Fractal Approach of the Nominal Exchange Rate Series Based on Hurst Exponent

P. Uthayakumar[1], M. Prabha[2]

Department of Mathematics, PSNA College of Engineering and Technology, Dindigul – 624 622, India

Abstract: ***The objective of this paper is to examine the behavior of the nominal exchange rate series between Indian Rupees and US Dollar by using fractal approach. Properties of the time series using this fractal method allows us to consider the important features ignored by the traditional time series analyses and focus on the utility of Hurst exponent intrinsic in financial time series in determining the persistency of Indian Rupees exchange rate at financial markets. This method allows us to analyse the properties of the time series and the utility of the Hurst exponent in financial time series is revealed the persistency of Indian rupees exchange rate in the market.***

Keywords: Fractal analysis, Rupees-Dollar exchange rate, Hurst exponents.

1. Introduction

An Exchange Rate is the rate at which the currency of one country can be exchanged to another. For the purpose of regulation of the level of imports and exports Exchange Rates are very essential for any country. If the national currency appreciates with respect to the foreign currency, trade in goods will be economical in the internal market and local firms would find that their overseas opponent's goods become more striking to consumers. If the nation has a concrete currency then its imports become more costly in the universal market, which results in lost effectiveness.

An exchange rate of₹ 1 = $ 0.02 means that anyone has to pay₹1 for$ 0.02 or to pay ₹ 60.184 for $ 1.Day by day the Exchange rate changes. Depends on the direction of the change, the changes of exchange rate can affect various investors in many ways. There are two important terms in the changes of exchange rate; they are 'strengthcurrency' and 'weakcurrency'. The two terms are being used in the foreign exchange market to describe the relative value and strength of the currency against the other currencies. As for as a developing country like India is considered, the Exchange rate plays a vital role in economic status and in the budget of the country and etc. The fractal theory which is first introduced by Benoit Mandelbrot in early 1960's. Domestic and foreign scholars have focused on the nonlinear characteristic test for the finance market. The fractal technique is a unified method, widely applies the rescaled range analysis (R/S analysis) and the fractal analysis from different approaches to assess the fractal characteristics of the time series. Using the Hurst exponent the currency exchange rate with non-periodic cycle length is analyzed by the R/S analysis.

The Rescaled range (R/S value) is a statistical measure of the variability of a time series introduced by the British hydrologist Harold Edwin Hurst. Its aim is to suggest an assessment of how the apparent variability of a time series changes with the length of the time-period being considered. The rescaled range (R/S value) is obtained from dividing the range of the values presented in a part of the time series by the standard deviation of the values over the same part of the time series.

2. Methodologies

2.1 The Hurst Exponent

H. E. Hurst studied the record that the Egyptians had of the Nile River from 622 A.D. to 1469 A.D. he was able to discover that the Nile river followed non-periodical cycles. To check the persistency he developed his own methodology using Einstein equation. The Hurst Exponent is directly correlated to the fractal dimension, which measures the smoothness of a surface. In this paper we use this method in measuring the smoothness of a time series.

The Hurst exponent is proposed by Hurst in 1951 in hydrological studies for the application to various research grounds. The application of Hurst exponent is more popular in the economic and financial researches. The Hurst method is most useful tool to check the persistency of the time series related to stock market, economic and finance data. The Hurst exponent is used as a measure of the long-term memory of a time series. The Hurst exponent gives intimation that the existing value of the time series depend on earlier values of the time series.

The relationship between the fractal dimension D, and the Hurst Exponent H, is given by: D = 2- H. The Hurst Exponent is the measure of the smoothness of fractal time series based on the asymptotic behavior of the rescaled range of the process. If the H-value lies between 0 and 0.5, we observe short memory phenomenon, i.e., it indicates anti-persistent behavior of the series, the series is said to have long memory phenomenon, i.e., it indicates persistent behavior of the time series. If the H-value is equal to 0.5, it indicates fractal Brownian motion or Random Walk. The control of the trend rises until H-value attains its upper maximum value of one.

2.2. The Hurst Exponent Estimation

The Hurst Exponent is estimated as follows:
Consider the time series,
X={X_1, X2, X3... Xn}
1. Calculate the mean:

$$m= \frac{1}{n}\sum_{t=1}^{n} X_i$$

2. Create a mean -adjusted series, which is differ from the mean (m):

$Y_t = X_t - m$ for t=1,2,….,n

3. Calculate the cumulative deviations Series Z:

$$Z_t = \sum_{i=1}^{t} Y_i \text{ for } t=1,2,\ldots,n$$

4. Create a range series R:

$R(n)=\max(Z_1,Z_2,\ldots,Z_n)-\min(Z_1,Z_2,\ldots,Z_n)$

5. Calculate the standard deviation of the series:

$$S(n)=\sqrt{\frac{1}{n}\sum_{i-1}^{n}(X_i - m)^2}$$

6. Calculate the rescaled range series (R/S):

$$(R/S)=\frac{R(n)}{S(n)}$$

Hurst exponent, H is defined by $H=\frac{\log(R/S)}{\log(T)}$ where n is the period of the sample and R/S is the standard deviation ratio.

By Power law,The Hurst exponent is attained as: $E\left(\frac{R(n)}{S(n)}\right) = K.n^h$ where k is a constant and we obtain Hurst exponent by the slope of the attained curve.

Hurst noticed that the value of H directly depends on the range to standard deviation ratio. This ratio provides a process of categorizing time series, which is very useful to identify the greater predictability markets. For a random time series fore casting technique will not be successful, So we check the predictability of the time series and the forecasting of a time series is to be done, and time series having some degree of predictability may be focused. Through the power law the Hurst exponent will be calculated. This traditional approach is commonly used in the series having unifractal or monofractal features.

