not of dimension greater than two, like cubes, because to get a dimension greater than two, say three, the respective entity must expand following an axe orthogonal to the two axes that guide the expansion of the dimension 2 entity, and to do so it must extend out of the dimension 2 entity. On this respect there is a popularized example with Flatland (the universe of dimension 2) in which it is said, as a proof of the existence of extra dimensions that are unobserved by us, that if a cylinder is passing through Flatland the flatlanders would not see the cylinder only its shadow, but what is obviated is just that the section of the cylinder that is in Flatland is a two dimensional section of it (Fig. 1), therefore in Flatland there are only entities with at most two dimensions no more, all the extra dimensions are at any timeout of Flatland. Thus we have the set E_{dn}: set of elements of the D-n entity.

$E_{dn}= \{ \alpha_{d0}, \ldots , \alpha_{dj}, \ldots , \alpha_{dn-1}, \alpha_{dn} / \nexists \alpha_{>dn}\}$

For a 2-D entity

$E_{d2} = \{\alpha_{d0}, \alpha_{d1}, \alpha_{d2} / \nexists \alpha_{>d2}\}$

3.2 In agree with Hurewicz and Wallman (1948) a space X has dimension ≤ n, dim X ≤ n, if X has D ≤ n at each of its points, therefore if there would be a point with D ≤n+r (with r ≥ 1) X must does not have D ≤ n. Also it is stablished under a proposition that if dim X = n, n finite, then X contains an m-dimensional subset for every m ≤ n, therefore if m would be greater than n (m > n) it would not be contained in X. Also it is shown the theorem III.1, which expresses that a subspace of a space of D ≤ n has D ≤ n, therefore it cannot be any subspace of D ≥ n. In addition we have in agree with Nagata (1965) theorem II. 6, which expresses that let R be a space, then dim R ≤ n if and only if for any locally finite open covering U of R there exists a locally finite open covering B with ordB≤ n+1, B<U. Therefore, if there would be a subspace P of R, with dim P ≤n+r (r ≥ 1) this will mean that there would exists a locally finite open covering C with ordC≤ n+1+r, C<Uby necessity, then from this it will result that dim R ≤n+r an absurd in addition that it would be in contradiction with ordB≤ n+1. Also it is shown in Isham (1999), Hicks (1965), and Spivak (1988) that a subset N of an m-manifold M is a C^∞ -submanifoldof M if every point of N lies in some chart (U, Ø) with

$$Ø (N \cap U) = Ø (U) \cap \Re^K \text{ where } 0 < K \leq m \qquad (1)$$

Therefore, if it would be a subset Sof an m – manifold being a C^∞- submanifold of M, but with the condition that the $\Re^K$in the left side of (1) would have a k = j = k + r, with r ≥ 1, this would mean that for some $j_s$we will have j > m; which is a contradiction an S cannot be in M (i.e., be a subset, a C^∞ - submanifold of M), otherwise M must has to increase its dimension. While it is also clear from 0 < k ≤ m in (1) that entities with dimension ≤ m can be in M.

3.3 As it is well established the dimensionality of the universe or our corner of it is four (by now it is conceded the four dimension, though it will be shown the no existence of time in a forthcoming paper, and this reduce the dimensions of nature to three) being space three dimensional, this is also supported by other articles (Callender, 2005; Greene, Kabat and Marnerides, 2012; Karch and Randall, 2005). In addition it seems that one of the probes has measured the dimensionality of space at a very far distance finding that it is three. As have been shown above in an entity of n dimensions it can only be entities of n or less dimensions and no more and of course this include their movements, i.e., they can move in n dimensions or less no more, therefore in our universe there cannot be entities of more than or moving in more than four dimensions, or perhaps only three. In addition, the tests that have been done so far have not found any tiny extra dimension (Hoyle et al., 2004; Adelberger et al., 2008; Kapner et al., 2008). An interesting idea is compactification but it relies on the reduction of the extra dimensions to a very tiny size making them unobservable by us (Font and Theisen, 2005 ; Greene, 1997; Kiritsis, 1997; Smolin 2013; Vafa, 1997), which means that the extra dimensions continue existing being this not possible in our spatially three dimensional universe, it does not matter how convoluted is the entity (a manifold or whatever) with the extra dimensions the fact is that they continue existing and as have been shown above the only way of an extra dimension to exist is by been out of our spatiallythree dimensional universe. Neither it matters if the dimensions are complex because this mean that the real dimension is the double of the complex dimension an being space-time a smooth manifold there would be extra dimensions that must be out of space-time (i.e., out of our universe).

4. Hypothesis

It seems that in nature there is not negative extension hence there is not negative dimension. There are only extension and dimension as such, but in general we tend to think as positive, this is because of our bias of thinking in a positive sense and because of this we might think in the possibility of a negative complement of an entity under study, but the real fact is that it is not positive and as a consequence it does not have a negative counterpart. Simply stated, an entity has extension greater than zero or it has extension zero which is gotten when all its points are merged one into each other becoming only one or the entity has only one element, but there is nothing that extends under zero (i.e., less that zero) being this the minimal extensional limit . Therefore, this is the same with dimension, being dimension zero the minimal dimensional limit. On this respect, we have what is expressed by Callender (2005) " negative dimensions are impossible according to any of the usual ways of understanding either topological or metrical dimension."

5. Conjecture

Since the way of getting an extra dimension from any previous one is by expanding through the addition of entities of that previous dimension following an axe with a determined angle with respect to the previous entity (points to get dimension one, lines to get dimension two, and so for). It seems that in nature there would be no more than three dimensions, because to get the next dimension with respect to a previous three dimensional entity, it would be necessary to add cubes or any other three dimensional entity making an angle with respect to the previous three dimensional entity and finishing with a four dimensional entity, but it seems that it is not possible to do this, in spite of taking any angle with respect to the previous three dimensional entity, because any of the entities that could be obtained will, ultimately, be another three dimensional entity.

6. CODA

We have created a world of fantasy of many dimensions based on calculations using mathematics but not doing mathematics, trying to make predictions with mathematical and statistical tools, forgetting sometimes that science is about nature, in consequence its basic task is to explain and understand it. Some researchers like Penrose (2004) are confounding things saying that degrees of freedom are dimensions, they are in some way related but two related things are not the same thing, in this way Penrose adds more fantasy to fantasy. The future awaits for new more realistic developments, on this respect an interesting work is the one by Mezincescu and Towsend (2011) who are trying to develop quantum three dimensional superstrings but regretfully it runs with the problem of interactions, perhaps it is necessary to put more effort in this direction.

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