Multiresponse Optimization in Response Surface Method by Aggressive DEA

Sony Sunaryo[1], Muhammad Sjahid Akbar[2], Haryono[3], Rendra Erdkhadifa[4]

[1, 2, 3, 4]Department of Statistics, Sepuluh Nopember Institute of Technology Surabaya, Indonesia

Abstract: ***This paper uses DEA method and response surface to optimize quality of GLS lamp. DEA is non-parametric method that used to evaluate relative efficiency. DEA which used was DEA CCR. This method will apply on case GLS lamp. The result of DEA method will be optimized with response surface. The results show the optimization on GLS lamps is better than the desirability for LTB criteria***

Keywords: DEA, Aggressive DEA**,** Response surface**,** optimized, GLS lamp

1. Introduction

Quality is conformity of the product that used to satisfy needs. Identifying a quality needs detail process. The quality is closely related to the industry process. Today, demand of industrial products tends to increase. So, some of quality variables need to be paid attention for resulting product that appropriately with customers demand. Suitability of the product is affected by the setting of variable quality (independent variable). The results of the variable settings will produce optimal quality products according to customer wishes (response variable). If a customer desire one response, then many methods that have been discussed. But if multi-response and optimized simultaneously, than are not easy to solve. There are several methods used for optimization multi-response, one of which is a desirability function [1-2].

Desirability function is an optimization method for multiresponses that introduced by Derringer and Suich [3]. This method optimized multireponses with changing into new single response. This response is linier combination from the old responses. Koksoy used Derringer and Suich's method for optimizing dual response [2]. On desirabilty function, each response assumed has the same priority. This method uses equal weight to each response. In real conditions, there is a response that has a higher priority than the other responses. This paper discusses the method of Data Envelopment Analysis (DEA) that using different weights on the response. For the next step, the optimization with using response surface methodology [4-5].

Data envelopment analysis is a mathematic methodology that used to estimate related multiple inputs and multiple outputs. Each variable has different weight with constructing efficiency in every calculating unit whom called decision making unit (DMU). DEA had been introduced by Charnes, Cooper, dan Rhodes was DEA CCR [6]. Al-Refaie dan Li show that CCR shows the model of the CCR allows generating efficiency values are less precise in setting unrealistic weighting. Because DEA CCR can be called input-oriented which assessing sum weighted equal with 1. DEA aggressive is development of DEA CCR. This method minimized from cross-efficiencies of other DMU and efficiency value is more than 1[7].

Several research on application of DEA include Ertay and Ruan determine optimal operator in CMS (cellular manufacturing system) [8]. Mousavi-Avval, Rafiee, Jafari, and Mohammadi on the use of energy for production of canola [9]. Azadeh, Ghaderi, Mirjalili, and Moghaddam conducted an analysis on a bank with a method combining DEA with Analytical Hierarchy Process (AHP) and Principal Components Analysis (PCA) [10]. Research on aggressive DEA, Al-Refaie and Li who solve cases multiresponses on the Taguchi method [7]. Angiz, Mustafa, and Kamali (2013) who explains that some applications in DEA ranking Decision Makin Units (DMUs) are important by using cross-Efficiencies [11].

This research uses DEA method and response surface which will be applied on case of optimization quality of GLS lamp. This case encourages Fatima to conduct research on how to define the components of PD and CML to obtain the best quality of the lamp. Fatima using the desirability function method to determine the optimum value [12].

The content of this paper is organized as follows: Section 2 describes DEA method. Section 3 describes response surface method. Section 4 provides an application of the method of DEA on selected case studies. Section 5 contains the results of the case solving. Section 6 describes comparison result of the research. Section 7 is conclusions.

2. Data Envelopment Analysis

DEA is non-parametric statistics technique programming that used to evaluate relative efficiency for homogenity decision making units (DMUs). The method of DEA which often used is DEA CCR. CCR model uses virtual multiplier that combining multiple inputs and multiple outputs into single index. Virtual multiplier that used is to generate sum of weighted output divided with sum of weighted input. Relative efficiency for each DMUs, θ_t to get each efficiency value can be evaluated by solving:

$$\max \theta_t = \frac{\sum_{j=1}^{n} v_j y_{jt}}{\sum_{k=1}^{p} u_k y_{kt}} \quad (1)$$

Clearly, constraints for (1)

$$\frac{\sum_{j=1}^{n} v_j y_{ji}}{\sum_{k=1}^{p} u_k y_{ki}} \leq 1$$

$$u_1, u_2, \ldots, u_p \geq 0$$

$$v_1, v_2, \ldots, v_n \geq 0$$

$$i = 1,2,\ldots,m \quad j = 1,2,\ldots,n \quad k = 1,2,\ldots,p$$

The model of DEA CCR can be changed into oriented input that setting sum of weighted input. For the first step is transforming CCR model into linier model.

