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Quadratic Fractional Optimization through FGP Approach

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Abstract: ***The purpose of this paper is to study the quadratic fractional optimization problem (QFOP) through fuzzy goal programming (FGP) methodology and propose an algorithm to find the optimal solution of it. Here, we construct two QOPs from the given QFOP such that one is of maximization and the other is of minimization subject to set of linear constraints. It is based on the FGP, which differ totally from transforming methodology. At the first stage of the methodology, we transform the QFOP into equivalent two-objectives QOP and the second stage of methodology; we transform the two objectives QOP into LPP. Finally, we give an illustrative example for the better understanding of the solution procedure.***

Keywords: Quadratic Optimization, Quadratic Fractional Optimization, FGP, Pay-off Matrix, LPP and Membership Function

1. Introduction

QFOP is a particular type of NLP problem in which the target function may be a ratio of two quadratic objective functions subject to a set of linear constraints. Such problems arise naturally in decision-making when several rates have to be compelled for the optimization at the same time, for example financial and corporate planning, production planning, hospital and health care planning. In real life problem, the concept of decision-making takes place in an environment in which the objectives and constraints are not glorious exactly. A fuzzy decision is a based on the intersection of membership functions of the goals and constraints. In the literature, quadratic fractional optimization has received much attention. It is the fundamental problem in optimization theory and practice. There were several results on quadratic fractional optimization and FGP such as P. Durga Prasad et al. [10] presented a method in which a fuzzy multi objective NLP is reduced to crisp using ranking function and then the crisp problem is solved by fuzzy programming technique. Sugiyarto S. et al. [11] proposed a computational procedure by using fuzzy approach to find the optimal solution of QOPs. Pal et al. [13] suggested a GP procedure for fuzzy multi-objective LFP problem. Shahroudi and Soltani [14] proposed use of GP for setting in Irans auto industry. Chao-Fang et al. [16] proposed a generalized varying domain optimization method using FGP for multi- objective optimization problem with priorities. Pramanik and Roy [15] gave a procedure for solving multi level programming problems in a large hierarchical decentralized organization through linear FGP approach. Most recently, Lachhwani K. [4] studied on multi-objective QFOP, which involves optimization of several objective functions in the form of a of numerator and denominator functions on the other view. Abdulrahim B. K. [1] studied on QFOP via Feasible Direction Development and Modified Simplex Method. Behrouz K. (2011) gave a method for solving fully fuzzy QOP with fuzzy numbers. Baky I. A. [17] has presented FGP algorithm for solving decentralized bi-level multi-objective problems with a single decision maker at the upper level and multiple decision makers at the lower level. Effati S. et al. [2] has introduced an interval valued LFP. Sen S. [8] presented a piecewise linear approximation method to solve fuzzy separable QOP. Li and Hu (2009) proposed a satisfactory optimization method based on GP for fuzzy multi-objective optimization problem with the aim of achieving the higher desirable satisfying degree. However, a very few of the researchers have considered objective QFOP specially. This situation inspired us to propose FGP approach to the solution of multi-objective QFOP. It optimizes more than one objective function in the form of a ratio in which denominator and numerator both contain linear and quadratic forms. We assume that the set of feasible solutions is a convex polyhedron with a finite number of extreme points and the denominator of the objective functions is non-zero in the constraint set. Pramanik S. et al. [6] used FGP technique to solve the multi-objective QOPs. Pandian P. and Jayalakshmi M. [12] proposed denominator objective restriction method, which is based on simplex method to solve LFP problem and Singh P. et al. [20] developed a method for solving multi-objective linear plus LFP problem. In order to extend this work, we present an algorithm to solve the QFOP. The excel solver optimization software is used to solved the illustrative numerical problem.

2. Formulation of the Problem

In many real life situations, the QFOP may be formulated in different models and can be solved with single objective in deterministic and crisp environment. The mathematical format of QFOP can be stated as:

Max $\Phi(x)= f_1/g_1 = (c_1^T x +\alpha)(c_2^T x + \beta) / (c_3^T x + \gamma)(c_4^T x + \delta)$, subject to $Ax \le b,\ x \ge 0$ (1)

where, $c_1, c_2, c_3, c_4 \in R^n$, $\alpha, \beta, \gamma, \delta \in R$, $A\in R^{mxn}$, $b\in R^m$ and the factors $(c_1^T x +\alpha)$, $(c_2^T x + \beta)$, $(c_3^T x + \gamma)$ and $(c_4^T x + \delta)$ are positive for all feasible solution and constraints set $S=\{x \in R^n : Ax \le b,\ x \ge 0\}$ of feasible solution is nonempty and bounded.

