

# Allocation of Transmission Loss and Transmission Usage Charge to Participants in Restructured Power System

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**Abstract:** *In the present open access restructured power system, it is necessary to develop an appropriate pricing and loss allocation scheme that can provide the useful economic information to the market participants. Transmission system is the most important key for competition in market. The main objective of the paper is to propose a fair, transparent and equitable method to charge transmission facility users and to allocate loss for users Here two methods has been proposed 1.Tracing Method 2. Z-bus Method. A numerical example is considered to demonstrate the proposed method.*

**Keywords:** Generation centre, Demand centre, Loss Factors, Gross flow, Net flow

## Nomenclature

I,j,k Index for bus GENCO/DISCO

C Cost to be collected for usage of line

$[A_u]$  Upstream distribution matrix

$[A_d]$  Downstream distribution matrix

$p_{i-m}^{gross}$  Gross line flow in a line i-m

$p_{i-m}^{net}$  Net flow in a line i-m

$\alpha_{gr}$  Set of transmission lines used by generator at bus r to supply the demands

$\alpha_{dr}$  Set of transmission lines which are used to feed the demand at bus r

$a_{jk}^i$  Electrical distance between bus i and line jk

$C^{Gi}$  Total transmission cost allocated to the generator located at bus i(\$/h)

$C^{Di}$  Total transmission cost allocated to the demand located at bus i(\$/h)

$U_{jk}^{Gi}$  Usage of line jk allocated to the generator located at bus i(W)

$U_{jk}^{Di}$  Usage of line jk allocated to the demand located at bus i(W)

$U_{jk}^i$  Usage of line jk associated with nodal current i(W)

## 1. Introduction

In the past, the transmission losses arising from vertically integrated utilities gave rise to the concern for reducing the costs for the overall system. Attempts were made to use all kinds of optimization methods to carry out the most efficient strategy of reducing the production cost to the minimum. However, the role of system operation has been changed after the deregulation of the power industry. Its principal obligation is now shifted to offer a secure and fair platform for all the market participants as well as to enhance electricity production efficiency through competition. The fair allocation of loss and usage charge becomes an important issue. Here tracing method and z-bus method has been proposed for loss allocation and allocation of usage charge.

The tracing method permits calculation of the real and reactive power contributions of individual generators and loads to flow in all network lines. hence it can be used to allocate the transmission loss and use-of-system charges to individual generators and loads. Electricity tracing is based on the proportional sharing rule. The rule of proportional sharing can be extended to all the network nodes and allows electricity to be tracked in the network by a series of recursive calculation. There are two algorithms in electricity tracing, Downstream tracing algorithm and upstream tracing algorithm. Downstream tracing algorithm is used to trace electricity from the generator to the load centre similarly upstream tracing algorithm is used to trace electricity from the load to the generation centre. Transmission usage pricing is done by determining contribution of participants by using upstream and downstream algorithm.

This paper uses loss factors which is representative of the impact of average system losses caused by participants. These loss factors computed with reference to a specific location for balancing generation, load and loss at a unique delivery/withdrawal point for energy called as "market centre". Two sets of average loss factors are calculated with reference to the market centre. In tracing method loss factor for genco and disco is different even though they are connected to the same bus. These loss factors are used to allocate transmission loss to participants. Z-bus method is based on the network Z-bus matrix, although all required computations exploit the sparse Y-bus matrix. One innovative feature and advantage of this method is that, it exploits the full set of network equations and does not require any simplifying assumptions. This method is based on a solved load flow solution. The method emphasizes current rather than power injections.

## 2. Transmission Loss Allocation

### 2.1 Tracing

Flow tracing method can be used to trace a flow from a generator to the load centre through various lines in the

network and is known as downstream tracking. Tracing method can also be used to trace a flow from a load to the generation centre through the various lines in the network and is known as upstream tracking. This flow can either be gross flow, average flow or net flow. In flow tracing each branch loss is assigned to one of the adjacent buses in the direction of tracking, and then redistributed among lines proportionally to the flows, again in the direction of tracking.

