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Soft Atoms and Soft Complements of Soft Lattices

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Abstract: ***Soft set theory was introduced by Molodtsov in 1999 as a mathematical tool for dealing with problems that contain uncertainty. Faruk Karaaslan et al.[6] defined the concept of soft lattices, modular soft lattices and distributive soft lattices over a collection of soft sets. In this paper, we define the concept of complemented soft lattices and complemented distributive soft lattices over a collection of soft sets, study their related properties and illustrate them with some examples. We also define the concept of soft Boolean algebras, soft atoms of soft lattices and discuss the theorems related to soft atoms. In addition, we establish representation theorem for finite soft Boolean algebras.***

Key words: Soft atoms, soft complements, complemented soft lattices, soft Boolean algebras.

1. Introduction

Soft set theory was introduced by Molodtsov [9] in 1999 as a mathematical tool for dealing with uncertainty. Maji et al.[8] defined some operations on soft sets and proved related properties. Irfan Ali et al.[5] studied some new operations in soft set theory. Li [7] , Nagarajan et al.[10] defined the soft lattices using soft sets. Faruk Karaaslan et al.[6] defined the concept of soft lattices over a collection of soft sets by using the operations of soft sets defined by Cagman et al.[1]. Nagarajan et al. [11] proved characterization theorems for modular and distributive soft lattices. Ridvan Sahin et al. [12] applied the notion of soft set theory of Molodtsov to the theory of Boolean algebras. In this paper, we define the concept of complemented soft lattices, complemented distributive soft lattices and soft Boolean algebras over a collection of soft sets. We study their related properties with some examples.We also define the concept of soft Boolean algebras, soft atoms of soft lattices and discuss the theorems related to soft atoms.In addition, we establish representation theorem for finite soft Boolean algebras. The readers are asked to refer Cagman et al.[1], Maji et al.[8] and Molodtsov [9] for basic definitions and results of soft set theory. Faruk Karaaslan et al.[6], Nagarajan et al.[10] and Nagarajan et al. [11] for results of soft lattices. Throughout this work, U refers to the initial universe, $P(U)$ is the power set of U, E is a set of parameters, $A \subseteq E$ and $S(U)$ is the set of all soft sets over U.

2. Complemented Soft Lattice

In this section, we give the definition of complemented soft lattices and study their related properties with some examples.

Definition 2.1 *Let $(L,\vee,\wedge)$ be a soft lattice and $f_A \in L$. If $f_A \le f_X$ for all $f_X \in L$, then f_A is called the least element of L. If $f_X \le f_A$ for all $f_X \in L$, then f_A is called the greatest element of L.* The least the and greatest elements of a soft lattice are the empty soft set $f_\varnothing$ and the universal soft set $f_{\tilde{E}}$ respectively.

Definition 2.2 *A soft lattice $(L,\vee,\wedge)$ is said to be a bounded soft lattice if L has both the least element $f_\varnothing$ and the greatest element $f_{\tilde{E}}$.*

Example 2.3 *Let* $U=\{u_1,u_2,u_3,u_4\}, E=\{e_1,e_2\}$, $A=\{e_1\}, B=\{e_2\}$ *where* $A,B \subseteq E$. *Assume that* $f_\varnothing=\varnothing$, $f_A=\{(e_1,\{u_1,u_2\})\}$, $f_B=\{(e_2,\{u_3,u_4\})\}$, $f_{\tilde{E}}=\{(e_1,\{u_1,u_2\}),(e_2,\{u_3,u_4\})\}$.

Then $L=\{f_\varnothing,f_A,f_B,f_{\tilde{E}}\} \subseteq S(U)$ is a soft lattice with the operations $\tilde{\cup}$ and $\tilde{\cap}$. In this soft lattice,the least element is $f_\varnothing$ and the greatest element is $f_{\tilde{E}}$. Thus $(L,\tilde{\cup},\tilde{\cap})$ is a bounded soft lattice. The Hasse diagram of it is given in figure 1

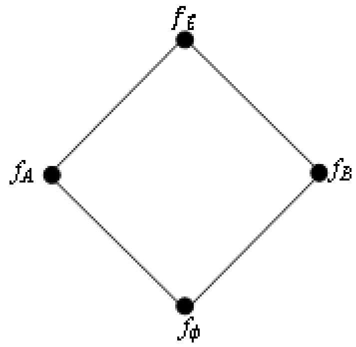

Figure 1

Definition 2.4 *A bounded soft lattice $(L,\vee,\wedge)$ is said to be a complemented soft lattice if for each $f_A \in L$ there exists an element $f_B \in L$ such that $f_A \wedge f_B = f_\varnothing$ and $f_A \vee f_B = f_{\tilde{E}}$.*