$$\max \theta_t = \sum_{j=1}^{n} v_j y_{jt} \quad (2)$$

constraints:

$$\sum_{k=1}^{p} u_k y_{kt} = 1$$

$$\sum_{j=1}^{n} v_j y_{ji} \leq \sum_{k=1}^{p} u_k y_{ki}$$

$$\sum_{j=1}^{n} v_j y_{ji} - \sum_{k=1}^{p} u_k y_{ki} \leq 0$$

3. Data Envelopment Analysis Aggressive

CCR model can give uncorrect efficiency value on mis-leading weighted. Cause of DEA CCR is oriented input which assessing sum of weighted equal with 1. Data envelopment analysis with aggressive formulation is completion of DEA CCR that optimizing model with minimizing cross-efficiencies of other DMUs. In this study, for solving multiresponses, to determine which being input and output based on following [7]:

1. If all of responses have smaller the better (STB) type, one of them is selected for being input.
2. If all of responses have nominal the better (NTB) type, to calculate quality loss for being input and and one of other responses for being output.

Quality loss formulated as:

$L_i = c\left(\frac{s_i^2}{\bar{y}_i^2}\right)$ where c is coefficient quality loss. $\bar{y}_i, s_i$ are average and deviation standard of replication for each DMUs.

3. If all of responses have three types, set STB and quality loss as input. Set larger the better (LTB) type as output.
4. If all of responses have STB and LTB type, set LTB type as output and STB type as input.

The model of DEA formulation aggressive for each is minimize [7].

$$\min \sum_{j=1}^{n}\left(v_{jt} \sum_{t \neq i} y_{ji}\right) - \sum_{k=1}^{p}\left(u_{kt} \sum_{t \neq i} y_{ki}\right) \quad (3)$$

constraints. $\sum_{k=1}^{p}\left(u_{kt} \sum_{t \neq i} y_{ki}\right) = 1$

$$\sum_{j=1}^{n} v_{jt} y_{ji} - \sum_{k=1}^{p} u_{kt} y_{ki} \leq \delta; \ \forall j \neq 0$$

$$\sum_{j=1}^{n} v_{jt} y_{jt} - E_{tt} \sum_{k=1}^{p} u_{kt} y_{kt} = 0$$

$$v_{jt}, u_{kt} \geq 0$$

Cross-efficiencies DMUs is calculated with optimal weighted both input and output. Cross-efficiencies, E_{ti}, can be evaluated by solving.

$$E_{ti} = \frac{\sum_{j=1}^{n} v_{jt} y_{ji}}{\sum_{k=1}^{p} u_{kt} y_{ki}} \quad (4)$$

Mean of cross-efficiencies is efficiency value DEA aggressive formulation for each DMUs.

$$e_i = \frac{\sum_{t \neq i} E_{ti}}{n-1} \quad (5)$$

4. Response Surface Methodology (RSM)

RSM is an optimization method. The character of this method is combination between experiment design and regression. The designs often used in some researches were Central Composite Design (CCD) and Box-Behnken Design. For the first step single response optimization in RSM is to analyze the first order model and then, to check appropriate model by signification of lack of fit (LOF). If LOF is significant or it's not appropriate model, for the next step is analyzing the second order model. Let the second order model be as in (6) [13].

$$\hat{y} = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i + \sum_{i=1}^{k} \hat{\beta}_{ii} x_i^2 + \sum_{i<j} \hat{\beta}_{ij} x_i x_j \quad (6)$$

$$\hat{y} = \hat{\beta}_0 + \mathbf{x}'\mathbf{b} + \mathbf{x}'\mathbf{B}\mathbf{x} \quad (7)$$

If the second order model is appropriate model, it will get correct parameter setting to find optimum response. Optimum condition can be found from derivatives $\frac{\partial \hat{y}}{\partial \mathbf{x}} = 0$.

So, in that case

$$\frac{\partial \hat{y}}{\partial \mathbf{x}} = \mathbf{b} + 2\mathbf{B}\mathbf{x} = 0 \quad (8)$$

Stationary point can be found with $\mathbf{x}_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b}$. Stationary point represent optimum response by $\hat{y}_s = \hat{\beta}_0 + \frac{1}{2}\mathbf{x}_s\mathbf{b}$. After analyze optimization, for the next step is checking residual assumption for each response.

