

Multiresponse Optimization in Response Surface Method by Aggressive DEA

Sony Sunaryo¹, Muhammad Sjahid Akbar², Haryono³, Rendra Erdkhadifa⁴

^{1,2,3,4}Department of Statistics, Sepuluh Nopember Institute of Technology Surabaya, Indonesia

Abstract: This paper uses DEA method and response surface to optimize quality of GLS lamp. DEA is non-parametric method that used to evaluate relative efficiency. DEA which used was DEA CCR. This method will apply on case GLS lamp. The result of DEA method will be optimized with response surface. The results show the optimization on GLS lamps is better than the desirability for LTB criteria

Keywords: DEA, Aggressive DEA, Response surface, optimized, GLS lamp

1. Introduction

Quality is conformity of the product that used to satisfy needs. Identifying a quality needs detail process. The quality is closely related to the industry process. Today, demand of industrial products tends to increase. So, some of quality variables need to be paid attention for resulting product that appropriately with customers demand. Suitability of the product is affected by the setting of variable quality (independent variable). The results of the variable settings will produce optimal quality products according to customer wishes (response variable). If a customer desire one response, then many methods that have been discussed. But if multi-response and optimized simultaneously, than are not easy to solve. There are several methods used for optimization multi-response, one of which is a desirability function [1-2].

Desirability function is an optimization method for multi-responses that introduced by Derringer and Suich [3]. This method optimized multi-responses with changing into new single response. This response is linear combination from the old responses. Koksoy used Derringer and Suich's method for optimizing dual response [2]. On desirability function, each response assumed has the same priority. This method uses equal weight to each response. In real conditions, there is a response that has a higher priority than the other responses. This paper discusses the method of Data Envelopment Analysis (DEA) that using different weights on the response. For the next step, the optimization with using response surface methodology [4-5].

Data envelopment analysis is a mathematic methodology that used to estimate related multiple inputs and multiple outputs. Each variable has different weight with constructing efficiency in every calculating unit whom called decision making unit (DMU). DEA had been introduced by Charnes, Cooper, dan Rhodes was DEA CCR [6]. Al-Refaie dan Li show that CCR shows the model of the CCR allows generating efficiency values are less precise in setting unrealistic weighting. Because DEA CCR can be called input-oriented which assessing sum weighted equal with 1. DEA aggressive is development of DEA CCR. This method minimized from cross-efficiencies of other DMU and efficiency value is more than 1[7].

Several research on application of DEA include Erta and Ruan determine optimal operator in CMS (cellular manufacturing system) [8]. Mo usavi-Avval, Ra fiee, J afari, and Mo hammadi on the use of energy for production of canola [9]. A zadeh, Ghaderi, Mirjalili, and Mo ghaddam conducted an analysis on a bank with a method combining DEA with Analytical Hierarchy Process (AHP) and Principal Components Analysis (PCA) [10]. Research on aggressive DEA, Al-Refaie and Li who solve cases multi-responses on the Taguchi method [7]. Angiz, Mustafa, and Kamali (2013) who explains that some applications in DEA ranking Decision Making Units (DMUs) are important by using cross-efficiencies [11].

This research uses DEA method and response surface which will be applied on case of optimization quality of GLS lamp. This case encourages Fatima to conduct research on how to define the components of PD and CML to obtain the best quality of the lamp. Fatima using the desirability function method to determine the optimum value [12].

The content of this paper is organized as follows: Section 2 describes DEA method. Section 3 describes response surface method. Section 4 provides an application of the method of DEA on selected case studies. Section 5 contains the results of the case solving. Section 6 describes comparison result of the research. Section 7 is conclusions.

