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Combined Effects of Thermal Radiation and MHD of Flow past an Accelerated Vertical Plate in a Rotating Fluid with Variable Temperature

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Abstract: ***An analysis is performed to study the combined effects of thermal radiation and MHD on unsteady flow past a uniformly accelerated vertical plate in a rotating fluid in the presence of thermal radiation. An exact solution is obtained for the axial and transverse components of the velocity by defining a complex velocity. The effects of magnetic field, rotation, radiation, free-convection parameters and components on the plate are discussed.***

Keywords: MHD, Thermal Radiation, rotation, accelerated vertical plate, heat transfer

1. Introduction

A very common industrial application of natural convection is free air cooling without the aid Of fans: this can happen on small scales (computer chips) to large scales. Such configuration is also encountered in several practical systems for industry based applications such as Heat exchanger devices, cooling of molten metal's, insulation systems, petroleum reservoirs, Desert coolers, wet bulb thermometers and frost formation. The effect of radiation is quite significant at high temperature. Radiative heat transfer plays an important role in manufacturing industries for the design of reliable equipment. Effects of various parameters on human body can be studied and appropriate suggestions can be given to the persons working in hazardous areas using the knowledge of MHD.

The unsteady free convection flow of an electrically conducting fluid past an accelerated infinite vertical plate with constant heat flux is investigated under the influence of uniform transverse magnetic field fixed relative to the fluid or to the plate in the presence of heat generation or absorption of exponentially accelerated plate and uniformly accelerated plate have been considered and discussed by L. Debnath and M. Narahari [3]. An analytical study is performed by Vijaya and G.V.Ramana Reddy [10] to investigate the effects of chemical reaction and radiation on unsteady MHD flow past an exponentially accelerated vertical plate with variable temperature and variable mass diffusion in the presence of applied transverse magnetic field. The radiation effects on unsteady flow of a viscous incompressible fluid past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion is considered in the presence of magnetic field and heat source was studied by P. Bala Anki Reddy etal.[2]. The combined effects of rotation and radiation on the unsteady hydrodynamic flow past an impulsively accelerated vertical plate with ramped plate temperature have been studied by M. Jana etal [4]. They have considered the fluid which is a gray, absorbing-emitting but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the analysis.

Narhari and M. Yunus Nayan [6] had performed an analytical study of free convection flow near an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and constant mass diffusion. D. Vijay Kumar and Ch. V. Ramana Murthy [11] aimed at the free convective flow past an impulsively started vertical plate with constant heat flux in the presence of thermal radiation. The effect of thermal radiation past an impulsively started infinite vertical plate under the influence of transverse magnetic field has been discussed by M. K. Mazumdar and R. K. Deka [5]. G. Palani and I.A. Abbas [7] had investigated the combined effects of MHD and Radiation on the free Convection flow past a semi-infinite vertical plate, when the fluid is compressible, viscous and electrically conducting. G. Vidyasagar and B. Ramana[9] had analyzed the combined effect of thermal diffusion and heat absorption on the MHD free convection heat and mass transfer flow of a viscous incompressible fluid past a continuously moving infinite plate. G. Palani and U. Srikanth [8] performed the analysis to study the MHD flow of an electrically conducting, incompressible, viscous fluid past a semi-infinite vertical plate with mass transfer, under the action of transversely applied magnetic field is carried out. The heat due to viscous dissipation and the induced magnetic field are assumed to be negligible.

However the combined effect of MHD and thermal radiation in an accelerated vertical plate was not discussed by any author so far. SO In this present paper we proposed to study the combined effects of thermal radiation and MHD of flow past an accelerated vertical plate in a rotating fluid with variable temperature. The dimensionless governing equations are solved by Laplace transform technique.

2. Basic Equations and Analysis

Consider the three dimensional flow of a viscous incompressible fluid induced by uniformly accelerated motion of an infinite vertical isothermal plate in a rotating fluid. On this plate, the x' -axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to x'-axis in the plane of the plate and z'-axis is normal to it. Both the fluid and the plate are in a state of rigid rotation with uniform angular velocity Ω' about the z'-axis. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium .A transverse magnetic field B_0 of uniform strength is applied normal to the plate in the z′ direction, The induced magnetic field and viscous dissipation is assumed to be negligible. Initially, the plate and fluid were at rest and with the same temperature. At time t'>0, the plate starts moving with a velocity ct' in its own plane in the vertical direction against gravitational field, in the presence of thermal radiation. At the same time the plate temperature is raised or lowered to T'_w which is there after maintained constant. Since the plate occupying the plane z'=0 is of infinite extent, all the physical quantities depend only on z' and t'. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} - 2\Omega v' = g\beta(T' - T'_\infty) + \nu\frac{\partial^2 u'}{\partial z'^2} - \sigma\frac{B_0^2}{\rho}u' \qquad (1)$$