5. Application

The data that used in this study was research from Fatima [12]. This research used central composite design with 13 units of GLS lamps. Type of GLS lamp is A55 100 W 230 V. Constant mounting length (X_1) and pool distance (X_2) are important components On GLS lamps. They are estimated to have influence against lumen (Y_1), wattage (Y_2), and life time (Y_3). Optimum response criterion of lumen and life time is larger the better, but response criterion of wattage is smaller the better. In that case we will use DEA aggressive. In the application of DEA, the response values are used to establish the efficiency which inputs and outputs are determined b y t he o ptimal r esponse c riteria. DEA weighted value is determined from t he r esults o f t he a nalysis with linear p rogramming. Efficiency value o ptimized with response surface methodology.

6. Result

The first step to get a ggressive DE A efficiency value is t o calculate a weighted value for e ach DMU. T able 1 sh ows weighted value with aggressive formulation for each DMUs. U1i is weighted input value. V 1i is weighted value for first output and V2i s weighted value for second output. Result of Analyze DE A a ggressive is i ndicated o n a ppendix 1 . e i is efficiency which calculated from average DMUs value. From the analysis the highest e fficiency value is 1 ,225 DMUi 1 3. The lo west val ue is 0 ,545 DMUi 12. Appendix 1 also informs about ranking o f a verage val ue which s howed b y ordinal value (OV) for knowing best performer. The ranking is got from t he lo west value to the highest value. B ased o n ordinal value, a nd t hen to an alyze optimization using response surface.

Table 1: Weighted Value Aggressive Formulation for Each DMUs

DMUs	*Weighted Aggressive Formulation*		
	Input	*Output*	
	U1i	V_{1i}	V_{2i}
1	0,0008305 0,	00005691	0
2 0	,0008311	0	0,00004205
3 0	,0008296	0	0,00006396
4 0	,0008300	0	0,00006276
5 0	,0008308	0	0,00004312
6 0	,0008303	0	0,00006579
7 0	,0008308	0	0,00006612
8 0	,0008294	0	0,00006462
9	0,0008308 0,	00005693	0
10	0,0008305 0	,00005691	0
11	0,0008306 0	,00005692	0
12	0,0008308 0	,00005693	0
13 0	,0008312	0	0,00004215

RSM model used to analyze is second-order model.

$$\hat{y} = 0,86254 - 0,03818x_1 - 0,0226x_2 - 0,10544x_1^2 + 0,07919x_2^2 + 0,13552x_1x_2$$

Table 2 ANOVA of Second-Order Model For Efficiency DEA *Aggressive*

Source	*df*	*SS*	*MS*	*F*	*P-value*
Regression	5 0,	227	0,045 1	,02	0,470
Residual Error	7	0,311	0,0444		
Lack of fit	3	0,132	0,0441	0,99	0,483
Pure Error	4	0,178	0,0446		
Total	12 0	,538			

The r esult o f an alyzing optimization in dicated t hat th e characteristic of optimum re sponse is saddle po int. F igure 2 indicate characteristic of response.

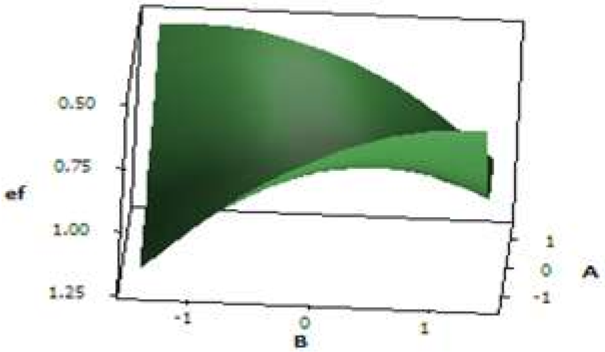

Figure 1: *Surface Plot* Second-Order Model Efficiency DEA *Aggressive*

Compatibility c an be s een from F igure 1 where t he s addle point c onditions indicated t he pa ttern o f curved image. Saddle po int condition c an be solved with r idge an alysis. Ridge a nalysis p roduce o ptimum efficiency value i s 1.113384 with C onstant mounting le ngth variable 0 .497573 and pool d istance v ariable 1 .323787. If converted at t he actual v alue for the po ol d istance i s 2 4.5024 mm an d constant mounting length 27.5819 m m. T he o ptimum conditions obtained in Lumen 14 13,6 Wattage 10 0 and L ife time 1756.

7. Comparison Result of Analysis

Comparison be tween D EA methods Aggressive with Desirability method will be shown in Table 3.