Now we can construct two single-objective QOPs from (1) as follows:

(QOP-I)

Max $f_1^{(N)}(x) = (c_1^T x + \alpha)(c_2^T x + \beta)$
subject to $Ax \le b$,
$x \ge 0$ (2)
and
Min $g_1(x) = (c_3^T x + \gamma)(c_4^T x + \delta)$
subject to $Ax \le b$,
$x \ge 0$

Due to the fact min $g_1(x)$ = max $\{-g_1(x) = f_1^{(D)}(x)\}$, we will focus our attention only on maximization problems as follows:

(QOP-II)

Max $f_1^{(D)}(x) = -(c_3^T x + \gamma)(c_4^T x + \delta)$
subject to $Ax \le b$,
$x \ge 0$ (3)

Many real life QOPs are in multi-objective in nature and optimized subject to a common set of linear constraints. The most general mathematical model of two objectives of QOP has formulated as follows:

Max $\Phi(x) = \{f_1^{(N)}(x), f_1^{(D)}(x)\}$
subject to $Ax \le b$,
$x \ge 0$ (4)

where $f_1^{(N)}$ and $f_1^{(D)}$ are the objective functions of (2) & (3) respectively.

3. FGP Approach to the QFOP

In the FGP problem formulation, the defined membership functions are converted into the membership goals and assigning the highest membership value (Unity) as the aspiration level to each of them. If an imprecise aspiration level to each of the objective functions, then these fuzzy objectives are termed as fuzzy goals. To define the tolerance limits for achievement of their aspired levels, they are characterizing by their associated membership functions.

Suppose that $\tilde{x}^{(N)} = (x_1^{(N)}, \dots, x_n^{(N)})$ and $\tilde{x}^{(D)} = (x_1^{(D)}, \dots, x_n^{(D)})$ are the best solutions of objective function $f_1^{(N)}(\tilde{x})$ and $f_1^{(D)}(\tilde{x})$ respectively subject to the system constraints. Let us assume that $f_1^{(N)}(\tilde{x}^*) = \max_{x \in S} f_i^{(N)}(\tilde{x})$ (i=1,2) and the values of objective function $f_i^{(N)}(\tilde{x})$, which are equal to or larger than $f_1^{(N)}(\tilde{x}^*)$, then the fuzzy goal of the objective function of the decision-making unit appears as :$f_1^{(N)}(\tilde{x}) \gtrsim f_1^{(N)}(\tilde{x}^*)$ and similarly we can assumed that $f_1^{(D)}(\tilde{x}) \gtrsim f_1^{(D)}(\tilde{x}^*)$. Now, using the individual optimal solutions we find the values of all the objective functions at each individual optimal solution point and formulate pay- off matrix as follows:

$$\begin{pmatrix} f_1^{(N)}(\tilde{x}_1) & f_1^{(D)}(\tilde{x}_1) \\ f_1^{(N)}(\tilde{x}_2) & f_1^{(D)}(\tilde{x}_2 \\ \dots & \dots \\ f_1^{(N)}(\tilde{x}_n) & f_1^{(D)}(\tilde{x}_n) \end{pmatrix}$$

let us assume that $z_l^{(N)}$ = Min $(f_1^{(N)}(\tilde{x}_1), f_1^{(N)}(\tilde{x}_2), \dots f_1^{(N)}(\tilde{x}_n))$ and $z_u^{(N)}$ = Max $(f_1^{(N)}(\tilde{x}_1), f_1^{(N)}(\tilde{x}_2), \dots f_1^{(N)}(\tilde{x}_n))$ can be considered as the lower and upper tolerance limits of the fuzzy goal to the objective functions in the first column of pay-off matrix. Similarly we find out lower and upper tolerance limits $z_l^{(D)}$ =Min$(f_1^{(D)}(\tilde{x}_1), f_1^{(D)}(\tilde{x}_2), \dots f_1^{(D)}(\tilde{x}_n))$ and $z_u^{(D)}$ = max $(f_1^{(D)}(\tilde{x}_1), f_1^{(D)}(\tilde{x}_2), \dots f_1^{(D)}(\tilde{x}_n))$ of the fuzzy goal to the objective functions in the second column of the pay-off matrix. The solutions usually are different because both objective functions are conflicting in nature, it can easily be assumed that all values larger than or equal to $z_u^{(N)}$ and $z_u^{(D)}$ are absolutely unacceptable. Therefore, the membership functions for the fuzzy goal of the decision-making unit has formulated as follows:

$$\begin{cases} 1, \text{ if } f_1^{(N)}(\tilde{x}) \gtrsim z_u^{(N)} \\ \mu_1^{(N)}(\tilde{x}) = (f_1^{(N)}(\tilde{x}) - z_l^{(N)})/(z_u^{(N)} - z_l^{(N)}), \text{ if } z_l^{(N)} \lesssim f_1^{(N)}(\tilde{x}) \lesssim z_u^{(N)} \\ 0, \text{ if } f_1^{(N)}(\tilde{x}) \lesssim z_l^{(N)} \end{cases}$$

$$\begin{cases} 1, \text{ if } f_1^{(D)}(\tilde{x}) \gtrsim z_u^{(D)} \\ \mu_2^{(D)}(\tilde{x}) = (f_1^{(D)}(\tilde{x})\ z_l^{(D)})/(z_u^{(D)} - z_l^{(D)}), \text{ if } z_l^{(D)} \lesssim f_1^{(D)}(\tilde{x}) \lesssim z_u^{(D)} \\ 0, \text{ if } f_1^{(D)}(\tilde{x}) \lesssim z_l^{(D)} \end{cases}$$

where $\tilde{x}_i^{(N)} = (x_{i1}^{(N)}, \dots, x_{in}^{(N)})$ and $\tilde{x}_i^{(D)} = (x_{i1}^{(D)}, \dots, x_{in}^{(D)})$ are the best solution of the $\mu_1^{(N)}(\tilde{x})$ and $\mu_2^{(D)}(\tilde{x})$ subject to the system constraints respectively and $\gtrsim$, $\lesssim$ indicate the fuzziness of the aspiration level. Now, we transform quadratic membership functions into linear membership functions by using first order Taylor's polynomial theorem at $\tilde{x} = \tilde{x}_i^{(N)}$ & $\tilde{x}_i^{(D)}$ and expanding these functions about the corresponding points as follows:

$$\mu_1^{(N)}(\tilde{x}) \cong \tilde{\mu}_1^{(N)}(\tilde{x}) = \mu_1^{(N)}(\tilde{x}) + \left[\sum_{i=1}^{n} (x_i - x_{i1}^{(N)})\ \partial\mu_1^{(N)}(\tilde{x})/\partial x_i\right] \text{ at } \tilde{x} = \tilde{x}_i^{(N)}$$

and

$$\mu_2^{(D)}(\tilde{x}) \cong \tilde{\mu}_2^{(D)}(\tilde{x}) = \mu_2^{(D)}(\tilde{x}) + \left[\sum_{i=1}^{n} (x_i - x_{i1}^{(D)})\ \partial\mu_2^{(D)}(\tilde{x})/\partial x_i \text{ at } \tilde{x} = \tilde{x}_i^{(D)}\right]$$

Hence the propose problem can be formulated as

Max $\psi(\tilde{x}) = (\tilde{\mu}_1^{(N)}(\tilde{x}) + \tilde{\mu}_2^{(D)}(\tilde{x}))/2$,
subject to $A\tilde{x} \le b$, $\tilde{x} \ge 0$ (5)

We know that the maximum value of $\psi(\tilde{x})$ is unity. The optimum solution of LPP (5) gives the efficient solution of the QFOP (1) and the values of the membership functions at the optimal points are determined, which give the satisfaction percentage for numerator and denominator of the objective function of (1) to the solution.

The proposed method proceeds as:

Step 1: Construct two single objective QOPs (2) & (3) and solve these problems by taking one of the objective functions at a time.

Step 2: from the above results, determine the corresponding values for every objective at each point. According to each solution and value of every objective, find out a pay-off matrix and then define upper and lower tolerance limits.

Step 3: Formulate the membership function of the objective function $f_1^{(N)}(x)$, $f_1^{(D)}(x)$ and transform these functions into linear membership functions by using first order Taylor's theorem.