$$\frac{\partial v'}{\partial t'} + 2\Omega u' = \nu\frac{\partial^2 v'}{\partial z'^2} - \sigma\frac{B_0^2}{\rho}v' \qquad (2)$$

$$\rho C_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \qquad (3)$$

The term $\frac{\partial q_r}{\partial z'}$ represents the change in the radiative flux with distance normal to the plate with the following initial and boundary conditions:

$$\begin{array}{llll} t' \le 0: & u' = 0, \quad v' = 0, & T' = T'_\infty, & \text{for all } z' \\ t' > 0: & u' = ct', \quad v' = 0, & T' = T'_w, & \text{at } z' = 0 \\ & u \to 0, \quad v' \to 0 & T \to T_\infty, & \text{as } z' \to \infty. \end{array} \qquad (4)$$

By Rosseland approximation, radiative heat flux of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z'} = -4a^*\sigma\left(T'^4_\infty - T'^4\right) \qquad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \qquad (6)$$

By using equations (5) and (6), equation (3) reduces to

$$\rho C_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + 16a^*\sigma T'^3_\infty(T'_\infty - T') \qquad (7)$$

On introducing the following dimensionless quantities

$$(u, v) = \frac{(u', v')}{(\nu c)^{\frac{1}{3}}}, \quad t = t'\left(\frac{c^2}{\nu}\right)^{\frac{1}{3}}, \quad z = z'\left(\frac{c}{\nu^2}\right)^{\frac{1}{3}}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$Gr = \frac{g\beta(T'_w - T'_\infty)}{c},$$

$$M = \frac{\sigma B_0^2 u'}{\rho} \qquad (8)$$

$$Pr = \frac{\mu C_p}{k}, \quad \Omega = \Omega'\left(\frac{\nu}{c^2}\right)^{\frac{1}{3}}, \quad R = \frac{16a^*\nu\,\sigma T'^3_\infty}{k}\left(\frac{\nu}{c^2}\right)^{\frac{1}{3}}.$$ and

the complex velocity $q = u + iv$, $i = \sqrt{-1}$ in equations (1) to (4), the equations relevant to the problem reduces to

$$\frac{\partial q}{\partial t} + 2i\Omega q = Gr\ \theta + \frac{\partial^2 q}{\partial z^2} - Mq, \qquad (9)$$

$$\frac{\partial\theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial z^2} - \frac{R}{Pr}\theta \qquad (10)$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{lllll} & q = 0, & \theta = 0, & \text{for all} & z \le 0 \,\&\, t \le 0 \\ t > 0: & q = t, & \theta = t, & at & z = 0 \\ & q = 0, & \theta \to 0, & as & z \to \infty. \end{array} \qquad (11)$$

All the physical variables are defined in the nomenclature. The solutions are obtained for the equations (9) to (10), subject to the boundary conditions (11), by Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{t}{2}\left[\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{\Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{\Pr} - \sqrt{at})\right]$$
$$- \frac{\eta\, Pr\sqrt{t}}{2\sqrt{R}}\left[\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{\Pr} - \sqrt{at}) - \exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{\Pr} + \sqrt{at})\right] \qquad (12)$$

$$q = \frac{1}{2}\left(t + \frac{Gr}{b^2(1 - Pr)} + \frac{tGr}{b(1 - \Pr)}\right)\left[\exp(2\eta\sqrt{mt})\,erfc(d1) + \exp(-2\eta\sqrt{mt})\,erfc(d2)\right]$$
$$- \frac{\eta\sqrt{t}}{2\sqrt{m}}\left(1 + \frac{Gr}{b(1 - \Pr)}\right)\left[\exp(-2\eta\sqrt{mt})\,erfc(d2) - \exp(2\eta\sqrt{mt})\,erfc(d1)\right]$$
$$- \frac{Gr\exp(bt)}{2b^2(1 - \Pr)}\left(\begin{array}{l}\left[\exp(2\eta\sqrt{(b + m)t})\,erfc(d3) + \exp(-2\eta\sqrt{(b + m)t})\,erfc(d4)\right] \\ - \exp(2\eta\sqrt{\Pr(b + a)t})\,erfc(d7) - \exp(-2\eta\sqrt{\Pr(b + a)t})\,erfc(d8)\end{array}\right)$$
$$- \frac{Gr}{2b^2(1 - \Pr)}(1 + tb)\left(\exp(2\eta\sqrt{Rt})\,erfc(d5) + \exp(-2\eta\sqrt{Rt})\,erfc(d6)\right) \qquad (13)$$
$$+ \frac{Gr}{b(1 - \Pr)}\frac{\eta\sqrt{t}\sqrt{\Pr}}{2\sqrt{a}}\left[\exp(2\eta\sqrt{Rt})\,erfc(d5) - \exp(-2\eta\sqrt{Rt})\,erfc(d6)\right]$$