2. Data Envelopment Analysis

DEA is non-parametric statistics technique programming that used to evaluate relative efficiency for homogeneity decision making units (DMUs). The method of DEA which often used is DEA CCR. CCR model uses virtual multiplier that combining multiple inputs and multiple outputs into single index. Virtual multiplier that used is to generate sum of weighted output divided with sum of weighted input. Relative efficiency for each DMUs, θ , to get each efficiency value can be evaluated by solving:

$$\max \theta_t = \frac{\sum_{j=1}^n v_j y_{jt}}{\sum_{k=1}^p u_k y_{kt}} \quad (1)$$

Clearly, constraints for (1)

$$\frac{\sum_{j=1}^n v_j y_{ji}}{\sum_{k=1}^p u_k y_{ki}} \leq 1$$

$$u_1, u_2, \dots, u_p \geq 0$$

$$v_1, v_2, \dots, v_n \geq 0$$

$$i = 1, 2, \dots, m \quad j = 1, 2, \dots, n \quad k = 1, 2, \dots, p$$

The model of DEA CCR can be changed into oriented input that setting sum of weighted input. For the first step is transforming CCR model into linear model.

$$\max \theta_t = \sum_{j=1}^n v_j y_{jt} \quad (2)$$

constraints:

$$\sum_{k=1}^p u_k y_{kt} = 1$$

$$\sum_{j=1}^n v_j y_{ji} \leq \sum_{k=1}^p u_k y_{ki}$$

$$\sum_{j=1}^n v_j y_{jt} - \sum_{k=1}^p u_k y_{kt} \leq 0$$

3. Data Envelopment Analysis Aggressive

CCR model can give uncorrect efficiency value on misleading weighted. Cause of DEA CCR is oriented input which assessing sum of weighted equal with 1. Data envelopment analysis with a aggressive formulation is completion of DEA CCR that optimizing model with minimizing cross-efficiencies of other DMUs. In this study, for solving multiresponses, to determine which being input and output based on following [7]:

1. If all of responses have smaller the better (STB) type, one of them is selected for being input.
2. If all of responses have nominal the better (NTB) type, to calculate quality loss for being input and and one of other responses for being output.

Quality loss formulated as:

$$L_i = c \left(\frac{s_i^2}{\bar{y}_i^2} \right) \text{ where } c \text{ is coefficient quality loss. } \bar{y}_i, s_i \text{ are}$$

average and deviation standard of replication for each DMUs.

3. If all of responses have three types, set STB and quality loss as input. Set larger the better (LTB) type as output.
4. If all of responses have STB and LTB type, set LTB type as output and STB type as input.

The model of DEA formulation a aggressive for each is minimize [7].

$$\min \sum_{j=1}^n \left(v_{jt} \sum_{i \neq j} y_{ji} \right) - \sum_{k=1}^p \left(u_{kt} \sum_{i \neq k} y_{ki} \right) \quad (3)$$

$$\text{constraints. } \sum_{k=1}^p \left(u_{kt} \sum_{i \neq k} y_{ki} \right) = 1$$

$$\sum_{j=1}^n v_{jt} y_{ji} - \sum_{k=1}^p u_{kt} y_{ki} \leq \delta; \quad \forall j \neq 0$$

$$\sum_{j=1}^n v_{jt} y_{jt} - E_{tt} \sum_{k=1}^p u_{kt} y_{kt} = 0$$

$$v_{jt}, u_{kt} \geq 0$$

Cross-efficiencies DMUs is calculated with optimal weighted both input and output. Cross-efficiencies, E_{ii} , can be evaluated by solving.

$$E_{ii} = \frac{\sum_{j=1}^n v_{jt} y_{ji}}{\sum_{k=1}^p u_{kt} y_{ki}} \quad (4)$$

Mean of cross-efficiencies is efficiency value DEA aggressive formulation for each DMUs.

$$e_i = \frac{\sum_{t \neq i} E_{ti}}{n-1} \quad (5)$$

4. Response Surface Methodology (RSM)

RSM is an optimization method. The character of this method is combination between experiment design and regression. The designs often used in some researches were Central Composite Design (CCD) and Box-Behnken Design. For the first step single response optimization in RSM is to analyze the first order model and then, to check appropriate model by significance of lack of fit (LOF). If LOF is significant or it's not appropriate model, for the next step is analyzing the second order model. Let the second order model be as in (6) [13].

$$\hat{y} = \hat{\beta}_0 + \sum_{i=1}^k \hat{\beta}_i x_i + \sum_{i=1}^k \hat{\beta}_{ii} x_i^2 + \sum_{i < j} \hat{\beta}_{ij} x_i x_j \quad (6)$$

$$\hat{y} = \hat{\beta}_0 + \mathbf{x}'\mathbf{b} + \mathbf{x}'\mathbf{B}\mathbf{x} \quad (7)$$

If the second order model is a appropriate model, it will get correct parameter setting to find optimum response. Optimum condition can be found from derivatives $\frac{\partial \hat{y}}{\partial \mathbf{x}} = 0$.