Step 4: Formulate the LPP model (5) and solve

4. Illustrative Example

Maximize $\Phi(x) = (x_2 + 1)(x_1 + x_2 + 3) / (x_1 + 4)(x_1+x_2 + 2)$
subject to $-x_1 + x_2 \le 1$,
$x_2 \le 2$,
$x_1 + 2x_2 \le 7$,
$x_1 \le 5$,
$x_{1,} x_2 \ge 0$ (6)

Table 4.1: Computational Results of the Problem (6)

Input Data					
	x_1	x_2	Total	Sign	Limits
Target Fun.Φ(x)	-	-	0.72		
Constraint I	-1	1	1	$\le$	1
Constraint II	0	1	2	$\le$	2
Constraint III	1	2	5	$\le$	7
Constraint IV	1	0	1	$\le$	5
Output Results					
	x_1	x_2	Φ(x)		
	1	2	0.72		

Following two QOPs can be constructed from the given problem:

(**QOP-I**) Maximize $f_1^{(N)}(x) = (x_2 + 1)(x_1 + x_2 + 3)$
subject to $-x_1 + x_2 \le 1$, $x_2 \le 2$,
$x_1 + 2x_2 \le 7$, $x_1 \le 5$,
$x_{1,} x_2 \ge 0$ (7)

Table 4.2: Computational Results of the Problem (7)

Inputs					
	x_1	x_2	Total	Sign	Limits
$f_1^{(N)}(x)$			24		
Constraint I	-1	1	-1	$\le$	1
Constraint II	0	1	2	$\le$	2
Constraint III	1	2	7	$\le$	7
Constraint IV	1	0	3	$\le$	5
Non-Neg.	≥ 0	≥ 0			
Outputs					
	x_1	x_2	$f_1^{(N)}(x)$	$f_1^{(D)}(x)$	
	3	2	24	-49	

(**QOP-II**)

Maximize $f_1^{(D)}(x) = -(x_1 + 4)(x_1+x_2 + 2)$
subject to $-x_1 + x_2 \le 1$, $x_2 \le 2$,
$x_1 + 2x_2 \le 7$, $x_1 \le 5$,
$x_{1,} x_2 \ge 0$ (8)

Table 4.3: Computational Results of the Problem (8)

Input results					
	x_1	x_2	Total	Sign	Limits
$f_1^{(D)}(x)$			-8		
Constraint I	-1	1	0	$\le$	1
Constraint II	0	1	0	$\le$	2
Constraint III	1	2	0	$\le$	7
Constraint IV	1	0	0	$\le$	5
Non-negativity	≥ 0	≥ 0			
Output results					
	x_1	x_2	$f_1^{(N)}(x)$	$f_1^{(D)}(x)$	
	0	0	3	-8	

If the fuzzy aspiration levels of the two objective functions are 24, and -8 , then find x in order to satisfy the following fuzzy goals appears as $f_1^{(N)}(\tilde{x}) \gtrsim 24$, $f_1^{(D)}(\tilde{x}) \gtrsim -8$. Hence $z_l^{(N)} = 3$, $z_u^{(N)} = 24$ and $z_l^{(D)} = -49$, $z_u^{(D)} = -8$, by expending the first order Taylor's theorem, quadratic membership functions about the points (3, 2) and (0, 0) in the feasible region, we have linear membership functions as follows:

$\mu_1^{(N)}(\tilde{x}) \cong \tilde{\mu}_1^{(N)}(\tilde{x}) = 0.1429\, x_1 + 0.523809\, x_2 - 0.47619$,
$\mu_2^{(D)}(\tilde{x}) \cong \tilde{\mu}_2^{(D)}(\tilde{x}) = -\, 0.14634\, x_1 - 0.09756\, x_2 + 1$

Hence, the LPP model is formulated as:

Max $\psi(\tilde{x}) = ½ (-0.0035x_1 + 0.4262\, x_2 + 0.52381)$
Subject to $-x_1 + x_2 \le 1$, $x_2 \le 2$, $x_1 + 2x_2 \le 7$, $x_1 \le 5$,
$x_{1,} x_2 \ge 0$ (9)

The optimal solution is x_1=1, x_2=2 and $\psi(\tilde{x}) = 0.686355 \le$ 1and values of membership functions are μ_1 =0.7143 and μ_2 =0.65846. The membership function values at (1, 2) indicate that the numerator and denominator of QOFP (6) are satisfied 71.43%, 65.85% respectively, for the obtained solution.