where $d1, d2 = \left[\eta \pm \sqrt{mt}\right]$, $d3, d4 = \left[\eta \pm \sqrt{(b + m)t}\right]$

$d5, d6 = \left[\eta\sqrt{\Pr} \pm \sqrt{at}\right]$, $d7, d8 = \left[\eta\sqrt{\Pr} \pm \sqrt{(a + b)t}\right]$

$a = \frac{R}{\Pr}$, $b = \frac{R - m}{1 - \Pr}$, $m = 2i\,\Omega + M$ *and* $\eta = \frac{z}{2\sqrt{t}}$

In equation (13), the argument of the complementary error function and error function is complex. Hence in order to obtain the u and v components of the velocity we have used the following formula due to Abramowitz and stegun (1964):

$$erf(a+ib) = erf(a) + \frac{\exp(-a^2)}{2a\pi}[1-\cos(2ab) + i\sin(2ab)] + \frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}[f_n(a,b) + ig_n(a,b)] + \varepsilon(a,b)$$

Where, f_n = 2a – 2a cosh (nb) cos(2ab) + n sinh(nb) sin(2ab),g_n = 2a cosh(nb) sin(2ab) + n sinh(nb) cos(2ab)

$$|\varepsilon(a,b)| \approx 10^{-16}|erf(a+ib)|$$

3. Discussion of Results

Using the above formula, expressions for u, v are obtained but they are omitted here to save the space. In order to get a physical view of the problem, these expressions are used to obtain the numerical values of u, v, τ_x and τ_y for different values of the parameter like rotation, radiation, and thermal Grashof number.

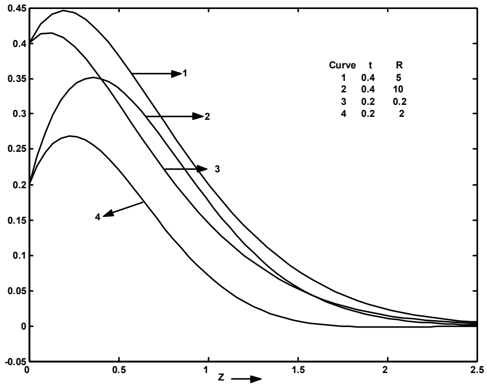
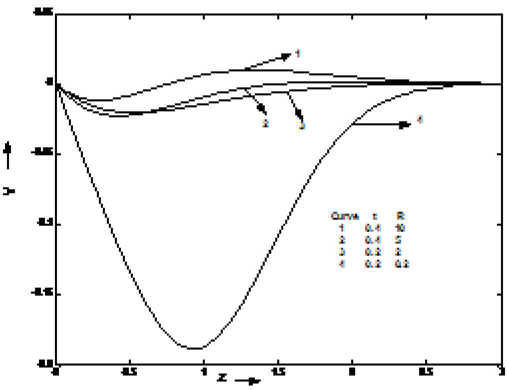
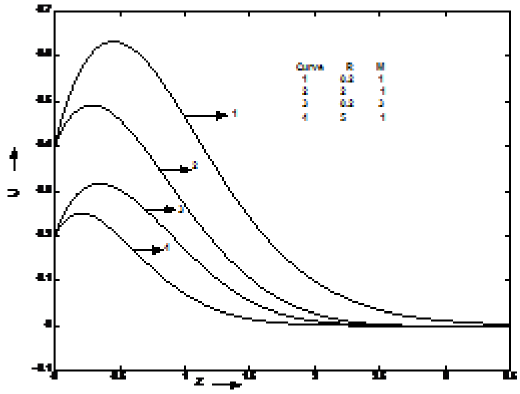


