Restricted Problem of (2+2) Bodies Where Oblate Primaries are Magnetic Dipoles and Infinitesimal Bodies are Electric Dipoles

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Abstract: *In this paper we have studied the (2+2) restricted model in which we have considered primaries as magnetic dipoles and also the shapes of the both primaries are taken as oblate spheroid. Primaries are moving in the circular orbits around their center of mass. The infinitesimal bodies' m_1, m_2 are taken as electric dipoles in the field of rotating magnetic dipoles with rotatary mean motion n. Here the effects of magnetic field of the primaries are neglected.*

Keywords: Restricted model, Oblate body, Magnetic dipoles, Electric dipoles, Infinitesimal bodies

1. Introduction

Many researchers have studied about the restricted problem of three bodies with different shapes of the primaries in the gravitational field. Szebehely(1967) has studied in his book about the restricted three body problem with primaries are spherical in shape. Whipple (1984) has found the equilibrium solutions of the restricted problem of 2+2 bodies with primaries as spherical in shape in gravitational field only. Goudas and Petsgouraki's (1985) considered minor bodies to be charged particles and have studies the equilibrium solutions in the field of two rotating magnetic dipoles. Mavraganis and Kalvorides (1987) have also studied the symmetric motion in the equatorial magnetic binary problem. Prasad et.al. (1996) have found the equations of motion in the (2+2) restricted model when two primaries are magnetic dipoles but spherical in shape and two infinitesimal bodies are taken as electric dipoles. Now, we also have studied the (2+2) restricted model when two primaries are magnetic dipoles but both are oblate in shape and infinitesimal bodies are taken as electric dipoles. Our problem is quite realistic and useful for space research.

2. Equations of Motion

Let P_1 and P_2 be two primaries of masses M_1 and M_2 with oblate shapes. We consider the barycentric rotating coordinate system OXYZ related to inertial system with angular velocity n about common z-axis. We have considered line joining the primaries as X-axis. The coordinates of P_1 and P_2 are (μ, 0, 0) and (1-μ, 0, 0) respectively. Let m_i and m_j are the infinitesimal bodies with electric dipoles having coordinates (x_i, y_i, z_i) and (x_j, y_j, z_j) respectively.

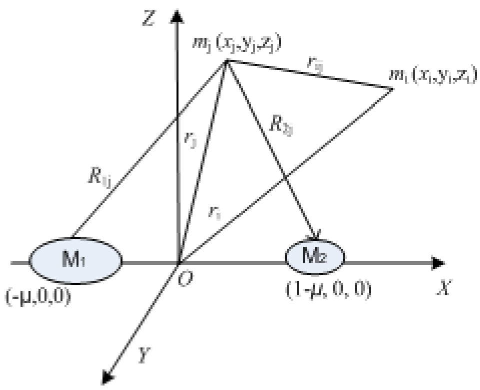

Figure 1: Configurations of the problem

We consider the electric dipole moment of infinitesimal bodies as

$$\vec{h_j} = h_{jx}\vec{i} + h_{jy}\vec{j} + h_{jz}\vec{k},$$

The position vector of dipole is

$$\vec{r_j} = x_j\vec{i} + y_j\vec{j} + z_j\vec{k},$$

Such that the distance between any two dipoles are given by

$$\vec{r_{ij}} = (x_j - x_i)\vec{i} + (y_j - y_i)\vec{j} + (z_j - z_i)\vec{k},$$

The distance of j[th] infinitesimal bodies from primaries are R_{1j} and R_{2j} respectively, then

$$\vec{R_{1j}} = (x_j + \mu)\vec{i} + y_j\vec{j} + z_j\vec{k},$$
$$\vec{R_{2j}} = (x_j + \mu - 1)\vec{i} + y_j\vec{j} + z_j\vec{k},$$

