Design of Decoupled PI Controller for Quadruple Tank System

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Abstract: ***This work is present review of various methods for the design of controllers for a Multiple-input Multiple-output system (MIMO). The quadruple tank process is laboratory equipment which has been used in control literature to illustrate many concepts in MIMO systems. The decentralized controller design, and decoupling controller design are analyzed and their performances are evaluated using IAE and ISE criteria. The main work of the project is to design the decentralized with decoupled PI for Quadruple tank process for both minimum and non-minimum phase. PI controller is used to control the process. In order to design the PI controller are basically emphasized on the tuning parameters of the PI. To get the tuning parameters, various tuning methods are used such as direct synthesis, Sequential Relay with ZN settings, IMC based PI method are used in this process for getting the PI parameter for both minimum and non-minimum phase.***

Keyword: MIMO, Decoupler, Decentralized control, direct synthesis, IMC-PID

1. Introduction

In process control industry, more than 95% of the control loops are of PI/PID (Proportional plus Integral /Proportional plus Integral plus Derivative) type. This is mainly attributed to its effectiveness and relatively simple structure, which can be easily understood and implemented in practice. Consequently, the research on PID control algorithm development and their applications is still a very active area; many formulas have been derived to tune the PID controllers over the years. Due to the high product quality and energy integration requirements, most of modern industry processes, however, are Multi-Input Multi-Output (MIMO) processes. For easier field implementation, it is desirable to apply well established single loop PID tuning principles to these MIMO processes. However, compared with Single-Input Single-Output (SISO) [9] counterparts, MIMO systems are more difficult to control due to the existence of interactions between input and output variables. Adjusting controller parameters of one loop affects the performance of the others, sometimes to the extent of destabilizing the entire system practical approach. Since controllers interact with each other in a decentralized control system, the performance of one loop cannot be evaluated without the information of the controllers of other loops. Before a fully interacting multivariable design is considered, it is useful to check if a completely decentralized design can achieve the desired performance objectives. The advantages of a fully decentralized controller to a centralized controller is that, it is simpler to understand, easier to maintained, stable in case of any loop failure and can be enhanced in a straight forward fashion when a plant is upgraded. The key simplifying assumption is that in completely decentralized control interactions are considered as disturbances. Though this is conceptually not correct as disturbances are independent inputs, in some cases this form gives satisfactory results. We treat the MIMO plant as a full multivariable system which can be refined into sets of smaller multivariable systems that show little interactions to each other. Because of such conveniences most of the real world examples use decentralized control. This Multi Input –Multi Output control strategy may be implemented for either time domain or Laplace transform models. For multi input-multi output systems such as Quadruple tank process, a commonly employed linear model takes the form.

2. Quadruplle Tank Process

In this section we derive a mathematical model for the quadruple-tank process from physical data. The quadruple tank experiments consist of four water tanks and two pumps [3] (see Fig.1. for a system schematic). The aim is to control the water level in the lower tanks (tank 1 and 2) with the two pumps. The inputs of the process to control are the input voltages of the pumps v1 and v2. The output are the corresponding water level in the tanks h1 and h2.The flow from pump j is split up in a part proportional to γj and a part proportional to 1- γj. The flows of the pumps are split up by valves. The flow of pump 1 goes into tanks 1 and 4 whereas pump 2 feeds tanks 2 and 3.

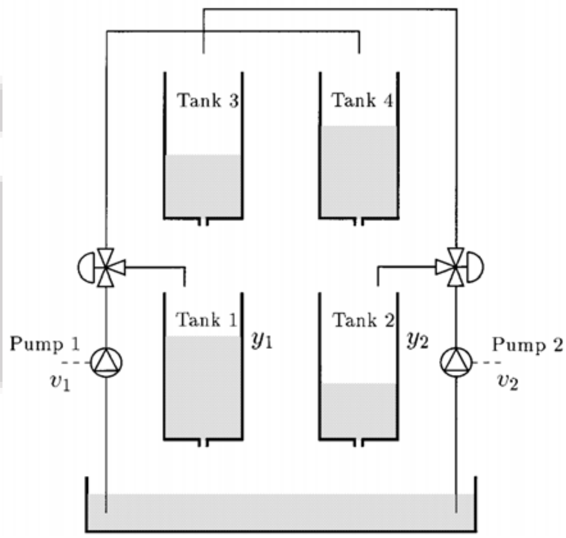


Figure 1: Schematic diagram of quadruple tank process

There are two valve section in the setup each consisting of two manually operated valves.[1] Changing the flow ratios of the valve sections makes the system minimum or non-minimum phase. The minimum and non-minimum phase mode can be achieved as