### 2.2 Downstream tracing

Loss factor for generators is calculated using gross flows. In a network of  $nb$  buses and  $nl$  transmission lines, the gross nodal flow at node  $j$  is  $P_j^{gross}$ . Gross flow is the total power through the node when the network with actual generation and no power loss in the network.

While considering inflows to the bus  $j$ , is expressed as,

$$P_j^{gross} = \sum_{m \in \alpha_j^{(u)}} |p_{j-m}^{gross}| + P_{gj}; j = 1 \dots nb \quad (1)$$

Relating line flow in the loss incorporated network to nodal flow,

$$P_{j-m}^{gross} = \frac{|p_{j-m}^{gross}|}{P_m^{gross}} * P_m^{gross} \quad (2)$$

$$C_{mj}^{gross} = \left( \frac{|p_{j-m}^{gross}|}{P_m^{gross}} \right) \quad (3)$$

Substituting equation (3) in (2),

$$P_{j-m}^{gross} = (C_{mj}^{gross}) * P_m^{gross} \quad (4)$$

Taking into account (4) and rearranging equation (1),

$$P_j^{gross} - \sum_{m \in \alpha_j^{(u)}} C_{mj}^{gross} P_m^{gross} = P_{gj} \quad (5)$$

In matrix form (5) can be written as,

$$[AU_u] \underline{p}^{gross} = \underline{p}_g \quad (6)$$

The elements of  $AU_u$  is equal to,

$$[AU_u]_{mj} = \begin{cases} 1 \text{ for } i = j \\ -C_{mj}^{gross} \text{ for } j \in \alpha_j^{(u)} \\ 0 \text{ otherwise} \end{cases} \quad (7)$$

The element  $j$  of the gross nodal flow vector is,

$$P_j^{gross} = \sum_{n=1}^{nb} [A_u^{-1}]_{jr} P_{gn} \quad (8)$$

The (8) shows the contribution of the  $n^{th}$  bus generator to the gross nodal flow  $j$ . The loss due to the line outflow in the line  $m-l$  from node  $m$  can be calculated using the proportional sharing, as

$$P_{m-l}^{loss} = \frac{p_{m-l}^{loss}}{P_m^{gross}} * P_m^{gross} \quad (9)$$

Substituting (8) in (9),

$$P_{m-l}^{loss} = \frac{p_{m-l}^{loss}}{P_m^{gross}} * \sum_{r=1}^{nb} [A_u^{-1}]_{mn} * P_{gn}; \quad (10)$$

The portion of network loss owing to the  $n^{th}$  bus generator due to the flow in line  $m-l$  is,

$$\frac{p_{m-l}^{loss}}{P_m^{gross}} * [A_u^{-1}]_{jr} * P_{gn} \quad (11)$$

The loss factor of the  $m^{th}$  bus generator is obtained by adding the contribution of the loss in all the transmission lines used by the generator to deliver energy to load centre and is expressed as,

$$LFG_m = \sum_{m \in \alpha_{gr}} \frac{P_m^{loss}}{P_m^{gross}} [A_u^{-1}]_{mn} \quad (12)$$

Loss allocated to  $n^{th}$  bus generator is obtained as follows,

$$Lg_n = LFG_n * P_{gn} \quad (13)$$

### 2.3 Upstream tracing

Similar to downstream tracing an upstream tracing is done using net flows to calculate the loss factors for the demand. The loss factor of the  $n^{th}$  bus demand is obtained by adding the contribution of the loss in all transmission lines used by demand to draw energy from generation centre and is expressed as,

$$LFD_n = \sum_{m \in \alpha_{dr}} \frac{P_m^{loss}}{P_m^{net}} [AD_d^{-1}]_{mn} \quad (14)$$