The Kinetic energy of infinitesimal dipoles

$$T = \tfrac{1}{2}\sum m_j[(\dot{x}_j - n\,y_j)^2 + (\dot{y}_j + n\,x_j)^2 + \dot{z}_j^2],$$

and the total potential energy of the system

$$V = V_g + U.$$

Where U is the potential due to dipole interaction between infinitesimal bodies. Or

$$U=\frac{1}{4\pi\varepsilon_0}\left(\frac{\overline{h_i}.\overline{h_j}}{r_{ij}^3}-\frac{3(\overline{h_i}.\overline{r_{ij}})(\overline{h_j}.\overline{r_{ij}})}{r_{ij}^5}\right),$$

$$=\frac{C}{r_{ij}^3}-\frac{1}{r_{ij}^5}[B_{11}(x_j-x_i)^2+B_{22}(y_j-y_i)^2+B_{33}(z_j-z_i)^2$$
$$+B_{12}(x_j-x_i)(y_j-y_i)+B_{13}(x_j-x_i)(z_j-z_i)$$
$$+B_{23}(y_j-y_i)(z_j-z_i)]$$

With the conditions that

$$\frac{1}{4\pi\varepsilon_0}=1,\ C=\overline{h_i}.\overline{h_j},$$

$$B_{11}=h_{ix}.h_{jx},$$
$$B_{22}=h_{iy}.h_{jy},\ B_{33}=h_{iz}.h_{jz},$$
$$B_{12}=[h_{ix}.h_{jy}+h_{iy}.h_{jx}],$$
$$B_{13}=[h_{ix}.h_{jz}+h_{iz}.h_{jx}],$$
$$B_{23}=[h_{iz}.h_{jy}+h_{iy}.h_{jz}].$$

Now,

$$V_g=-\sum_{j=1}^{2}\left(\frac{Gm_jM_1}{R_{1j}}+\frac{Gm_jM_1\sigma_1}{2R_{1j}^3}+\frac{Gm_jM_2}{R_{2j}}+\frac{Gm_jM_2\sigma_2}{2R_{2j}^3}+\sum_{i=1}^{2}\frac{Gm_jm_i}{2r_{ij}}\right),\qquad i\neq j.$$

a_1, a_2, b_1 and b_2 are the major and the minor axes of the oblate body.

Total Potential

$$V=-G\sum_{j=1}^{2}m_j\left(\frac{M_1}{R_{1j}}+\frac{M_1\sigma_1}{2R_{1j}^3}+\frac{M_2}{R_{2j}}+\frac{M_2\sigma_2}{2R_{2j}^3}+\sum_{i=1}^{2}\frac{m_i}{2r_{ij}}\right)$$
$$+\frac{C}{r_{ij}^3}-\frac{1}{r_{ij}^5}[B_{11}(x_j-x_i)^2+B_{22}(y_j-y_i)^2+B_{33}(z_j-z_i)$$
$$+B_{12}(x_j-x_i)(y_j-y_i)+B_{13}(x_j-x_i)(z_j-z_i)$$
$$+B_{23}(y_j-y_i)(z_j-z_i)].$$

The dipole moments are not function of time therefore they do not generate magnetic induction. So we have neglected the potential energy of any magnetic interaction. Hence, Lagrangian of the system is given by

$$L=T-V,$$

$$=\sum_{j=1}^{2}\frac{m_j}{2}[(\dot{x}_j-n\,y_j)^2+(\dot{y}_j+n\,x_j)^2+\dot{z}_j^2]$$
$$+G\sum_{j=1}^{2}m_j\left(\frac{M_1}{R_{1j}}+\frac{M_1\sigma_1}{2R_{1j}^3}+\frac{M_2}{R_{2j}}+\frac{M_2\sigma_2}{2R_{2j}^3}+\sum_{i=1}^{2}\frac{m_i}{2r_{ij}}\right)$$
$$-\frac{C}{r_{ij}^3}+\frac{1}{r_{ij}^5}[B_{11}(x_j-x_i)^2+B_{22}(y_j-y_i)^2+B_{33}(z_j-z_i)^2$$
$$+B_{12}(x_j-x_i)(y_j-y_i)+B_{13}(x_j-x_i)(z_j-z_i)$$
$$+B_{23}(y_j-y_i)(z_j-z_i)].$$