Minimum Phase: **1 < (γ 1 + γ 2) < 2**
Non-minimum Phase: **0 < (γ 1 + γ 2) < 1**

Mass balance and Bernolli's law yields [1] non-linear plant equation as following

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}\upsilon_1$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_4}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}\upsilon_2$$
$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}\upsilon_2$$
$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}\upsilon_1$$
$$y_1(t) = k_c h_1(t) + e_1(t)$$
$$y_2(t) = k_c h_2(t) + e_2(t) \tag{1}$$

The process transfer function of the quadruple-tank process is given by for both minimum and non-minimum phase [1]. The linearized state-space is then given by

$$\frac{dh}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} h + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 K_2}{A_2} \\ 0 & \frac{(1-\gamma_2)K_2}{A_3} \\ \frac{(1-\gamma_1)K_1}{A_4} & 0 \end{bmatrix} \upsilon$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & kc & 0 & 0 \end{bmatrix} h$$

Where the time constant are

$$T_i = \frac{A_i}{a_i}\sqrt{2h_i^0}, i = 1,\ldots 4. \tag{2}$$

The corresponding transfer matrix is

$$G(S) = \begin{bmatrix} \frac{\gamma_1 c_1}{1+sT_1} & \frac{(1-\gamma_2)c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \tag{3}$$

Where c_1 and c_2 are,

$$c_1 = \frac{T_1 K_1 K_C}{A_1}, c_2 = \frac{T_2 K_2 K_C}{A_2} \tag{4}$$

Using the 2, 3, 4 equation find the transfer function for minimum and non minimum phase and using the parameters shown in [1]

For Minimum Phase

$$G_{P11} = \frac{2.57}{62.7s+1}, G_{P12} = \frac{1.5}{(23.8s+1)(62s+1)}$$
$$G_{P21} = \frac{1.4}{(30s+1)(90s+1)}, G_{p22} = \frac{2.8}{90s+1}$$
$$G-(s) = \begin{bmatrix} \frac{2.57}{62.7s+1} & \frac{1.5}{(23.8s+1)(62s+1)} \\ \frac{1.4}{(30s+1)(90s+1)} & \frac{2.8}{90s+1} \end{bmatrix} \tag{5}$$

For Non-Minimum Phase

$$G_{P11} = \frac{1.52}{63s+1}, G_{P12} = \frac{2.33}{(39s+1)(63s+1)}$$
$$G_{P21} = \frac{2.67}{(56s+1)(91s+1)}, G_{p22} = \frac{1.6}{91s+1}$$
$$G+(s) = \begin{bmatrix} \frac{1.52}{63s+1} & \frac{1.5}{(39s+1)(63s+1)} \\ \frac{2.67}{(56s+1)(91s+1)} & \frac{1.6}{91s+1} \end{bmatrix} \tag{6}$$

3. Decoupled Control System Design

In case RGA shows that there is a strong interaction for all the possible pairings, to minimize the interaction we have to design a Decoupler [2]. Consider a two loop system shown in Fig.1. The pairing of input-output is done by RGA analysis. Consider a multivariable control problem consisting of the design of a linear controller C for a linear stable process G. For simplicity assume the process has two inputs and two outputs, so that G is a transfer function. It shows the 2*2 system which has four processes G_{P11}, G_{P12}, G_{P21}, G_{P22}. The cross decoupler are used in the block diagram such as T_{12} and T_{21}. G_{C1} and G_{C2} is the controller to control the process. The two output of the system is Y_1 and Y_2.

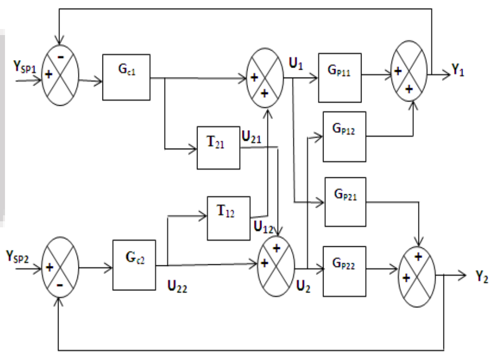


Figure 2: Block diagram for decoupled system

$$G(S)=\begin{bmatrix} G_{P11}(S) & G_{P12}(S) \\ G_{P21}(S) & G_{P22}(S) \end{bmatrix} \quad (7)$$