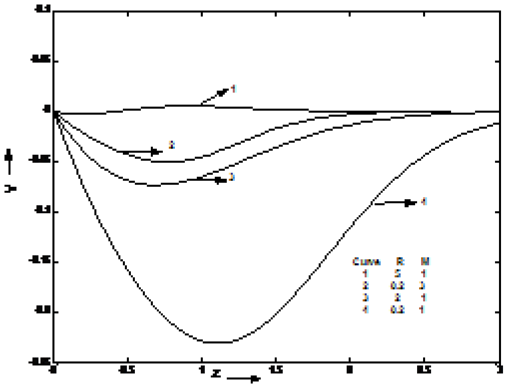
Fig1:Primary velocity profiles for different R



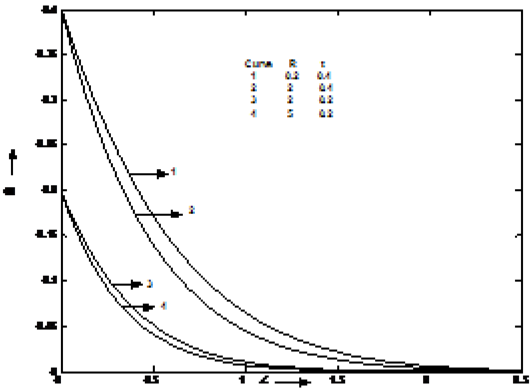
The primary velocity profiles of air for different values radiation parameter are shown in Fig. 1. It is observed that the primary velocity increases with decreasing radiation parameter R in cooling of the plate. This shows that primary velocity decreases in the presence of high thermal radiation. It is also observed that greater cooling of the plate, due to free convection currents, increases the primary velocity of the plate. in Fig.2.the effect of radiation parameter R increases the secondary velocity.



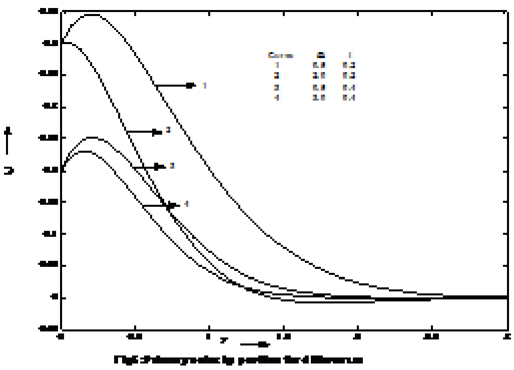
The primary velocity profiles of air for different values radiation parameter and magnetic parameter are shown in Fig. 3. It is observed that the primary velocity increases with decreasing radiation parameter R and Magnetic parameter M in cooling of the plate.



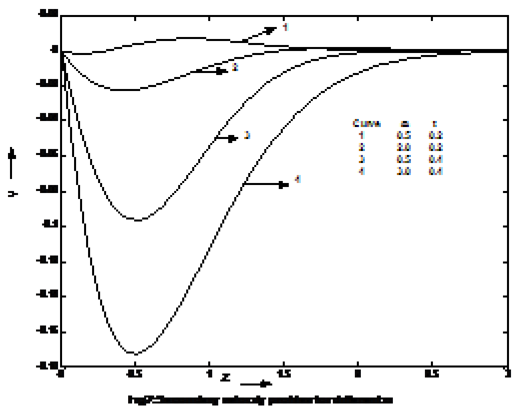
The secondary velocity profiles of air for different values of the magnetic field and radiation parameter are shown in Fig.4, the effect of magnetic field and radiation increases the secondary velocity v.



The temperature profiles for air (Pr = 0.71)are calculated for different values of thermal radiation parameter from Equation (12) and these are shown in Fig.5. The effect of thermal radiation is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.



The primary velocity profiles of air for different values of the rotation parameter are shown in Fig. 6. It depicts that the primary velocity increases with decreasing rotation parameter Ω. It is also observed that greater cooling of the plate, due to free convection currents, increases the primary velocity of the plate in this case too. . Fig. 7. shows the effect of rotation on v which is just reverse to that of radiation parameter. Further, greater cooling of the plate, due to free- convection currents, decreases the secondary velocity of the plate.



4. Conclusions

Theoretical analysis is performed to study flow past an accelerated infinite vertical plate with uniform temperature, in the presence of thermal radiation in a rotating fluid. The dimensionless governing equations are solved by Laplace-transform technique. The conclusions of the study are as follows:

(i) The influence of the magnetic field, radiation or rotation parameter on primary flow has a retarding effect for cooling of the plate.
(ii) As time advances, the influence of the radiation or rotation parameter on primary flow increases velocity but the trend is reversed in secondary flow.
(iii) The secondary velocity is enhanced with the raise in thermal radiation and opposite phenomenon occurs with the rotation parameter.

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