The Lagrangian equations of motion are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_j}\right)-\frac{\partial L}{\partial x_j}=0,$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_j}\right)-\frac{\partial L}{\partial y_j}=0,$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}_j}\right)-\frac{\partial L}{\partial z_j}=0,$$

After Simplification, we get the equations of motion

$$\ddot{x}_j-2n\dot{y}_j-n^2x_j=-G\left[\frac{M_1(x_j+\mu)}{R_{1j}^3}+\frac{3M_1\sigma_1(x_j+\mu)}{2R_{1j}^5}+\frac{M_2(x_j+\mu-1)}{R_{2j}^3}+\frac{3M_2\sigma_2(x_j+\mu-1)}{2R_{2j}^5}\right]$$

Where

$$-G\frac{m_{3-j}(x_j-x_i)}{2r_{ij}^3m_j}+$$
$$\frac{(3C+2B_{11})(x_j-x_i)+[B_{12}(y_j-y_i)+B_{13}(z_j-z_i)]}{r_{ij}^5m_j}$$
$$-\frac{5B_{123}^{ji}(x_j-x_i)}{r_{ij}^7m_j},$$

$$B_{123}^{ji}=B_{11}(x_j-x_i)^2+B_{22}(y_j-y_i)^2+B_{33}(z_j-z_i)^2$$
$$+B_{12}(x_j-x_i)(y_j-y_i)+B_{13}(x_j-x_i)(z_j-z_i)$$
$$+B_{23}(y_j-y_i)(z_j-z_i).$$

Similarly, we can write the y and z components,

$$\ddot{y}_j+2n\dot{x}_j-n^2y_j=-G\left[\frac{M_1y_j}{R_{1j}^3}+\frac{3M_1\sigma_1y_j}{2R_{1j}^5}+\frac{M_2y_j}{R_{2j}^3}+\frac{3M_2\sigma_2y_j}{2R_{2j}^5}\right]$$

$$-G\frac{m_{3-j}(y_j-y_i)}{2r_{ij}^3m_j}+$$

$$\frac{(3C+2B_{22})(y_j-y_i)+[B_{21}(x_j-x_i)+B_{23}(z_j-z_i)]}{r_{ij}^5 m_j} - \frac{5B_{123}^{ji}(y_j-y_i)}{r_{ij}^7 m_j},$$

$$\ddot{z}_j = -G[\frac{M_1 z_j}{R_{1j}^3}+\frac{3M_1\sigma_1 z_j}{2R_{1j}^5}+\frac{M_2 z_j}{R_{2j}^3}+\frac{3M_2\sigma_2 z_j}{2R_{2j}^5}] - G\frac{m_{3-j}(z_j-z_i)}{2r_{ij}^3 m_j}+$$

$$\frac{(3C+2B_{33})(z_j-z_i)+[B_{31}(x_j-x_i)+B_{32}(y_j-y_i)]}{r_{ij}^5 m_j} - \frac{5B_{123}^{ji}(z_j-z_i)}{r_{ij}^7 m_j}$$

Let the distance between the primaries be unity and $M_1 + M_2$, the unit of mass and the unit of time is so chosen so as to make G = 1 and hence n will also be unity.