Example 2.5 *Let* $U=\{u_1,u_2,u_3,u_4,u_5,u_6\}, E=\{e_1,e_2,e_3\}$, $P=\{e_1\}, Q=\{e_2\}, R=\{e_3\}, S=\{e_1,e_2\}$, $T=\{e_1,e_3\}, V=\{e_2,e_3\}$ *where* $P,Q,R,S,T,V\subseteq E$. *Assume that* $f_\varnothing=\varnothing$, $f_P=\{(e_1,\{u_1,u_2\})\}$, $f_Q=\{(e_2,\{u_3,u_4\})\}$, $f_R=\{(e_3,\{u_5,u_6\})\}$, $f_S=\{(e_1,\{u_1,u_2\}),(e_2,\{u_3,u_4\})\}$, $f_T=\{(e_1,\{u_1,u_2\}),(e_3,\{u_5,u_6\})\}$, $f_V=\{(e_2,\{u_3,u_4\}),(e_3,\{u_5,u_6\})\}$, $f_{\tilde{E}}=\{(e_1,\{u_1,u_2\}),(e_2,\{u_3,u_4\}),(e_3,\{u_5,u_6\})\}$. Then $L=\{f_\varnothing,f_P,f_R,f_R,f_S,f_T,f_V,f_{\tilde{E}}\}\subseteq S(U)$ is a soft lattice with the operations $\tilde{\cup}$ and $\tilde{\cap}$. Here the soft complement of f_P is f_V, the soft complement of f_Q is f_T, and the soft complement of f_R is f_S. Thus $(L,\tilde{\cup},\tilde{\cap})$ is a complemented soft lattice. The Hasse diagram of it is given in figure 2.

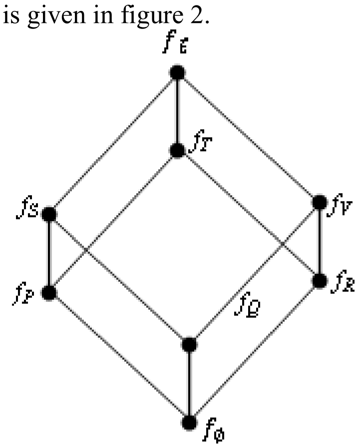

Figure 2

Remark 2.6 *If* f_A *is a soft complement of* f_B, *then* f_B *is a soft complement of* f_A.

Remark 2.7 *If* L *has* $f_\varnothing$ *and* $f_{\tilde{E}}$, *then* $f_\varnothing\wedge f_{\tilde{E}}=f_\varnothing$ *and* $f_\varnothing\vee f_{\tilde{E}}=f_{\tilde{E}}$ *and so* $f_\varnothing$ *is a soft complement of* $f_{\tilde{E}}$ *and* $f_{\tilde{E}}$ *is a soft complement of* $f_\varnothing$.

Lemma 2.8 *If* $(L,\vee,\wedge)$ *is a finite soft chain with more than two elements then* L *is not soft complemented.*

Proof. Let $f_A\in L$ such that $f_A\neq f_\varnothing$ and $f_A\neq f_{\tilde{E}}$. Suppose $f_B\in L$. Since L is a chain, either $f_A\leq f_B$ or $f_B\leq f_A$. Case(i) Let $f_A\leq f_B$. Then $f_A\vee f_B=f_B$ and $f_A\wedge f_B=f_A$.Therefore f_B can not be a soft complement of f_A .Case(ii) Let $f_B\leq f_A$. Then $f_A\vee f_B=f_A$ and $f_A\wedge f_B=f_B$. Therefore f_B can not be a soft complement of f_A. Thus no element $f_B\in L$ be a soft complement of f_A. Hence L in not a complemented soft lattice.

Theorem 2.9 *If* $(L,\vee,\wedge)$ *is a complemented distributive soft lattice, then every element of* L *has a unique soft complement.*

Proof. Let us suppose that $f_A\in L$ has two soft complements f_{B_1} and f_{B_2}. Then by the definition of the soft complement and $f_A\wedge f_{B_2}=f_\varnothing, f_A\vee f_{B_2}=f_{\tilde{E}}$.