The controller to be designed is a static decoupler combined with a decentralized controller with set point weighting. The control law can be written as

$$\begin{bmatrix} U_1(S) \\ U_2(S) \end{bmatrix}=\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} G_{c1}Y_{SP1}-G_{C1}Y_1 \\ G_{c2}Y_{SP2}-G_{c2}Y_2 \end{bmatrix} \quad (8)$$

where U is the control signal, Y is the process output and Y_{SP} is the reference. The decoupler is a constant matrix

$$T=\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (9)$$

The controllers are of the form of

$$G_{Ci}(s)=K_{Pi}+\frac{k_{Ii}}{s}+k_{Di}(s) \quad (10)$$

for PI controller K_{Di}=0.

When loop1 is closed, then too overcome the interaction signal effect from loop1, we add decoupler or cross controller T_{21} as shown in Fig.1.That is the following relation can be established.

$$T_{21}=-\frac{G_{P21}}{G_{P22}} \quad (11)$$

It is to be noted that the transfer function of the control valve for loop1 is clubbed together with G_{P11} and that of control value for loop2 is clubbed with G_{P22} for convenience. Similarly, the cross controller T_{12} used to nullify the interaction from loop2 to loop1, is given by

$$T_{12}=-\frac{G_{P12}}{G_{P11}} \quad (12)$$

Consider quadruple tank process is a multivariable system that has been taken the process has the following transfer function G(s) matrix shown in (5) and (6) and get the decoupled matrix for minimum phase (13) and non minimum phase (14)

Decoupled matrix for minimum phase

$$G_{P11}=\frac{2.57}{62.7s+1},G_{P12}=\frac{1.5}{(23.8s+1)(62s+1)}$$
$$G_{P21}=\frac{1.4}{(30s+1)(90s+1)},G_{p22}=\frac{2.8}{90s+1}$$
$$T_{12}=-\frac{0.57692}{1+23s},T_{21}=-\frac{0.5}{1+30s}$$
$$G-(s)=\begin{bmatrix} 1 & -\frac{0.57692}{1+23s} \\ -\frac{0.5}{1+30s} & 1 \end{bmatrix} \quad (13)$$

Decoupled matrix for non minimum phase

$$G_{P11}=\frac{1.52}{63s+1},G_{P12}=\frac{2.33}{(39s+1)(63s+1)}$$
$$G_{P21}=\frac{2.67}{(56s+1)(91s+1)},G_{p22}=\frac{1.6}{91s+1}$$
$$T_{12}=-\frac{1.6666}{1+39s},T_{21}=-\frac{1.5625}{1+52s}$$
$$G+(s)=\begin{bmatrix} 1 & -\frac{1.6666}{1+39s} \\ -\frac{1.5625}{1+52s} & 1 \end{bmatrix} \quad (14)$$

4. Controller Design

The design method is compared with the various tuning method [10] of PI controller design approaches discussed in this chapter

1. The direct synthesis method
2. The relay based auto-tuning approach proposed by wang and
3. The IMC based PID tuning method

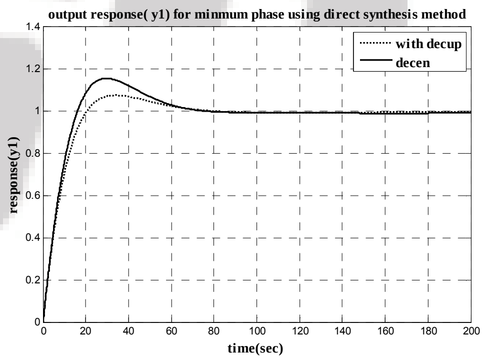
A. Direct Synthesis Method

It turns out that we are not limited by the desired closed loop response, if the system is minimum phase(the process does not have RHP zeros or time delays .the following section present the direct synthesis method for minimum phase systems, and also non-minimum phase systems in the subsequent section[5].

Table 1: Controller Parameter by Direct Synthesis Method

	Minimum Phase		*Non-Minimum Phase*	
PI	Loop1	Loop2(Y2)	Loop1	Loop2
K_P	2.39	3.21	1.36	0.24
τ_I	62	90	102	147

The controller parameters of PI controller such as k_p and k_I is obtained by direct synthesis method of quadruple tank process for both minimum and non minimum phase trasnfer function is shown in the table 1.