Loss allocated to  $n^{th}$  bus demand is obtained as follows,

$$Ld_n = LFD_n * P_{dn} \quad (15)$$

### 2.4 Z-Bus Method

The Z-bus loss allocation method uses a solved power flow to systematically distribute transmission losses,  $P_{loss}$  among the participants according to,

$$P_{loss} = \sum_{m=1}^n L_m \quad (16)$$

The loss component  $L_m$  is the fraction of the system losses allocated to the net injection at bus  $m$ . The system real losses can be expressed in terms of through Z and I.

$$P_{loss} = \text{real}\{\sum_{m=1}^n V_m I_m^*\} \quad (17)$$

The system losses can be expressed, through Z and I,

$$P_{loss} = \text{real}\{\sum_{m=1}^n I_m^* (\sum_{j=1}^n Z_{mj} I_j)\} \quad (18)$$

Equation (18) is separated into two summations, one due to resistance R and other to the reactance matrix, X.

$$P_{loss} = \text{real}\{\sum_{m=1}^n I_m^* (\sum_{j=1}^n R_{mj} I_j)\} + \text{real}\{\sum_{m=1}^n I_m^* (\sum_{j=1}^n X_{mj} I_j)\} \quad (19)$$

Second summation is equal to zero so the equation becomes,

$$P_{loss} = \text{real}\{\sum_{m=1}^n I_m^* (\sum_{j=1}^n R_{mj} I_j)\} \quad (20)$$

The loss component associated with bus  $m$  can be expressed as,

$$L_m = \text{real}\{\sum_{m=1}^n I_m^* (\sum_{j=1}^n R_{mj} I_j)\} \quad (21)$$

System losses in loss term depend on complex bus current injections.

This step assigns to each individual bus  $m$ , the responsibility of paying for  $L_m$  at the market marginal price. If the given bus  $m$  has both generation,  $P_{gm}$  and demand,  $P_{dm}$ , the allocated loss component is divided as follows

$$\gamma_m = \frac{P_{gm}}{P_{gm} - P_{dm}} \quad (22)$$

Loss allocated to generator at bus  $m$  is  $\gamma_m L_m$  and loss allocated to demand at bus  $k$  is  $(1-\gamma_m) L_m$ .

## 3. Transmission Usage Cost Allocation

### 3.1 Tracing Method

Downstream tracing

Downstream algorithm is used to charge the generators for usage of transmission network.

$$DG_{i-m,k} = \frac{P_{i-m}^{(gross)} [A_u^{-1}]_{ik}}{P_i^{(gross)}} \quad (23)$$

This equation defines  $DG_{i-m,k}$  as the tracking-based generation distribution factor is a proportion of generation due to  $k$ -th generator flows in line  $i-m$ . The tracking-based distribution factors are positive because they are based on

topological analysis of network flows and represent the contribution of a particular generation in the total line flow. Now the transmission charge of the  $k$ -th generator can be calculated by summing up all contributions of the generator in all the lines of the network,

$$U_{Gk} = \sum_{l \in \alpha_{gr}} DG_{i-l,k} * C_{i-l,k} \quad (24)$$

Upstream tracing

Upstream algorithm is used to charge the loads for usage of transmission network.

$$D_{i-m,k} = \frac{P_{i-m}^{(net)} [A_d^{-1}]_{ik}}{P_i^{(net)}} \quad (25)$$

This equation determines  $DD_{i-m,k}$  as the tracking based distribution factor that is the portion of  $k$ -th load demand that flows in line  $i-m$ . The tracking-based factor is always positive. The total usage of the network, by  $k$ -th load is calculated by summing up the contribution of the load in all the lines,

$$U_{Dk} = \sum_{l \in \alpha_{dr}} DD_{i-j,k} * C_{i-j,k} \quad (26)$$

### 3.2 Z- Bus Method

The method uses the power flow solution to calculate how line flow depends on nodal current. This result is then used to allocate network costs to generators and demands.