Now
$$\begin{aligned}f_{B_1}&=f_{B_1}\wedge f_{\tilde{E}}=f_{B_1}\wedge(f_A\vee f_{B_2})\\&=(f_{B_1}\wedge f_A)\vee(f_{B_1}\wedge f_{B_2})=f_\varnothing\vee(f_{B_1}\wedge f_{B_2})\\&=(f_{B_2}\wedge f_A)\vee(f_{B_2}\wedge f_{B_1})=f_{B_2}\wedge(f_A\vee f_{B_1})\\&=f_{B_2}\wedge f_{\tilde{E}}=f_{B_2}.\end{aligned}$$

Hence $f_{B_1}=f_{B_2}$ which contradicts the assumption that $f_A\in L$ has two different soft complements f_{B_1} and f_{B_2}. Thus the soft complement of any $f_A\in L$ is unique.

Theorem 2.10 *Let* $(L,\vee,\wedge)$ *be a complemented distributive soft lattice. For* $f_A,f_B\in L$, *the following are equivalent. (i)* $f_A\leq f_B$, *(ii)* $f_A\wedge f_B^{\tilde{c}}=f_\varnothing$, *(iii)* $f_A^{\tilde{c}}\vee f_B=f_{\tilde{E}}$, *(iv)* $f_B^{\tilde{c}}\leq f_A^{\tilde{c}}$.

Proof. $(i)\Rightarrow(ii)$: Let $f_A\leq f_B$.

Then
$$\begin{aligned}&f_A\vee f_B=f_B\\&\Rightarrow(f_A\vee f_B)\wedge f_B^{\tilde{c}}=f_\varnothing\\&\Rightarrow(f_A\wedge f_B^{\tilde{c}})\vee(f_B\wedge f_B^{\tilde{c}})=f_\varnothing\\&\Rightarrow f_A\wedge f_B^{\tilde{c}}=f_\varnothing \text{ as } f_B\wedge f_B^{\tilde{c}}=f_\varnothing.\end{aligned}$$

$(ii)\Rightarrow(iii)$: $f_A\wedge f_B^{\tilde{c}}=f_\varnothing$
$\Rightarrow(f_A\wedge f_B^{\tilde{c}})^{\tilde{c}}=f_{\tilde{E}}\Rightarrow f_A^{\tilde{c}}\vee(f_B^{\tilde{c}})^{\tilde{c}}=f_{\tilde{E}}$
$\Rightarrow f_A^{\tilde{c}}\vee f_B=f_{\tilde{E}}$.

$(iii)\Rightarrow(iv)$: $f_A^{\tilde{c}}\vee f_B=f_{\tilde{E}}\Rightarrow(f_A^{\tilde{c}}\vee f_B)\wedge f_B^{\tilde{c}}=f_B^{\tilde{c}}$
$\Rightarrow(f_A^{\tilde{c}}\wedge f_B^{\tilde{c}})\vee(f_B\wedge f_B^{\tilde{c}})=f_B^{\tilde{c}}\Rightarrow f_A^{\tilde{c}}\wedge f_B^{\tilde{c}}=f_B^{\tilde{c}}$
$\Rightarrow f_B^{\tilde{c}}\leq f_A^{\tilde{c}}$.

$(iv)\Rightarrow(i)$: $f_B^{\tilde{c}}\leq f_A^{\tilde{c}}\Rightarrow f_A^{\tilde{c}}\wedge f_B^{\tilde{c}}=f_B^{\tilde{c}}$
$\Rightarrow(f_A^{\tilde{c}}\wedge f_B^{\tilde{c}})^{\tilde{c}}=(f_B^{\tilde{c}})^{\tilde{c}}\Rightarrow f_A\vee f_B=f_B$
$\Rightarrow f_A\leq f_B$.

3. Soft Boolean Algebra

Soft Boolean Algebras are special type of soft lattices. In this section, We define soft Boolean algebras with some of its properties.

Definition 3.1 *A complemented distributive soft lattice is said to be a soft Boolean algebra or a soft Boolean lattice with least element $f_\varnothing$ and greatest element $f_{\tilde{E}}$.*

Example 3.2

Let $U=\{u_1,u_2,u_3,u_4,u_5,u_6\}, E=\{e_1,e_2,e_3\}$, $A=\{e_1\}, B=\{e_2\}, C=\{e_3\}$

where $A,B,C\subseteq E$. *Assume that* $f_\varnothing=\varnothing$

$f_A=\{(e_1,\{u_1,u_2\})\}$, $f_B=\{(e_2,\{u_3,u_4\})\}$,

$f_C=\{(e_3,\{u_5,u_6\})\}$,

$\{f_A,f_B\}=\{(e_1,\{u_1,u_2\}),(e_2,\{u_3,u_4\})\}$,

$\{f_A,f_C\}=\{(e_1,\{u_1,u_2\}),(e_3,\{u_5,u_6\})\}$,

$\{f_B,f_C\}=\{(e_2,\{u_3,u_4\}),(e_3,\{u_5,u_6\})\}$,

$\{f_A,f_B,f_C\}=\{(e_1,\{u_1,u_2\}),(e_2,\{u_3,u_4\}),(e_3,\{u_5,u_6\})\}$.