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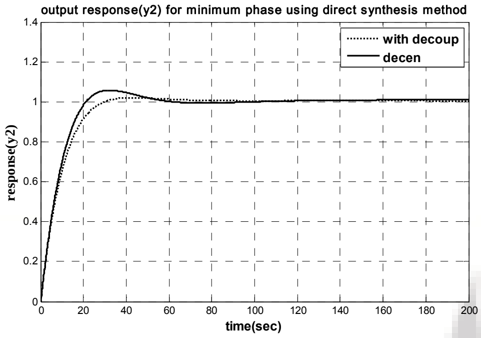

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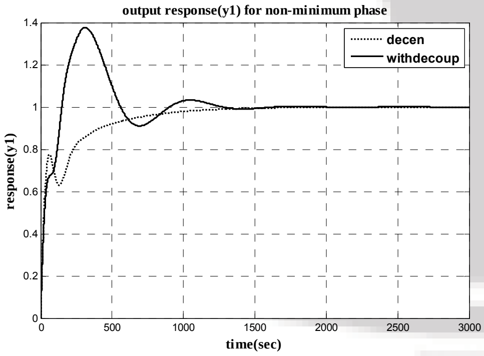
Figure 3 (a) & (b): Output response of y1 and y2 for Minimum phase by direct synthesis method



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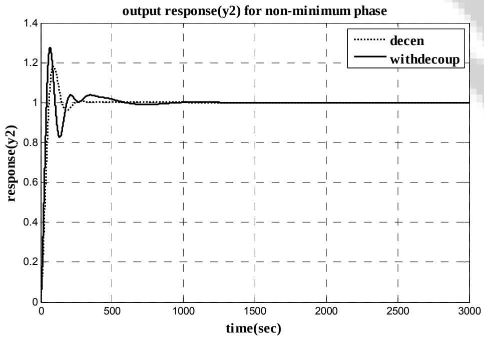

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Figure 4 (a) & (b): Output response of y1and y2 for Non-Minimum phase by direct synthesis

B. Relay Based Sequential Method

Based on the concept of sequential design, a simple method is proposed for MIMO Auto tuning [5] [7] for the purpose of illustration let us consider a 2X2 system with known pairing under decentralized control. Initially a relay is placed between y & u while loop 2 is on manual i.e. open loop following the relay feedback test. A controller can be designed from the ultimate gain and ultimate frequency. The next step is to perform a relay feedback test between y & u while loop I is on automatic. Then a controller can be designed for loop II. Thereafter loop II is on automatic and relay feedback test is performed in loop I. this procedure is repeated iteratively till controller parameters converges. This usually occurs in 3-4 iterations for 2X2 systems.

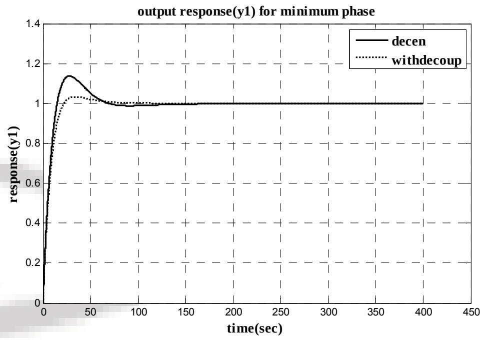
Steps:

1. Using a constant input, say u_o, determine the corresponding steady state output, and say y_o.
2. Next, change the set point to be y_o. Switch the controller to relay mode and choose a small value for h.
3. This will implement the following algorithm:

$$u0 + h, \text{if} \ \ e \geq 0 \ \text{ and } \ u0 - h, \text{if} \ \ e \leq 0$$

From the plot, measure the amplitude of the output cycle, a. Using both a and h, the ultimate gain, K_u, can then be approximated by

$$k_u = \left(\frac{4}{p_i}\right) * \left(\frac{h}{a}\right) \tag{15}$$



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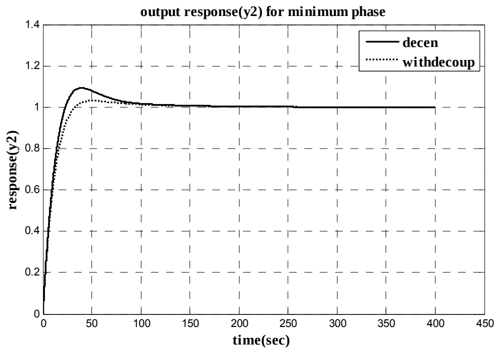