Table 3: Comparison Result of Analysis

Methods	*Response*			*Factor*	
	Lumen	*Wattage*	*Life time*	*Constant mounting length*	*pool distance*
	Y_1	Y_2	Y_3	X_1	X_2
Desirability	1381,20	99,3962	1428,18 2	4,1	28,5
DEA *Aggressive*	1413,60	100,052	1755,99 24	,5	27,58

The r esults show t hat using the method of DEA i s the criterion LTB. T he result increases 1 4,923% for lu men response var iable a nd in creases 22,95% f or lif e ti me response variable. On the other hand, Desirability gave better result than DEA aggressive.

8. Conclusion

This pa per has d iscusses ab out o ptimization multi-response. The m ethod used r esponse surface with DE A aggressive. Optimization re sults with this method co mpared with t he method de sirability function (previous study). Ba sed on the discussion at the previous section, it can be some conclusions as follows:

1) DEA aggressive g ave co mbination be tween co nstant mounting length 24,5 mm and pool distcance 27,58.
2) It results lumen variable 1413,6 lm, wattage variable 100 w, and life time variable 1755 hours.
3) LTB c riteria DEA method p roduces b etter t han desirability method , O therwise ST B criteria d esirability method produces better than DEA method.

This study has been obtained setting Constant mounting and pool length distance to get optimum of lumens, wattage and life time. The result of optimization using this method does not guarantee global optimization. Future work will focus on determining the global optimization on this issue.

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Author Profile

Sony Sunaryo received his bachelor degree in Statistics Department from Sepuluh Nopember Institute of Technology (ITS) and received his master degree in Statistics department from Bogor Agricultural University, Indonesia. He finished his doctoral program in Bogor Agricultural University, He is a lecturer and also working as head of industrial laboratory in Statistics Department, ITS.

Muhammad Sjahid Akbar received his S.Si degree in Statistics department from Sepuluh Nopember Institute of Technology (ITS), Indonesia and received his master degree in Statistics department from Bogor Agricultural University, Indonesia. He is student doctoral program in Statistics department, ITS Indonesia

Haryono received his bachelor degree Mathematic department from Sepuluh Nopember Institute of Technology (ITS), Indonesia received his M.Sc in Industrial and system Engineering, Frorida, USA. He is a lecturer in Statistics department, ITS Indonesia.

Rendra Erdkhadifa received his S.Si degree in Statistics department from Sepuluh Nopember Institute of Technology (ITS), Indonesia. He also was graduated master program in Statistics department, ITS Indonesia as cumlaude.

Appendix 1: Cross-efficiencies Matrix DEA Aggressive

DMUi / DMUt	1	2	3 4	5		6	7	8	9	10	11	12	13
1		0,979	0,927	0,903	0,881	0,942 0,	940 0	,939	0,936 0	,937 0,	956	0,968	1
2 0,	429		0,608	0,604	0,859	0,602 0,	598 0	,610	0,335 0	,429 0,	428	0,249 0	,998
3	0,654 1,	524		0,921	1,309	0,917 0	,911 0	,929	0,511 0,	654 0	,653	0,379 1	,521
4	0,641 1,	494	0,909		1,284	0,899 0	,893 0,	911	0,501 0	,641 0	,640	0,371 1,	491
5	0,440 1,	026	0,624	0,620		0,617 0	,613 0,	625	0,344 0	,440 0	,439	0,255 1,	024
6	0,672 1,	566	0,952	0,946	1,346		0,936 0	,955	0,525 0,	672 0	,671	0,389 1	,563
7	0,675 1	,573	0,957	0,950	1,351	0,946		0,959	0,528 0,	675 0	,674	0,391 1,	569
8	0,661 1	,540	0,937	0,930	1,323	0,926	0,920		0,517 0,	661 0	,659	0,383 1,	536
9	0,956	0,979	0,927	0,903	0,881	0,942 0	,940 0,	939		0,937	0,956 0	,968	1
10	0,956	0,979	0,927	0,903	0,881	0,942 0	,940 0,	939	0,936		0,956 0,	968	1
11	0,956	0,979	0,927	0,903	0,881	0,942 0,	940 0	,939	0,936 0	,937		0,968 1	
12	0,956	0,979	0,927	0,903	0,881	0,942 0,	940 0	,939	0,936 0	,937 0,	956		1
13	0,430	1,002	0,610	0,606	0,861	0,603 0	,599 0,	611	0,336 0,	430 0	,429	0,249	
e_i	0,702	1,218	0,853	0,841	1,062	0,843 0	,847 0,	858	0,612 0	,696 0	,702	0,545 1,	225