Then $\mathsf{A}=\{f_A,f_B,f_C\}$ and $\mathsf{P}(\mathsf{A})=\{f_\varnothing,\{f_A\},\{f_B\},\{f_C\},\{f_A,f_B\},\{f_B,f_C\},\{f_A,f_C\},\{f_A,f_B,f_C\}\}\subseteq S(U)$. Hence $\mathsf{P}(\mathsf{A})\subseteq S(U)$ is a soft lattice with the operations $\tilde{\cup}$ and $\tilde{\cap}$. The Hasse diagram of the soft lattice $(\mathsf{P}(\mathsf{A}),\tilde{\cup},\tilde{\cap})$ is given in figure 3.

This soft lattice $(\mathsf{P}(\mathsf{A}),\tilde{\cup},\tilde{\cap})$ is a complemented distributive soft lattice with least element $f_\varnothing$ and a greatest element A. Hence the soft lattice $(\mathsf{P}(\mathsf{A}),\tilde{\cup},\tilde{\cap})$ is a soft Boolean algebra.

Remark 3.3 *Every element f_A in a soft Boolean algebra has a unique soft complement. The unique soft complement of f_A is denoted by $f_A^{\tilde{c}}$.*

Theorem 3.4 *Let $(\mathsf{B},\vee,\wedge,^{\tilde{c}},f_\varnothing,f_{\tilde{E}})$ be a soft Boolean algebra. Then for all $f_A\in\mathsf{B}$,*
(i) $(f_A^{\tilde{c}})^{\tilde{c}}=f_A$, (ii) $f_\varnothing^{\tilde{c}}=f_{\tilde{E}}$ and $f_{\tilde{E}}^{\tilde{c}}=f_\varnothing$.

Proof. (i)Let $f_A^{\tilde{c}}$ be the soft complement of $f_A\in\mathsf{B}$. Then $f_A\vee f_A^{\tilde{c}}=f_{\tilde{E}}$ and $f_A\wedge f_A^{\tilde{c}}=f_\varnothing$. By commutativity, $f_A^{\tilde{c}}\vee f_A=f_{\tilde{E}}$ and $f_A^{\tilde{c}}\wedge f_A=f_\varnothing$. This implies that f_A is the soft complement of $f_A^{\tilde{c}}$. That is, $(f_A^{\tilde{c}})^{\tilde{c}}=f_A$.
(ii)We know that $f_\varnothing\vee f_{\tilde{E}}=f_{\tilde{E}}$ and $f_\varnothing\wedge f_{\tilde{E}}=f_\varnothing$. This implies that $f_{\tilde{E}}$ is the soft complement of $f_\varnothing$. That is, $f_{\tilde{E}}^{\tilde{c}}=f_\varnothing$. By the principle of duality, $f_\varnothing^{\tilde{c}}=f_{\tilde{E}}$.

Remark 3.5 *A soft Boolean algebra will generally be denoted by $(\mathsf{B},\vee,\wedge,^{\tilde{c}},f_\varnothing,f_{\tilde{E}})$. where $\mathsf{B}\subseteq S(U)$. The two operations $\vee$ (join) and $\wedge$ (meet) are binary operations on B and soft complementation is a unary operations on B. The corresponding soft poset will be denoted by $(\mathsf{B},\leq)$. The bounds of the soft lattice are $f_\varnothing$ and $f_{\tilde{E}}$ where $f_\varnothing$ is the least element and $f_{\tilde{E}}$ is the greatest element.*

Note 3.6 *As a soft Boolean algebra should contain $f_\varnothing$ and $f_{\tilde{E}}$, every soft Boolean algebra has atleast two elements.*

Theorem 3.7 ***De Morgan's Law:*** *Let $(\mathsf{B},\vee,\wedge,^{\tilde{c}},f_\varnothing,f_{\tilde{E}})$ be a soft Boolean algebra. Then for any $f_A,f_B\in\mathsf{B}$,* (i) $(f_A\wedge f_B)^{\tilde{c}}=f_A^{\tilde{c}}\vee f_B^{\tilde{c}}$ (ii) $(f_A\vee f_B)^{\tilde{c}}=f_A^{\tilde{c}}\wedge f_B^{\tilde{c}}$