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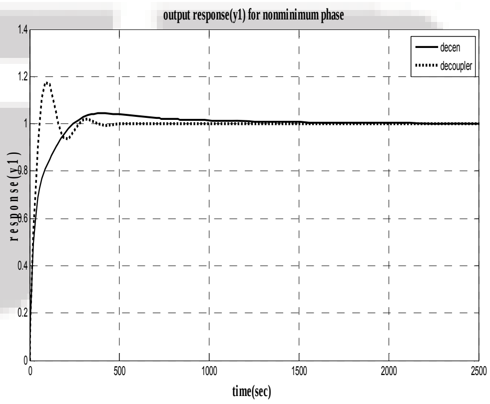
Figure 5 (a) & (b): Output response of y1and y2 for minimum phase



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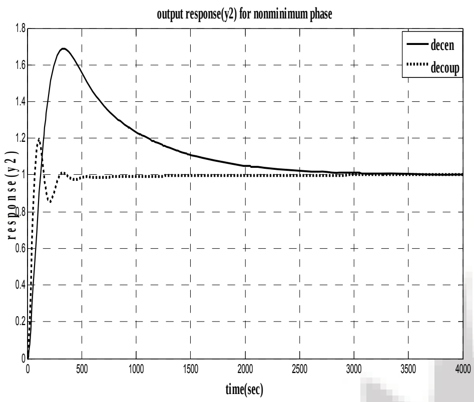

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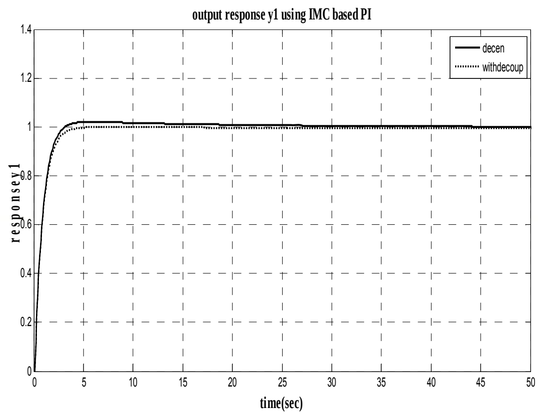
Figure 6 (a) & (b): Output response of y1and y2 for non-minimum phase by relay method

C. IMC Based PID Tuning

The IMC structure can be rearranged to form a standard feedback control system that can easily handle open loop unstable system as not the case with IMC [5, 11, and 12]. This modification of the IMC design procedure is developed to improve the input disturbance rejection. The IMC based PID structure which uses a standard feedback structure uses the process model in an implicit manner i.e. PID tuning parameters are often adjusted based on the transfer function model but it is not always clear how the process model affects the tuning decision. In the IMC procedure the controller Qc(s) is directly based on the good part of the process transfer function. Also the IMC formulation generally results in only one tuning parameter, the close loop time constant (filter tuning factor). The IMC based PID tuning parameters are then the function of this time constant. The selection of the closed loop time constant is directly related to the robustness (sensitivity to the modular of the closed loop system). Also, for open loop unstable processes it is necessary to implement the IMC strategy in standard feedback form, because the IMC suffers from internal stability problems. Though the IMC based PID controller will not give the same performance when there are process time delays because the IMC based PID procedures uses an approximation for the dead time. But if the process has no time delays and the inputs do not hit a constraint then the IMC based PID [4] controller give the same performance as does the IMC.

Table 2: Controller Parameter by IMC Based Tuning Method

	Minimum Phase		*Non-Minimum Phase*	
PI	Loop1	Loop2	Loop1	Loop2
K_P	238	321.42	210	284.3
K_I	3.842	3.57	3.14	3.125



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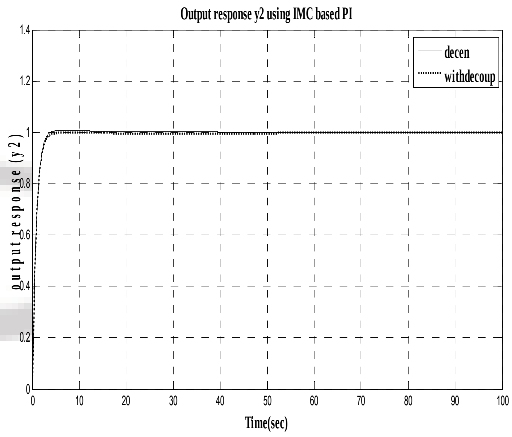