Consider the complex power

$$S_{ji} = V_j I_{ji}^* \quad (27)$$

Using the Z-bus matrix, the voltage at node  $k$  is given by

$$V_k = \sum_{i=1}^n Z_{ki} I_i \quad (28)$$

The current through the line  $ij$  is

$$I_{ij} = (V_i - V_j) y_{i-j} + V_i y_{i-j}^{sh} \quad (29)$$

Substituting (28) in (29)

$$I_{jk} = (\sum_{i=1}^n Z_{ji} I_i - \sum_{i=1}^n Z_{ki} I_i) y_{j-k} + \sum_{i=1}^n Z_{ji} I_i y_{j-k}^{sh} \quad (30)$$

Rearranging (30)

$$I_{jk} = \sum_{i=1}^n [(Z_{ji} - Z_{ki}) y_{j-k} + Z_{ji} y_{j-k}^{sh}] I_i \quad (31)$$

First term in (31) is constant as it depends only on network parameters.

$$I_{jk} = \sum_{i=1}^n e_{jk}^i I_i \quad (32)$$

Where

$$e_{jk}^i = (Z_{ji} - Z_{ki}) y_{j-k} + Z_{ji} y_{j-k}^{sh} \quad (33)$$

Substituting (32) in (27)

$$S_{jk} = V_j \sum_{i=1}^n (e_{jk}^i I_i)^* \quad (34)$$

Then, the active power flow through line  $jk$  is

$$P_{jk} = \text{real} \{ \sum_{i=1}^n V_j e_{jk}^i I_i^* \} \quad (35)$$

Usage of line  $jk$  due to nodal current  $i$  as the absolute value of the active power flow component

$$UB_{jk}^i = |P_{jk}^i| \quad (36)$$

The total usage of line  $jk$  is then

$$U_{jk} = \sum_{i=1}^n U_{jk}^i \quad (37)$$

If bus  $i$  contains only generation, the usage allocated to generator  $i$  to line  $jk$  is

$$UG_{jk}^{Gi} = U_{jk}^i \quad (38)$$

On the other hand if bus  $i$  contains only demand, the usage allocated to demand  $i$  to line  $jk$  is

$$UD_{jk}^{Di} = U_{jk}^i \quad (39)$$

If bus  $i$  contains both generation and demand, the usage allocated to the generation at bus  $i$  to line  $jk$  is

$$UG_{jk}^{Gi} = \left[ \frac{P_{Gi}}{P_{Gi} + P_{Di}} \right] U_{jk}^i \quad (40)$$

and the usage allocated to the demand at bus  $i$  to line  $jk$  is

$$UD_{jk}^{Di} = \left[ \frac{P_{Di}}{P_{Gi} + P_{Di}} \right] U_{jk}^i \quad (41)$$

The corresponding cost rate for line  $jk$  is

$$R = C_{jk} / U_{jk} \quad (42)$$

The cost of line  $jk$  allocated to the generator located at bus  $i$  is

$$CG_{jk}^{Gi} = R_{jk} U_{jk}^{Gi} \quad (43)$$

The cost of line  $jk$  allocated to the demand at bus  $i$  is

$$CD_{jk}^{Di} = R_{jk} U_{jk}^{Di} \quad (44)$$

The cost allocated to the generator at bus  $i$  is

$$CGB^{Gi} = \sum_{(j,k) \in \alpha_L} R_{jk} U_{jk}^{Gi} \quad (45)$$

Similarly, the total transmission cost allocated to the demand located at bus  $i$  is

$$CDB^{Di} = \sum_{(j,k) \in \alpha_L} R_{jk} U_{jk}^{Di} \quad (46)$$

## 4. Results and Discussion

To illustrate the working of loss allocation methods we consider the pjm five-bus system depicted in fig. 1. Line data for five-bus system is given in table 1.