So, in that case

$$\frac{\partial \hat{y}}{\partial \mathbf{x}} = \mathbf{b} + 2\mathbf{B}\mathbf{x} = 0 \quad (8)$$

Stationary point can be found with $\mathbf{x}_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b}$. Stationary

point represent optimum response by $\hat{y}_s = \hat{\beta}_0 + \frac{1}{2}\mathbf{x}_s'\mathbf{b}$. After analyze optimization, for the next step is checking residual assumption for each response.

5. Application

The data that used in this study was research from Fatima [12]. This research used central composite design with 13 units of GLS lamps. Type of GLS lamp is A55 100 W 230 V. Constant mounting length (X_1) and pool distance (X_2) are important components On GLS lamps. They are estimated to have influence against lumen (Y_1), wattage (Y_2), and life time (Y_3). Optimum response criterion of lumen and life time is larger the better, but response criterion of wattage is smaller the better. In that case we will use DEA aggressive. In the application of DEA, the response values are used to establish the efficiency which inputs and outputs a re

determined by the optimal response criteria. DEA weighted value is determined from the results of the analysis with linear programming. Efficiency value optimized with response surface methodology.

6. Result

The first step to get aggressive DEA efficiency value is to calculate a weighted value for each DMU. Table 1 shows weighted value with aggressive formulation for each DMUs. U_{1i} is weighted input value. V_{1i} is weighted value for first output and V_{2i} is weighted value for second output. Result of Analyze DEA aggressive is indicated on appendix 1. e_i is efficiency which calculated from average DMUs value. From the analysis the highest efficiency value is 1,225 DMU_i 13. The lowest value is 0,545 DMU_i 12. Appendix 1 also informs about ranking of average value which showed by ordinal value (OV) for knowing best performer. The ranking is got from the lowest value to the highest value. Based on ordinal value, and then to analyze optimization using response surface.

Table 1: Weighted Value Aggressive Formulation for Each DMUs

DMUs	Weighted Aggressive Formulation		
	Input	Output	
	U_{1i}	V_{1i}	V_{2i}
1	0,0008305 0	00005691	0
2 0	,0008311	0	0,00004205
3 0	,0008296	0	0,00006396
4 0	,0008300	0	0,00006276
5 0	,0008308	0	0,00004312
6 0	,0008303	0	0,00006579
7 0	,0008308	0	0,00006612
8 0	,0008294	0	0,00006462
9	0,0008308 0	00005693	0
10	0,0008305 0	,00005691	0
11	0,0008306 0	,00005692	0
12	0,0008308 0	,00005693	0
13 0	,0008312	0	0,00004215

RSM model used to analyze is second-order model.

$$\hat{y} = 0,86254 - 0,03818x_1 - 0,0226x_2 - 0,10544x_1^2 + 0,07919x_2^2 + 0,13552x_1x_2$$

Table 2 ANOVA of Second-Order Model For Efficiency DEA Aggressive

Source	df	SS	MS	F	P-value
Regression	5 0	227	0,045 1	,02	0,470
Residual Error	7	0,311	0,0444		
Lack of fit	3	0,132	0,0441	0,99	0,483
Pure Error	4	0,178	0,0446		
Total	12 0	,538			

The result of analyzing optimization indicated that the characteristic of optimum response is saddle point. Figure 2 indicate characteristic of response.

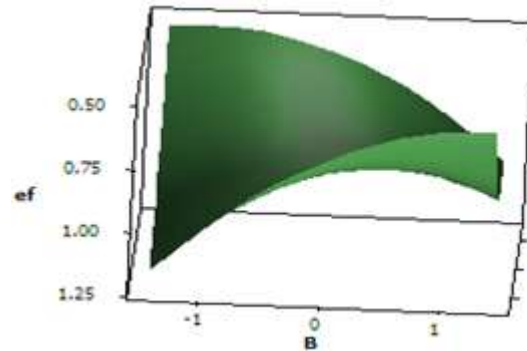


Figure 1: Surface Plot Second-Order Model Efficiency DEA Aggressive

Compatibility can be seen from Figure 1 where the saddle point conditions indicated the pattern of curved image. Saddle point condition can be solved with ridge analysis. Ridge analysis produce optimum efficiency value is 1.113384 with Constant mounting length variable 0.497573 and pool distance variable 1.323787. If converted at the actual value for the pool distance is 2.45024 mm and constant mounting length 27.5819 mm. The optimum conditions obtained in Lumen 1413,6 Wattage 100 and Life time 1756.