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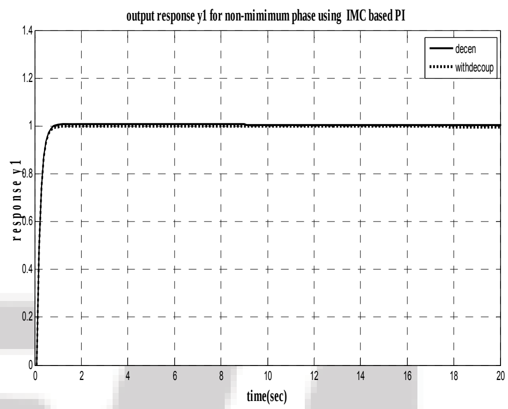
Figure 6 (a) & (b): Output response of y1and y2 for minimum phase by IMC method



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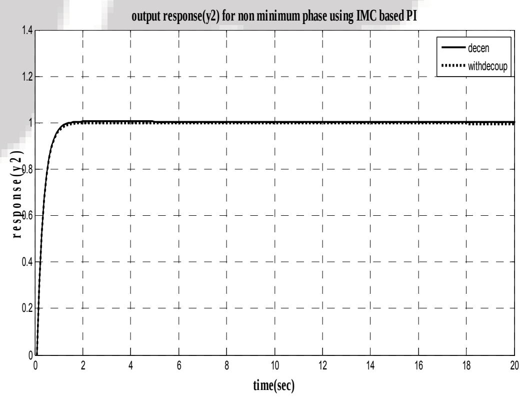

Figure 7 (a) & (b): Output response of y1and y2 for non-minimum phase by IMC method

5. Performance Analysis

A good control performance can be achieved with the proper tuning of the controller constants, but poor performance and controller instability can result from poor choice of values. Before the controller can be tuned, the control problem must be completely defined and the algorithm should be compatible with the type of controller implementation. A well-tuned controller should provide satisfactory performance for one or more measures of the behavior of the controller variable. To evaluate the performance of the controller we can use the dynamic performance criteria like IAE, ISE. These criteria are based on the entire response of the process, unlike other simple criteria that use the isolated characteristics of the response (Decay ratio, settling time etc).ISE is used for suppressing large errors and IAE is a tradeoff between both and generally gives good response.

Table 3: Performance analysis for Minimum Phase system

PI	TYPE	IAE		ISE	
		Y1	Y2	Y1	Y2
A.	Decent	10.2	9.7	4.3	4.6
	Decoupler	11.2	9.5	4.4	4.3
B.	Decent	10.3	3.75	13.4	5.24
	Decoupler	7.78	3.68	12.1	5.4
C.	Decent	0.14	0.11	0.11	0.04
	Decoupler	0.11	0.048	0.117	0.048

Table 3 and table 4 shows the performance analysis for minimum phase and non minimum phase. The performance analysis compares the decentralized controller and decentralized with decoupled controller for both minimum and non-minimum phase based on ISE and IAE for both loop1 and loop2. (A) Represent the direct synthesis method in this method decentralized controller gives the better performance for both minimum and non-minimum phase. (B) Represent the relay based sequential method for both minimum and non minimum phase analysis in this method decentralized with decoupler shows the better performance for both the loops y1 and y2.(c) represent the IMC based PID tuning for both minimum and non-minimum phase which gives the better performance in both the methods.

Table 4: Performance analysis for non-minimum phase system

PI	TYPE	IAE		ISE	
		Y1	Y2	Y1	Y2
A.	Decent	143	45	38.9	22.8
	Decoupler	175	53	53.4	20.12
B.	Decent	77	608	19.8	263
	Decoupler	40	76.8	15	31.69
C.	Decent	0.26	0.33	0.074	0.134
	Decoupler	0.22	0.33	0.07	0.135

6. Conclusion

In this paper the Quadruple processes such as 'two input two output', system have been studied, analyzed and simulate. When compared to all other tuning method of PI controller the IMC based PI tuning method gives the better performance for both minimum and non minimum phase of the four tank process and it gives the low value of ISE and IAE. There is no overshoot occurring in both minimum and non minimum phase by IMC based PI controller and also it settles fast when compared to other methods.

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