Proof. Let $(\mathsf{B},\vee,\wedge,^{\tilde{c}},f_\varnothing,f_{\tilde{E}})$ be a soft Boolean algebra and $f_A,f_B\in\mathsf{B}$. Then

$(f_A\wedge f_B)\vee(f_A^{\tilde{c}}\vee f_B^{\tilde{c}})$
$=(f_A\vee(f_A^{\tilde{c}}\vee f_B^{\tilde{c}}))\wedge(f_B\vee(f_A^{\tilde{c}}\vee f_B^{\tilde{c}}))$
$=((f_A\vee f_A^{\tilde{c}})\vee f_B^{\tilde{c}})\wedge((f_B\vee f_B^{\tilde{c}})\vee f_A^{\tilde{c}})$
$=(f_{\tilde{E}}\vee f_B^{\tilde{c}})\wedge(f_{\tilde{E}}\vee f_A^{\tilde{c}})=f_{\tilde{E}}\wedge f_{\tilde{E}}=f_{\tilde{E}}$ and

$(f_A\wedge f_B)\wedge(f_A^{\tilde{c}}\vee f_B^{\tilde{c}})$
$=((f_A\wedge f_B)\wedge f_A^{\tilde{c}})\vee((f_A\wedge f_B)\wedge f_B^{\tilde{c}})$
$=((f_A\wedge f_A^{\tilde{c}})\wedge f_B)\vee(f_A\wedge(f_B\wedge f_B^{\tilde{c}}))$
$=(f_\varnothing\wedge f_B)\vee(f_A\wedge f_\varnothing)=f_\varnothing\vee f_\varnothing=f_\varnothing$. Thus

$f_A^{\tilde{c}}\vee f_B^{\tilde{c}}$ is the soft complement of $f_A\wedge f_B$. That is $(f_A\wedge f_B)^{\tilde{c}}=f_A^{\tilde{c}}\vee f_B^{\tilde{c}}$.
By the principle of duality, we have $(f_A\vee f_B)^{\tilde{c}}=f_A^{\tilde{c}}\wedge f_B^{\tilde{c}}$.

4. Soft Atoms of Soft Lattices

In this section, we define the soft atoms of soft lattices. We show that a finite soft Boolean algebra has exactly 2^n elements for some positive integer n. Moreover, any two soft Boolean algebras of order 2^n are isomorphic to each other.

Definition 4.1 *Let f_A and f_B be two elements in a soft lattice. The element f_B is said to be a cover for f_A if $f_A\leq f_B, f_A\neq f_B$ and there is no element f_C in the soft lattice such that $f_A<f_C<f_B$. If f_B covers an element*

f_A, *we denote it by* $f_A \prec f_B$.

Definition 4.2 *An element which covers the least element* $f_\varnothing$ *is said to be a soft atom of the soft lattice.*

Remark 4.3 *Let* $f_A, f_B \in L$ *and* f_A *be a soft atom in* L. *If* $f_A \neq f_B$, *then* $f_A \wedge f_B \neq f_A$.
As f_A *is a soft atom* $f_A \wedge f_B \leq f_A \Rightarrow f_A \wedge f_B = f_\varnothing$.
In particular f_A *and* f_B *are two distinct soft atoms, then* $f_A \wedge f_B = f_\varnothing$.

Theorem 4.4 *Let* B *be a finite soft Boolean algebra. If* $f_B \neq f_\varnothing$ *is an element in* B, *then there exists a soft atom* f_A *such that* $f_A \leq f_B$.

Proof. If f_B itself is a soft atom, then we take $f_A = f_B$. If f_B is not a soft atom, as B is finite, we can find a soft chain $f_\varnothing \prec f_{B_1} \prec \ldots \prec f_{B_n} \prec f_B$ satisfying $f_\varnothing \prec f_{B_1}, f_{B_i} \prec f_{B_{i+1}}$ for all $i = 1,2,\ldots,n-1$ and $f_{B_n} \prec f_B$. So f_{B_1} is a soft atom such that $f_{B_1} \leq f_B$ and we take $f_A = f_{B_1}$.

Theorem 4.5 *Let* B *be a finite soft Boolean algebra and* $f_B \neq f_\varnothing$ *in* B. *Let* $f_{A_1}, f_{A_2}, \ldots, f_{A_k}$ *be all the soft atoms of* B *such that* $f_{A_i} \leq f_B$ *for all i= 1,2,....k. Then* $f_B = f_{A_1} \vee f_{A_2} \vee \ldots \vee f_{A_k}$.