7. Comparison Result of Analysis

Comparison between DEA methods Aggressive with Desirability method will be shown in Table 3.

Table 3: Comparison Result of Analysis

Methods	Response			Factor	
	Lumen	Wattage	Life time	Constant mounting length	pool distance
	Y_1	Y_2	Y_3	X_1	X_2
Desirability	1381,20	99,3962	1428,182	4,1	28,5
DEA Aggressive	1413,60	100,052	1755,9924	,5	27,58

The results show that using the method of DEA is the criterion LTB. The result increases 14,923% for lumen response variable and increases 22,95% for life time response variable. On the other hand, Desirability gave better result than DEA aggressive.

8. Conclusion

This paper has discusses about optimization multi-response. The method used response surface with DEA aggressive. Optimization results with this method compared with the method desirability function (previous study). Based on the discussion at the previous section, it can be some conclusions as follows:

- 1) DEA aggressive gave combination between constant mounting length 24,5 mm and pool distance 27,58.
- 2) It results lumen variable 1413,6 lm, wattage variable 100 w, and life time variable 1755 hours.
- 3) LTB criteria DEA method produces better than desirability method, Otherwise STB criteria desirability method produces better than DEA method.

This study has been obtained setting Constant mounting and pool length distance to get optimum of lumens, wattage and life time. The result of optimization using this method does not guarantee global optimization. Future work will focus on determining the global optimization on this issue.

References

- [1] Khuri, A. I., and Cornell, J. A., "Response Surfaces Designs and Analysis". New York: Marcel Dekker, 1996.
- [2] Köksoy, O., "Dual Response Optimization: The Desirability Approach", *International Journal of Industrial Engineering*, 12(4), pp. 335-342, 2005
- [3] Derringer, G. and Suich, R., "Simultaneous Optimization of Several Response Variables", *Journal Quality Technology*, vol. 12, no. 4, pp. 214-218, 1980
- [4] Doyle, J. and Green, R., "Efficiency and Cross-Efficiency in DEA: Derivations, Meanings and Uses". *The Journal of the Operational Research Society*, vol. 45, no. 5, pp. 567-578, 1994.
- [5] Ali, A. A., Lerme, C. S., and Seiford, L. M., "Components of Efficiency Evaluation in Data Envelopment Analysis". *European Journal of Operational Research*, 80, pp. 462-473. 1995.
- [6] Charnes, A., Cooper, W. W., and Rhodes, E., "Measuring The Efficiency Of Decision Making Units", *European Journal of Operational Research*, 2, pp. 429-444, 1978.
- [7] Al-Refaie, A., Li, M. H., "Solving the Multi-response Problem in Taguchi Method by Aggressive Formulation in DEA". *Proceedings of the World Congress on Engineering, London, Vol II, 2008.*
- [8] Ertay, T., Ruan, D., "Data Envelopment Analysis Based Decision Model For Optimal Operator Allocation in CMS". *European Journal of Operational Research*, 164, pp. 800-810, 2005.
- [9] Mousavi-Avval, S. H., Raftaei, S., Jafari, A., & Mohammadi, A., "Improving Energy Use Efficiency of Canola Production Using Data Envelopment Analysis (DEA) Approach". *Karaj : Department of Agricultural Machinery Engineering, Faculty of Agricultural Engineering and Technology, University of Tehran, 2011.*
- [10] Azadeh, A., Ghadiri, S. F., Mirjalili, M., and Moghaddam, M., "Integration of Analytic Hierarchy Process and Data Envelopment Analysis for Assessment and Optimization of Personnel Productivity In A Large Industrial Bank". *Expert Systems with Applications*, 38, pp. 5212-5225, 2011.
- [11] Angiz, M. Z., Mustafa, A., & Kamali, M. J., "Cross-ranking of Decision Making Units in Data Envelopment Analysis". *Applied Mathematical Modelling*, vol. 37, issues. 1-2, pp. 398-405, 2013.
- [12] Fatima, A., "Optimasi Lumen, Wattage Dan Life Time Pada Lampu GLS Dengan Menggunakan Pendekatan Fungsi Desirability pada Metode Multiresponse Surface (Studi Kasus: P.T.P Hilips Indonesia)". *Surabaya: Undergraduate Program, Department of Statistics, Sepuluh Nopember Institute of Technology, Indonesia, 2012.*
- [13] Montgomery, D. C., "Design and Analysis of Experiments 5th edition". New York: John Wiley, Sons. 1997.
- [14] Park, S. H., "Robust Design and Analysis for Quality Engineering". London: Chapman & Hall, 2-6 Boundary Row, London SE1 8HN, UK., 1996