Let $\mu = M_2,\ (1-\mu) = M_1$ and $\mu_i = m_i, i = 1, 2$ then the equations of motion in the dimensionless quantities be

$$\ddot{x}_j - 2\dot{y}_j - x_j = -[\frac{(1-\mu)(x_j+\mu)}{R_{1j}^3}+\frac{3(1-\mu)\sigma_1(x_j+\mu)}{2R_{1j}^5}+\frac{\mu(x_j+\mu-1)}{R_{2j}^3}+\frac{3\mu\sigma_2(x_j+\mu-1)}{2R_{2j}^5}]$$

$$-\frac{\mu_{3-j}(x_j-x_i)}{2r_{ij}^3\mu_j}+\frac{(3C+2B_{11})(x_j-x_i)+[B_{12}(y_j-y_i)+B_{13}(z_j-z_i)]}{r_{ij}^5\mu_j}-\frac{5B_{123}^{ji}(x_j-x_i)}{r_{ij}^7\mu_j},$$

$$\ddot{y}_j + 2\dot{x}_j - y_j = -[\frac{(1-\mu)y_j}{R_{1j}^3}+\frac{3(1-\mu)\sigma_1 y_j}{2R_{1j}^5}+\frac{\mu y_j}{R_{2j}^3}+\frac{3\mu\sigma_2 y_j}{2R_{2j}^5}]$$

$$-\frac{\mu_{3-j}(y_j-y_i)}{2r_{ij}^3\mu_j}+\frac{(3C+2B_{22})(y_j-y_i)+[B_{21}(x_j-x_i)+B_{23}(z_j-z_i)]}{r_{ij}^5\mu_j}-\frac{5B_{123}^{ji}(y_j-y_i)}{r_{ij}^7\mu_j},$$

$$\ddot{z}_j = -[\frac{(1-\mu)z_j}{R_{1j}^3}+\frac{3(1-\mu)\sigma_1 z_j}{2R_{1j}^5}+\frac{\mu z_j}{R_{2j}^3}+\frac{3\mu\sigma_2 z_j}{2R_{2j}^5}]$$

$$-\frac{\mu_{3-j}(z_j-z_i)}{2r_{ij}^3\mu_j}+\frac{(3C+2B_{33})(z_j-z_i)+[B_{31}(x_j-x_i)+B_{32}(y_j-y_i)]}{r_{ij}^5\mu_j}-\frac{5B_{123}^{ji}(z_j-z_i)}{r_{ij}^7\mu_j}.$$

Putting j =1, i = 2 and j = 2, i =1, we get the differential equations of motion of two electric dipoles with respect to two oblate primaries as magnetic dipoles in our restricted problem as

$$\ddot{x}_1 - 2\dot{y}_1 - x_1 = -[\frac{(1-\mu)(x_1+\mu)}{R_{11}^3}+\frac{3(1-\mu)\sigma_1(x_1+\mu)}{2R_{11}^5}+\frac{\mu(x_1+\mu-1)}{R_{21}^3}+\frac{3\mu\sigma_2(x_1+\mu-1)}{2R_{21}^5}]$$

$$-\frac{\mu_2(x_1-x_2)}{2r_{12}^3\mu_1}+\frac{(3C+2B_{11})(x_1-x_2)+[B_{12}(y_1-y_2)+B_{13}(z_1-z_2)]}{r_{12}^5\mu_1}-\frac{5B_{123}^{12}(x_1-x_2)}{r_{12}^7\mu_1},$$

$$\ddot{y}_1 + 2\dot{x}_1 - y_1 = -[\frac{(1-\mu)y_1}{R_{11}^3}+\frac{3(1-\mu)\sigma_1 y_1}{2R_{11}^5}+\frac{\mu y_1}{R_{21}^3}+\frac{3\mu\sigma_2 y_1}{2R_{21}^5}]$$

$$-\frac{\mu_2(y_1-y_2)}{2r_{12}^3\mu_1}+\frac{(3C+2B_{22})(y_1-y_2)+[B_{21}(x_1-x_2)+B_{23}(z_1-z_2)]}{r_{12}^5\mu_1}-\frac{5B_{123}^{12}(y_1-y_2)}{r_{12}^7\mu_1},$$