Proof. Let $f_B \neq f_\varnothing$ in B. Define $\mathsf{A}(f_B) = \{f_A \in \mathsf{B} : f_\varnothing \prec f_A$ and $f_A \leq f_B\}$. By theorem 4.4, $\mathsf{A}(f_B) \neq f_\varnothing$. As B itself is finite, $\mathsf{A}(f_B)$ is a finite soft set. Let $\mathsf{A}(f_B) = \{f_{A_1}, f_{A_2}, \ldots, f_{A_k}\}$ and $f_C = f_{A_1} \vee f_{A_2} \vee \ldots \vee f_{A_k}$. As each $f_{A_i} \leq f_B$, we have $f_C \leq f_B$. We claim that $f_B \leq f_C$. It is enough to show that $f_B \wedge f_C^{\tilde{c}} = f_\varnothing$.
If $f_B \wedge f_C^{\tilde{c}} \neq f_\varnothing$, then $\mathsf{A}(f_B \wedge f_C^{\tilde{c}}) \neq f_\varnothing$. Consider a soft atom f_A of B such that $f_A \leq f_B \wedge f_C^{\tilde{c}}$ (by theorem 4.4). Then $f_A \leq f_C^{\tilde{c}}$ and $f_A \leq f_B$. As $f_A \leq f_B$ and f_A is a soft atom, $f_A \in \mathsf{A}(f_B)$. So $f_A = f_{A_i}$ for some i and $f_A \leq f_C$. As $f_A \leq f_C$ and $f_A \leq f_C^{\tilde{c}}$ we have $f_A \leq f_C \wedge f_C^{\tilde{c}} = f_\varnothing$ which is a contradiction to f_A is a soft atom. Thus $f_B \wedge f_C^{\tilde{c}} = f_\varnothing$ and hence $f_B \leq f_C$ so $f_B = f_{A_1} \vee f_{A_2} \vee \ldots \vee f_{A_k}$.

Theorem 4.6 *Let* B *be a finite soft Boolean algebra and* $f_B \neq f_\varnothing$ *in* B. *If* $f_{A_1}, f_{A_2}, \ldots, f_{A_k}$ *and* $f_{B_1}, f_{B_2}, \ldots, f_{B_m}$ *are soft atoms of* B *such that* (i) $f_{A_1}, f_{A_2}, \ldots, f_{A_k}$ are distinct (ii) $f_{B_1}, f_{B_2}, \ldots, f_{B_m}$ are distinct and (iii) $f_B = f_{A_1} \vee f_{A_2} \vee \ldots \vee f_{A_k} = f_{B_1} \vee f_{B_2} \vee \ldots \vee f_{B_m}$ then $k = m$ and $\{f_{A_1}, f_{A_2}, \ldots, f_{A_k}\} = \{f_{B_1}, f_{B_2}, \ldots, f_{B_m}\}$.

Proof. By (i), (ii) and (iii) $f_{A_1}, f_{A_2}, \ldots, f_{A_k}$ are distinct soft atoms of B and $f_{B_1}, f_{B_2}, \ldots, f_{B_m}$ are distinct soft atoms of B such that $f_B = f_{A_1} \vee f_{A_2} \vee \ldots \vee f_{A_k} = f_{B_1} \vee f_{B_2} \vee \ldots \vee f_{B_m}$. Then each $f_{A_i} \leq f_B$ and each $f_{B_t} \leq f_B$. So $f_{A_i} = f_{A_i} \wedge f_B = f_{A_i} \wedge (f_{B_1} \vee f_{B_2} \vee \ldots \vee f_{B_m})$ $= (f_{A_i} \wedge f_{B_1}) \vee (f_{A_i} \wedge f_{B_2}) \vee \ldots \vee (f_{A_i} \wedge f_{B_m})$. As $f_{A_i} \neq f_\varnothing$, we can find j such that $f_{A_i} \wedge f_{B_j} \neq f_\varnothing$. As both f_{A_i} and f_{B_j} are soft atoms and $f_{A_i} \wedge f_{B_j} \neq f_\varnothing, f_{A_i} = f_{B_j}$. Thus each f_{A_i} is same as some f_{B_j}. Hence $k \leq m$. Similarly $f_{B_t} = f_{B_t} \wedge f_B = f_{B_t} \wedge (f_{A_1} \vee f_{A_2} \vee \ldots \vee f_{A_k})$ $= (f_{B_t} \wedge f_{A_1}) \vee (f_{B_t} \wedge f_{A_2}) \vee \ldots (f_{B_t} \wedge f_{A_k})$. As $f_{B_t} \neq f_\varnothing, f_{B_t} \wedge f_{A_l} \neq f_\varnothing$ for some f_{A_l}. As both f_{B_t} and f_{A_l} are soft atoms, $f_{B_t} = f_{A_l}$. Thus for each f_{B_t} there exists f_{A_l} such that $f_{B_t} = f_{A_l}$. Therefore, $m \leq k$.
Thus $k \leq m, m \leq k \Rightarrow k = m$.
Hence $\{f_{A_1}, f_{A_2}, \ldots, f_{A_k}\} = \{f_{B_1}, f_{B_2}, \ldots, f_{B_m}\}$.