Table 4.4: Computational Results of the Reduced LP Problem (9)

Inputs					
	x_1	x_2	Total	Sign	Limits
$\psi(\tilde{x})$	-	-	0.686355		
Constraint I	-1	1	1	$\le$	1
Constraint II	0	1	2	$\le$	2
Constraint III	1	2	5	$\le$	7
Constraint IV	1	0	1	$\le$	5
Non-Negativity	$\ge$	$\ge$			
Outputs					
	x_1	x_2	$\psi(\tilde{x})$	μ_1	μ_2
	1	2	0.686355	0.7143	0.65846
Objective function of original problem = 0.72					

5. Results Analysis

Target objective function (Max)				
	Name	Original Value		
	Total	0.686355		
adjustable variables				
	Name	Original Value		
	x_1	1		
	x_2	2		
constraints				
	Name	Total Value	Status	Slack
	Constraint II	2	Binding	0
	Constraint III	1	Not Binding	4
	Constraint I	1	Binding	0
	Constraint IV	5	Not Binding	2
	x_1	1	Not Binding	1
	x_2	2	Not Binding	2

Var.	F.V.	R. Cost	O.Coef.	Allow. Incr.	Allow.Decr.
x_1	1	0	-0.00175	0.00175	0.21135
x_2	2	0	0.2131	1E+30	0.21135
Cons.	F.Value	S. Price	Con. R.H.S	Allow. Incr.	Allow. Decr.
II	2	0.21135	2	0.666666667	1
III	1	0	5	1E+30	4
I	1	0.00175	1	1	2
IV	5	0	7	1E+30	2

Target Name	Value						
Ob.fun	0.6864						
Adjust.			Low	Tar.		Upp	Tar.
Name	Value		Lim	Resu		Lim.	Res.
x_1	1		1	0.686		3	0.683
x_2	2		0	0.260		2	0.686

6. Generalization into Multi Objective Quadratic Fractional Optimization Problem

The generalization of single objective to multi-objective QFOP is as:

Max $\Phi(x) = \{\Phi_1(x), \Phi_2(x), \ldots. \Phi_k(x)\}$
subject to $Ax \le b, x \ge 0$ (10)

where $\Phi_i(x)=(c^{(i)}_1{}^Tx+\alpha^{(i)})(c^{(i)}_2{}^Tx+\beta^{(i)})/(c^{(i)}_3{}^Tx+\gamma^{(i)})(c^{(i)}_4{}^Tx+\delta^{(i)})$ and $c^{(i)}_1, c^{(i)}_2, c^{(i)}_3, c^{(i)}_4 \in R^n$, $\alpha^{(i)}, \beta^{(i)}, \gamma^{(i)}, \delta^{(i)} \in R$, $A \in R^{m\times n}$, $b \in R^m$ for all $i = 1,2,\ldots.k$. The factors $(c^{(i)}_1{}^Tx + \alpha^{(i)})$, $(c^{(i)}_2{}^Tx + \beta^{(i)})$, $(c^{(i)}_3{}^Tx + \gamma^{(i)})$ and $(c^{(i)}_4{}^Tx + \delta^{(i)})$ are positive for all feasible solution and constraints set $S = \{x \in R^n : Ax \le b, x \ge 0\}$ of feasible solution is nonempty and bounded. Now we can construct 2k multi-objective QOPs from the above problem (10) as follows:

Max $f_i^{(N)}(x) = (c^{(i)}_1{}^Tx + \alpha^{(i)})\ (c^{(i)}_2{}^Tx + \beta^{(i)})$
subject to $Ax \le b, x \ge 0$ (11)
and
Max $f_i^{(D)}(x) = -\ (c^{(i)}_3{}^Tx + \gamma^{(i)})\ (c^{(i)}_4{}^Tx + \delta^{(i)})$ subject to $Ax \le b$, $x \ge 0$ (12)

Many real life QOPs are multi objective in nature and are to be optimized subject to a common set of linear constraints. The more general mathematical model of multi-objective of QFOP (10) has formulated as follows:

Max $\Phi(x) = \{f_1^{(N)}, f_1^{(D)}, f_2^{(N)}, f_2^{(D)} \ldots. f_i^{(N)}, f_i^{(D)}\}$
subject to $Ax \le b, x \ge 0$ (13)

where $f_i^{(N)}$ and $f_i^{(D)}$ are the i^{th} objective functions of the problem where $i = 1,2,\ldots.k$ and $x \in R^n$ is the set of decision variables and finally apply the above procedure for $i= 1,2\ldots..k$.