Author Profile



Sony Sunaryo received his bachelor degree in Statistics Department from Sepuluh Nopember Institute of Technology (ITS) and received his master degree in Statistics department from Bogor Agricultural University, Indonesia. He finished his doctoral program in Bogor Agricultural University, He is a lecturer and also working as head of industrial laboratory in Statistics Department, ITS.



Muhammad Sjahid Akbar received his S.Si degree in Statistics department from Sepuluh Nopember Institute of Technology (ITS), Indonesia and received his master degree in Statistics department from Bogor Agricultural University, Indonesia. He is student doctoral program in Statistics department, ITS Indonesia



Haryono received his bachelor degree Mathematics department from Sepuluh Nopember Institute of Technology (ITS), Indonesia received his M.Sc in Industrial and System Engineering, Florida, USA. He is a lecturer in Statistics department, ITS Indonesia.



Rendra Erdkhadifa received his S.Si degree in Statistics department from Sepuluh Nopember Institute of Technology (ITS), Indonesia. He also was graduated master program in Statistics department, ITS Indonesia as cumlaude.

Appendix 1: Cross-efficiencies Matrix DEA Aggressive

DMU _i \ DMU _t	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	0,979	0,927	0,903	0,881	0,942 0	0,940 0	0,939	0,936 0	0,937 0	0,956	0,968	1
2	0,429	1	0,608	0,604	0,859	0,602 0	0,598 0	0,610	0,335 0	0,429 0	0,428	0,249 0	0,998
3	0,654 1	0,524	1	0,921	1,309	0,917 0	0,911 0	0,929	0,511 0	0,654 0	0,653	0,379 1	0,521
4	0,641 1	0,494	0,909	1	1,284	0,899 0	0,893 0	0,911	0,501 0	0,641 0	0,640	0,371 1	0,491
5	0,440 1	0,026	0,624	0,620	1	0,617 0	0,613 0	0,625	0,344 0	0,440 0	0,439	0,255 1	0,024
6	0,672 1	0,566	0,952	0,946	1,346	1	0,936 0	0,955	0,525 0	0,672 0	0,671	0,389 1	0,563
7	0,675 1	0,573	0,957	0,950	1,351	0,946	1	0,959	0,528 0	0,675 0	0,674	0,391 1	0,569
8	0,661 1	0,540	0,937	0,930	1,323	0,926	0,920	1	0,517 0	0,661 0	0,659	0,383 1	0,536
9	0,956	0,979	0,927	0,903	0,881	0,942 0	0,940 0	0,939	1	0,937	0,956 0	0,968	1
10	0,956	0,979	0,927	0,903	0,881	0,942 0	0,940 0	0,939	0,936	1	0,956 0	0,968	1
11	0,956	0,979	0,927	0,903	0,881	0,942 0	0,940 0	0,939	0,936 0	0,937	1	0,968 1	
12	0,956	0,979	0,927	0,903	0,881	0,942 0	0,940 0	0,939	0,936 0	0,937 0	0,956	1	
13	0,430	1,002	0,610	0,606	0,861	0,603 0	0,599 0	0,611	0,336 0	0,430 0	0,429	0,249	1
e _i	0,702	1,218	0,853	0,841	1,062	0,843 0	0,847 0	0,858	0,612 0	0,696 0	0,702	0,545 1	0,225