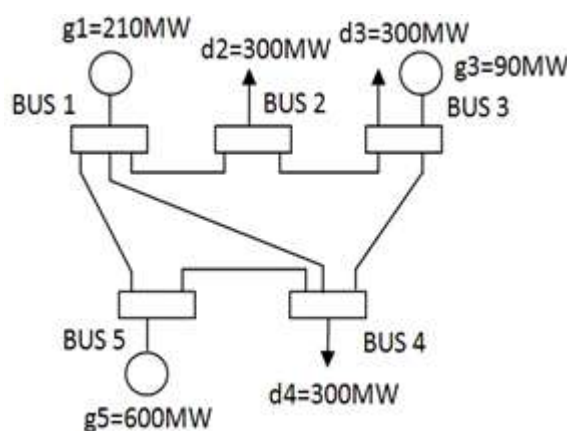


Figure 1: Pjm five-bus system

Table 1: line data

Line no	From bus	To bus	R(p.u)	X(p.u)	shunt data	Tap ratio
1	1	2	0.0281	0.0281	0.01	1
2	1	4	0.0304	0.0304	0.01	1
3	5	1	0.00064	0.00064	0.01	1
4	2	3	0.00108	0.00108	0.01	1
5	4	3	0.00297	0.00297	0.01	1
6	5	4	0.00297	0.00297	0.01	1

Table 2: loss allocated to generators

Tracing method(MW)	Z bus method(MW)
0.91746	0.9080
0	0
0	-0.4733
0	0
3.0196	3.7901
Total = 3.93706	Total = 4.2248

Table 3: loss allocated to demands

Tracing method(MW)	Z bus method(MW)
0	0
1.6684	1.5587
0.94879	1.5777
1.3199	0.3847
0	0
Total = 3.93709	Total = 3.5211

Total loss in load flow solution is 7.874MW, the total loss allocated in tracing method is 7.87 MW and the total loss

allocated in Z bus method is 7.7459. Loss allocated to generator at bus 3 in Z-bus method is negative because the generator helps the flow in the line, where as the loss allocated in tracing method is zero in bus 3 because the generator is near to the load centre. The loss allocated to demand at 4<sup>th</sup> bus is higher because the load is far away from the generation centre. Tracing method traces the actual usage of line and allocates the loss to the participants. Tracing method is fair method than Z bus method as it does not allocate any negative loss to participants.

To illustrate the working of transmission cost allocation method we consider sample four-bus system depicted in fig. 2. The values of resistances and reactances:0.01275 and 0.097 p.u., respectively is same for all lines, and the shunt admittance is same for all the five lines: 0.4611 p.u., cost for each line is considered to be proportional to its series reactance of line. Voltage at all buses is taken as 1.00.

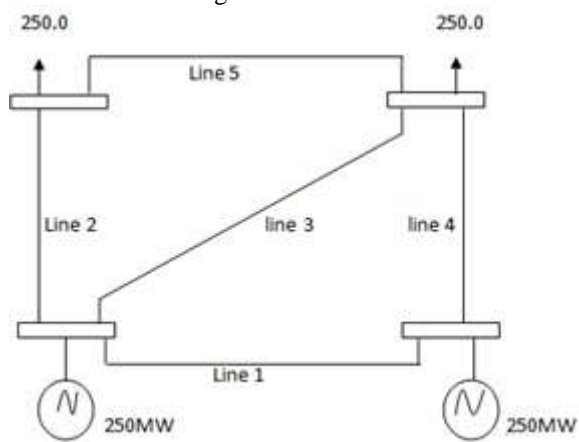


Figure 2: four-bus system

Table 4: transmission usage cost for generators

Tracing methos(\$)	Zbus method(\$)
213.0265	131.08
157.33	182.5
0	0
0	0

Table 5: transmission usage cost for demands

Tracing method(\$)	Zbus method(\$)
0	0
0	0
39.2722	77.92
75.5462	93.55

Total cost collected from participants in tracing method is 485.1749 where as the total cost allocated in Z bus method is 485.05.

## 5. Conclusion

Transmission usage cost and loss allocation is done by using tracing method and Zbus method. And a numerical example is considered to demonstrate both methods. From the example it is clear that the loss allocation and transmission usage charging by proposed methods is in equitable and fair.

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