Corollary 4.7 *Let* B *be a finite soft Boolean algebra and every* $f_B \neq f_\varnothing$ *in* B *can be expressed as a join of soft atoms and this expression is unique.*

Proof. By theorems 4.4, 4.5 and 4.6 the proof follows.

Definition 4.8 *Let* B_1 *and* B_2 *be soft Boolean algebras. A mapping* $\eta : \mathsf{B}_1 \to \mathsf{B}_2$ *is said to be a soft Boolean homomorphism from* B_1 *into* B_2 *if* η *is a soft lattice homomorphism and for all* $f_X \in \mathsf{B}_1$, *we have* $\eta(f_X^{\tilde{c}}) = (\eta(f_X))^{\tilde{c}}$.

Definition 4.9 *A soft Boolean homomorphism is said to be a soft Boolean isomorphism if it is bijective.*

Theorem 4.10 ***Representation theorem for finite soft Boolean algebras:*** *Let* B *be a finite soft Boolean algebra and let* A *be the set of all soft atoms of* B. *Then* B *is soft isomorphic to* $\mathsf{P}(\mathsf{A})$.

Proof. To each $f_B \neq f_\varnothing$ in B, let $\mathsf{A}(f_B) = \{f_A \in \mathsf{B} : f_\varnothing \prec f_A$ and $f_A \leq f_B\}$ and let $\mathsf{A}(f_\varnothing) = f_\varnothing$. Then for all $f_B \neq f_\varnothing, \mathsf{A}(f_B) \neq f_\varnothing$ and $\mathsf{A}(f_B) \subseteq \mathsf{A}$ where $\mathsf{A} = \mathsf{A}(f_{\tilde{E}})$ is the set of all soft atoms in B. Now we define a map $\eta : \mathsf{B} \to \mathsf{P}(\mathsf{A})$ given by $\eta(f_B) = \mathsf{A}(f_B)$ for all $f_B \in \mathsf{B}$. We now show that the map is a soft Boolean isomorphism.

i) η is one-to-one:

If $\eta(f_{B_1}) = \eta(f_{B_2})$, for $f_{B_1}, f_{B_2} \in \mathsf{B}$, then $\eta(f_{B_1}) = \eta(f_{B_2}) = \{f_{A_1}, f_{A_2}, \ldots, f_{A_k}\}$.

Then by theorem 4.6, $f_{B_1} = f_{A_1} \vee f_{A_2} \vee \ldots \vee f_{A_k} = f_{B_2}$.

So $\eta(f_{B_1}) = \eta(f_{B_2}) \Rightarrow f_{B_1} = f_{B_2}$.

ii) η is onto: Let $C \in \mathsf{P}(\mathsf{A})$. If $C = f_\varnothing$, then $\eta(f_\varnothing) = \mathsf{A}(f_\varnothing) = f_\varnothing = C$.

If $C \neq f_\varnothing$, let $C = \{f_{C_1}, f_{C_2}, \ldots, f_{C_m}\}$. Let $f_B = f_{C_1} \vee f_{C_2} \vee \ldots \vee f_{C_m}$. Then each $f_{C_i} \leq f_B$ and so $f_{C_i} \in \mathsf{A}(f_B)$ for all $i = 1, 2, \ldots, m$. If $f_A \in \mathsf{A}(f_B)$, then $f_A \leq f_B$ and

$$f_\varnothing \neq f_A = f_A \wedge f_B = f_A \wedge (f_{C_1} \vee f_{C_2} \vee \ldots \vee f_{C_m}) = (f_A \wedge f_{C_1}) \vee (f_A \wedge f_{C_2}) \vee \ldots (f_A \wedge f_{C_m}).$$

So $f_A \wedge f_{C_j} \neq f_\varnothing$ for atleast one j. As f_A and f_{C_j} are soft atoms and $f_A \wedge f_{C_j} \neq f_\varnothing$, $f_A = f_{C_j}$. Thus if $f_A \in \mathsf{A}(f_B)$, then $f_A \in C \Rightarrow \mathsf{A}(f_B) \subseteq C$. As $f_B = f_{C_1} \vee f_{C_2} \vee \ldots \vee f_{C_m}, C \subseteq \mathsf{A}(f_B)$. Thus we have $C = \mathsf{A}(f_B)$ and $\eta(f_B) = C$. Hence η is onto. iii) $\eta(f_P \wedge f_Q) = \eta(f_P) \cap \eta(f_Q)$: for all $f_P, f_Q \in \mathsf{B}$. If either $f_P = f_\varnothing$ or $f_Q = f_\varnothing$, we have $\eta(f_P \wedge f_Q) = f_\varnothing = \eta(f_P) \cap \eta(f_Q)$.