7. Conclusion

In this paper, an effort has made to extend FGP approach to solve quadratic fractional and multi- objective QFOP. In this approach, we convert it into linear FGP problem and solve. The main advantage of this technique is that it is easy, efficient to solve and requires less time as compared to other methods. We therefore, hope that this approach may be used as an effective tool for solving QFOP and hence time and labor may be saved.

References

[1] Abdulrahim B. K., “Solving Quadratic Fractional Programming Problem via Feasible Direction Development and Modified Simplex Method”, Journal of Zankoy Sulaimani- part A (JZS-A), 15(2), pp. 45-52, 2013.

[2] Effati S. et al., “Solving the Interval Valued LFP Problem”, American Journal of Computational Mathematics, 2, pp. 51-55, 2012.

[3] Gani A. N. et al., “The Principal Pivoting Method for Solving Fuzzy Quadratic Programming”, International Journal of Pure and Applied mathematics, 85(3), pp. 405-414, 2013.

[4] Lachhwani K., “FGP Approach to Multi-Objective Quadratic Fractional Programming Problem”, International Journal of Industrial Mathematics, 6(1), pp. 49-57, 2014.

[5] Pandey P. and Punnen A. P., “A Simplex Algorithm for Piecewise LFP Problems,” European Journal of Operational Research, 178, No. 2, pp. 343-358, 2007.

[6] Pramanik S. and et al. “Multi-Objective Quadratic Programming Problem Based on FGP”, International Journal of Pure and Applied Science and Technology, 6(1), pp. 45-53, 2011.

[7] Schaible S., “Fractional Programming: Applications and Algorithms,” European Journal of Operational research, Vol. 7, No. 2, pp. 111-120, 1981.

[8] Sen S. et al., "A Piecewise Linear Approximation Method to Solve Fuzzy Separable Quadratic Programming Problem", International Journal of Advanced Computer Research, 3(1), pp. 230-235, 2013.
[9] Zimmermann H. J. "Fuzzy Programming and Linear Programming with Several Objective Function", Fuzzy Sets and Systems, 1 pp. 45-55, 1965.
[10] Durga Prasad P. et al., "Solving Fuzzy Multi Objective Non-linear Programming Problem Using Fuzzy Programming Technique", International Journal of Engineering Science and Innovative Technology (IJESIT), Volume 2, Issue 5, pp. 137-142, 2013.
[11] Sugiyarto S. et al., "Optimization Model on Quadratic Programming Problem with Fuzzy", Journal of Mathematics and System Science, 2, pp. 464-468, 2012.
[12] Pandian P. & Jayalakshmi M., "On Solving LFP Problems", Modern Applied Science, 7(6), pp. 1913-1852, 2013.
[13] Pal B.B. et al., "A GP Procedure for Fuzzy Multi-Objective LFP Problem", Fuzzy Sets and Systems, 139 pp. 395-405, 2003.
[14] Shahroudi K., and Soltani S., "Application of GP for Target Setting in Irans Auto Industry", International Journal of Industrial Mathematics, 5, pp. 205-216, 2013.
[15] Pramanik S. and Roy T. K., "FGP Approach to Multi-Level Programming Problem", European Journal of Operational Research, 176, pp. 1151-1166, 2007.
[16] Chao-Fang et al. "A FGP Approach to Multi-Objective Optimization Problem with Priorities", European Journal of Operational Research, 176, pp. 1319-1333, 2007.
[17] Baky I. A., "FGP Algorithm for Solving Decentralized Bi-Level Multi-Objective Programming Problem", Fuzzy Sets and Systems, 160, pp. 2701-2713, 2009.
[18] Chakraborty M. and Gupta S., "Fuzzy Mathematical Programming for Multi-Objective LFP Problem", Fuzzy Sets and Systems, 125, pp. 335-342, 2005.
[19] K. Lachhwani, "FGP Approach to Multi-Objective Quadratic Programming Problem", Proc. National Academy of Sciences India, Section A, 82, pp. 317-322, 2012.
[20] Singh P. et al., "Fuzzy Method for Multi-Objective Linear Plus LFP Problem", International Mathematical Forum, 5(60), pp. 2971-2983, 2010.

Abbreviations Meaning

QOPs - Quadratic Optimization Problems
LPP - Linear Programming Problem
LFP - Linear Fractional Programming
GP - Goal programming
FGP - Fuzzy Goal Programming
QFOP - Quadratic Fractional Optimization Problem
NLP - Non Linear Programming

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