3. Application

The exchange rate series of Indian Rupees to US Dollar from 2004 to 2014 is considered for the analysis. The INR to USD Exchange rate evolution graph is given in the Figure 1.In figure 1, the various times of increase and decreases of Indian Rupees are given in the prevalent economic condition.

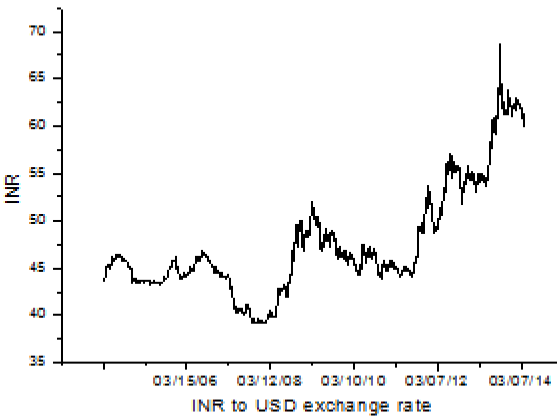

Figure 1: Evolution of the INR-USD exchange rate,(04/12/2004 to 04/09/2014)

The logarithmic difference of the exchange rate $r_i=\log(X_i)-\log(X_{i-1})$, is applied to analyze the scaling behavior of the exchange rate series X_i.

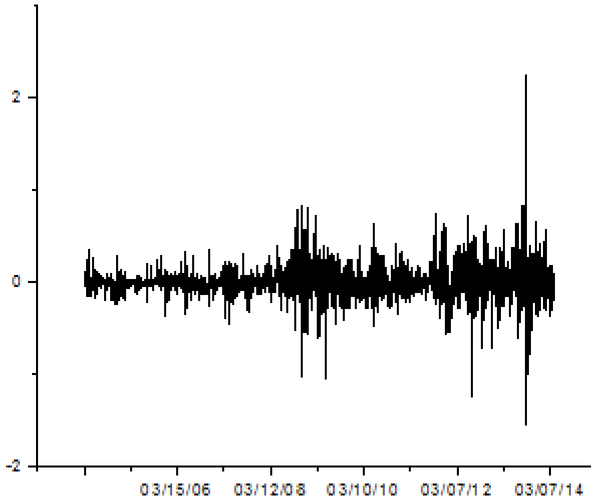

Figure 2: Evolution of the INR-USD exchange rate, in logarithmic difference (04/12/2004 to 04/09/2014)

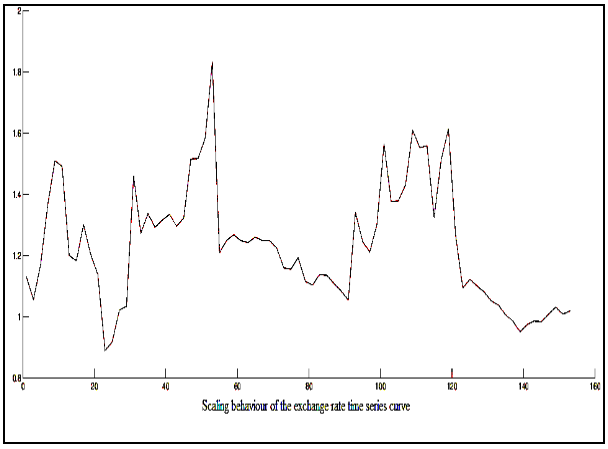

Figure 3: Scaling Behavior of the Exchange Rate time series curve

From the fractal analysis of the exchange rate series curve the following data is obtained.

Mean	Median	Range	Std.Deviation	Minimum	Maximum	Dimension	Correlation Coefficient
47.592	45.82	29.68	5.817	39.11	68.79	1.239	0.99913

Hurst Exponent Calculation Table:

Number of Observations(N)	Rescaled Range(R/S)	$Log_2(N)$	$Log_2(R/S)$
2000	47.671	10.966	5.575
1000	32.640	9.966	5.029
500	22.945	8.966	4.520
250	16.474	7.966	4.042
125	11.336	6.966	3.503

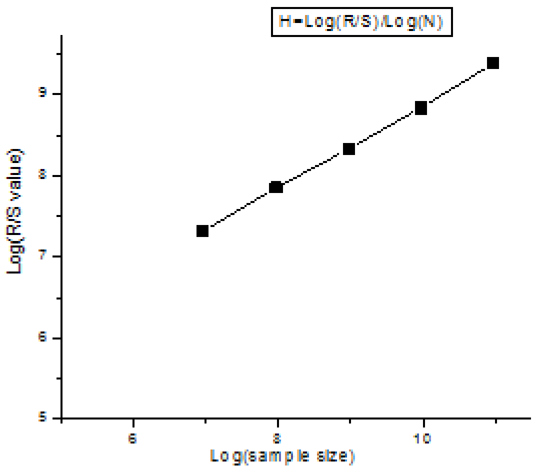

Figure 4: The Hurst Exponent

The Value of the Hurst Exponent is measured as 0.5041 which is approximately equal to 0.5, as shown in following Figure 4.Hence, the INR/USD exchange market exhibits Brownian motion of the exchange rate time series. The series indicates the performance of Random walk; it means that there is a no correlation between the present and future values.

4. Conclusion

In this paper the Hurst Exponent method is applied to obtain the exchange rate of INR/USD is a random walk t. The measured value of Hurst Exponent is nearly 0.5041.This is nearly the Hurst value 0.5, this shows that INR exchange market indicates Brownian motion of the exchange rate time series. In future, the correlation analysis among USD, EURO and INR in a three dimensional manner can be analyzed. Hence it is not easy for the Depositors to predict the future value actions or the future profit limits from INR/USD exchange market data.

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