So assume that both $f_P \neq f_\varnothing$ and $f_Q \neq f_\varnothing$. Then there exists a soft atom satisfying $f_A \leq f_P \wedge f_Q$. Then $f_A \leq f_P$ and $f_A \leq f_Q$ that is, $f_A \in \eta(f_P)$ and $f_A \in \eta(f_Q)$. So, $f_A \in \eta(f_P \cap f_Q) \Rightarrow f_A \in \eta(f_P) \cap \eta(f_Q)$. So, $\eta(f_P \wedge f_Q) \subseteq \eta(f_P) \cap \eta(f_Q)$.

If $\eta(f_P) \cap \eta(f_Q) = f_\varnothing$, then $\eta(f_P \wedge f_Q) = f_\varnothing$ and the equation is satisfied. Assume that $\eta(f_P) \cap \eta(f_Q) \neq f_\varnothing$. If $f_A \in \eta(f_P) \cap \eta(f_Q)$, then $f_A \in \mathsf{A}(f_P)$ and $f_A \in \mathsf{A}(f_Q)$ and so $f_A \leq f_P$ and $f_A \leq f_Q \Rightarrow f_A \leq f_P \wedge f_Q$.

As f_A is soft atom, $f_A \subseteq f_P \wedge f_Q, f_A \in \mathsf{A}(f_P \wedge f_Q) = \eta(f_P \wedge f_Q)$. So, $\eta(f_P) \cap \eta(f_Q) \subseteq \eta(f_P \wedge f_Q)$. Thus $\eta(f_P \wedge f_Q) = \eta(f_P) \cap \eta(f_Q)$. Similarly we can prove $\eta(f_P \vee f_Q) = \eta(f_P) \cup \eta(f_Q)$.

iv.) $\eta(f_Q^{\tilde{c}}) = \mathsf{A} \setminus \eta(f_Q)$: for all $f_Q \in \mathsf{B}$.

Let $f_Q \in \mathsf{B}$. Then $\mathsf{A} = \eta(f_{\tilde{E}}) = \eta(f_Q \vee f_Q^{\tilde{c}}) = \eta(f_Q) \cup \eta(f_Q^{\tilde{c}})$ & $f_\varnothing = \eta(f_\varnothing) = \eta(f_Q \wedge f_Q^{\tilde{c}}) = \eta(f_Q) \cap \eta(f_Q^{\tilde{c}})$. This means that $\eta(f_Q^{\tilde{c}}) = \mathsf{A} \setminus \eta(f_Q)$ From (i), (ii), (iii) and (iv), η is a soft Boolean isomorphism.

Corollary 4.11 *Every finite soft Boolean algebra has* 2^n *elements for some positive integer n.*

Proof. Let B be a finite soft Boolean algebra. Then if A is the set of all soft atoms of B and let $O(\mathsf{A}) = n$. Then there are exactly 2^n elements in $\mathsf{P}(\mathsf{A})$ and by theorem 4.10, B is soft isomorphic to soft Boolean algebra $\mathsf{P}(\mathsf{A})$. So $O(\mathsf{B}) = O(\mathsf{P}(\mathsf{A})) = 2^n$ where $O(\mathsf{A}) = n$. Hence B has exactly 2^n elements where n is the number of elements in A.

Corollary 4.12 *Any two soft Boolean algebras of order* 2^n *are isomorphic to each other.*

Proof. By theorem 4.10, every soft Boolean algebra of order 2^n is soft isomorphic to $\mathsf{P}(\mathsf{A})$ where A is the set of all soft atoms and $O(\mathsf{A}) = n$. Hence any two soft Boolean algebras of order 2^n are soft isomorphic to each other

5. Conclusion

In this paper, we defined the concept of complemented soft lattices, complemented distributive soft lattices and soft Boolean algebras over a collection of soft sets and discussed their related properties and illustrated them with some examples.We also defined the soft atoms of soft lattices and discussed the theorems related to soft atoms. Further, we have given representation theorem for finite soft Boolean algebras.We are studying about these soft lattices and are expected to give some more results.

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