$$\ddot{z}_1 = -[\frac{(1-\mu)z_1}{R_{11}^3}+\frac{3(1-\mu)\sigma_1 z_1}{2R_{11}^5}+\frac{\mu z_1}{R_{21}^3}+\frac{3\mu\sigma_2 z_1}{2R_{21}^5}]$$

$$-\frac{\mu_2(z_1-z_2)}{2r_{12}^3\mu_1}+\frac{(3C+2B_{33})(z_1-z_2)+[B_{31}(x_1-x_2)+B_{32}(y_1-y_2)]}{r_{12}^5\mu_1}-\frac{5B_{123}^{12}(z_1-z_2)}{r_{12}^7\mu_1}.$$

and

$$\ddot{x}_2 - 2\dot{y}_2 - x_2 = -[\frac{(1-\mu)(x_2+\mu)}{R_{12}^3} + \frac{3(1-\mu)\sigma_1(x_2+\mu)}{2R_{12}^5} + \frac{\mu(x_2+\mu-1)}{R_{22}^3} + \frac{3\mu\sigma_2(x_2+\mu-1)}{2R_{22}^5}]$$

$$-\frac{\mu_1(x_2-x_1)}{2r_{12}^3\mu_2} + \frac{(3C+2B_{11})(x_2-x_1)+[B_{12}(y_2-y_1)+B_{13}(z_2-z_1)]}{r_{12}^5\mu_2} - \frac{5B_{123}^{12}(x_2-x_1)}{r_{12}^7\mu_2},$$

$$\ddot{y}_2 + 2\dot{x}_2 - y_2 = -[\frac{(1-\mu)y_2}{R_{12}^3} + \frac{3(1-\mu)\sigma_1 y_2}{2R_{12}^5} + \frac{\mu y_2}{R_{22}^3} + \frac{3\mu\sigma_2 y_2}{2R_{22}^5}]$$

$$-\frac{\mu_1(y_2-y_1)}{2r_{12}^3\mu_2} + \frac{(3C+2B_{22})(y_2-y_1)+[B_{21}(x_2-x_1)+B_{23}(z_2-z_1)]}{r_{12}^5\mu_2} - \frac{5B_{123}^{21}(y_2-y_1)}{r_{12}^7\mu_2},$$

$$\ddot{z}_2 = -[\frac{(1-\mu)z_2}{R_{12}^3} + \frac{3(1-\mu)\sigma_1 z_2}{2R_{12}^5} + \frac{\mu z_2}{R_{22}^3} + \frac{3\mu\sigma_2 z_2}{2R_{22}^5}]$$

$$-\frac{\mu_1(z_2-z_1)}{2r_{12}^3\mu_2} + \frac{(3C+2B_{33})(z_2-z_1)+[B_{31}(x_2-x_1)+B_{32}(y_2-y_1)]}{r_{12}^5\mu_2} - \frac{5B_{123}^{21}(z_2-z_1)}{r_{12}^7\mu_2}.$$

$$\ddot{x}_2 - 2\dot{y}_2 - x_2 = -[\frac{(1-\mu)(x_2+\mu)}{R_{12}^3} + \frac{3(1-\mu)\sigma_1(x_2+\mu)}{2R_{12}^5} + \frac{\mu(x_2+\mu-1)}{R_{22}^3} + \frac{3\mu\sigma_2(x_2+\mu-1)}{2R_{22}^5}]$$

3. Conclusion

In this paper, we have studied about the equations of motion of the infinitesimal bodies which are taken as electric dipole in the influence of oblate primaries but not influencing them. We have neglected the effects of magnetic interaction of the primaries because magnetic force field is negligible in comparison to the gravitational force field. The equations of motion, which we have found are similar to the equations of motion found by Prasad et. al. (1996) but differs by the oblateness factors.

Practically we can consider these models as Sun-earth (oblate primaries as magnetic dipoles) and Asteroids (infinitesimal bodies as electric dipoles). These models are very-very useful in our space research.

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