

Existence Results for Quasilinear Delay Integro-differential Equations with Nonlocal Conditions Via Measures of Noncompactness

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Abstract: We study the existence of mild solutions for quasilinear delay integro differential equations with nonlocal Cauchy problem in Banach spaces. The results are established by using Hausdorff's measure of non-compactness.

Keywords: Mild solution, nonlocal conditions; Hausdorff's measure of noncompactness

1. Introduction

The notion of a measure of non-compactness turns out to be a very important and useful tool in many branches of mathematical analysis. The notion of a measure of weak compactness was introduced by De Blasi [9] and was subsequently used in numerous branches of functional analysis and the theory of differential and integral equations. El-Sayed [11] proves the existence theorem of monotonic solutions for a nonlinear functional integral equation of convolution type by Hausdorff measure of non-compactness.

Fan et al [12] discussed semi linear differential equations with nonlocal condition using measure of non-compactness. Balachandran and Ilammaran [2] studied an existence theorem for functional integral equations with deviating arguments by using the measure of weak noncompactness.

The study of abstract nonlocal initial value problems was initiated by Byszewski [6, 7]. Balachandran and Paul Samuel [3] give the existence and uniqueness of mild and classical solutions for quasi linear delay integro differential equations when f , g and k satisfy Lipschitz-type conditions. The problem of existence of solutions of quasi linear evolution equations in Banach space has been studied by several authors [3, 8, 10, 14]. Pazy [14] considered the following quasi linear equation with local condition of the form

$$u'(t) + A(t, u)u(t) = 0, 0 < t < T$$

$$u(0) = u_0,$$

and discussed the mild and classical solutions by using the fixed point argument. In this paper, we shall consider the problem of the existence of mild solutions of quasi linear delay integro differential equations with non local condition of the form

$$u'(t) + A(t, u)u(t) = f(t, u(\alpha(t)))$$

$$+ \int_0^t g(t, s, u(\beta(s)))ds, t \in [0, T], (1)$$

$$u(0) + h(u) = u_0, (2)$$

Where $A : [0, T] \times X \rightarrow X$ are continuous functions in Banach space X , $u_0 \in X$, $f : [0, T] \times X \rightarrow X$, $g : \Delta \times X \rightarrow X$, $h : C([0, T]; X) \rightarrow X$ and α, β are given functions, Here $\Delta = \{t, s, 0 \leq s \leq t \leq T\}$. The results obtained in this paper generalize the results of [2, 3, 9, 10].

2. Preliminaries

Let X be a Banach space with norm $\|\cdot\|$. Let $C([0, T]; X)$ be the space of X -valued continuous functions on $[0, T]$ with the norm $\|u\| = \sup\{\|u(t)\|, t \in [0, T]\}$ for $u \in C([0, T]; X)$, and denoted $L(0, T; X)$ by the space of X -valued Bochner integrable functions on $[0, T]$ with the norm $\|u\|_L = \int_0^T \|u(t)\| dt$. The Hausdorff's measure of non compactness χ_Y is defined by $\chi_Y(B) = \inf\{r > 0, B \text{ can be covered by finite number of balls with radii } r\}$ for bounded set B in a Banach space Y .

Lemma 2.1 [4]. Let Y be a real Banach space and $B, E \subseteq Y$ be bounded, with the following properties:

- 1) B is pre compact if and only if $\chi_X(B) = 0$.
- 2) $\chi_Y(B) = \chi_Y(\bar{B}) = \chi_Y(\text{conv } B)$, where \bar{B} and $\text{conv } B$ means the closure and convex hull of B respectively.
- 3) $\chi_Y(B) \leq \chi_Y(E)$, where $B \subseteq E$.
- 4) $\chi_Y(B + E) \leq \chi_Y(B) + \chi_Y(E)$, where $B + E = \{x + y : x \in B, y \in E\}$.
- 5) $\chi_Y(B \cup E) \leq \max\{\chi_Y(B), \chi_Y(E)\}$.
- 6) $\chi_Y(\lambda B) \leq |\lambda| \chi_Y(B)$ for any $\lambda \in \mathbb{R}$.
- 7) If the map $F : D(F) \subseteq Y \rightarrow Z$ is Lipschitz continuous with constant k the $\chi_Y(FB) \leq k \chi_Y(B)$ for any bounded subset $B \subseteq D(F)$, where Z is Banach space.
- 8) $\chi_Y(B) = \inf\{d_Y(B, E); E \subseteq Y \text{ is finite valued, where } d_Y(B, E) \text{ means the nonsymmetric (or symmetric) Hausdorff distance between } B \text{ and } E \text{ in } Y\}$.
- 9) If $\{W_n\}_{n=1}^{+\infty}$ is a decreasing sequence of bounded closed nonempty subset of Y and $\lim_{n \rightarrow \infty} \chi_Y(W_n) = 0$, then $\bigcap_{n=1}^{+\infty} W_n$ is nonempty and compact in Y .

Lemma 2.2 (Darbo-Sadovskii [4]). If $W \subseteq Y$ is bounded closed and convex, the continuous map $F : W \rightarrow W$ is a χ_Y contraction, then the map F has at least one fixed point in W . In this we denote by χ the Hausdorff's measure of non-compactness of X and denote χ_c by the Hausdorff's measure of non compactness of $C([a, T]; X)$. To discuss the existence, we need the following Lemmas in this paper.

Lemma 2.3 [4]. If $W \subseteq C([0, T]; X)$ is bounded, Then $\chi(W(t)) \leq \chi_c(W)$ for all $t \in [0, T]$, where $W(t) = \{u(t); u \in W\} \subseteq X$. Furthermore if W is equi-continuous on $[a, T]$, then $\chi(W(t))$ is continuous on $[a, T]$ and $\chi_c(W) = \sup\{\chi(W(t)), t \in [a, T]\}$.

Lemma 2.4 [13]. If $\{u_n\}_{n=1}^\infty \subset L^1(a, T; X)$ is uniformly integrable, then the function $\chi(\{u_n\}_{n=1}^\infty)$ is measurable and

$$\chi(\{\int_0^t u_n(s) ds\}) \leq 2 \int_0^t \chi(\{u_n\}_{n=1}^\infty) ds \quad (3)$$

Lemma 2.5 [4]. If $W \subseteq C([0, T]; X)$ is bounded and equi-continuous, then then $\chi(W(t))$ is continuous and

$$\chi(\{\int_0^t W(s) ds\}) \leq \int_0^t \chi(W(s)) ds \quad (4)$$

for all $t \in [0, T]$, where $\int_0^t W(s) ds = \{\int_0^t u(s) ds; u \in W\}$.

The C_0 - semigroup $U_u(t, s)$ is said to be equi continuous for $t > 0$ for all bounded set B in X . The following lemma is obvious.

Lemma 2.6. If the evolution family $\{U_u(t, s)\}_{0 \leq s \leq t \leq T}$ is equi-continuous and $\eta \in L(0, T; R^+)$, then the set $\{\int_0^t U_u(t, s)u(s)ds\}, \|u(s)\| \leq \eta(s)$ for i.e $s \in [0, T]$ is equi-continuous for $t \in [0, T]$.

From [8], we know that for any fixed $\in C([0, T]; X)$, there exist a unique continuous function $U_u : [0, T] \times [0, T] \rightarrow B(X)$ defined on $[0, T] \times [0, T]$ such that

$$U_u(t, s) = \int_s^t A_u U_u(w, s) dw \quad (5)$$

where $B(X)$ denote the Banach space of bounded linear operators from X to X with the norm $\|F\| = \sup\{\|Fu\|; \|u\| = 1\}$, and I stands for the identity operator on X , $A_u(t) = A(t, u(t))$. We have $U_u(t, t) = I$, $U_u(t, s)U_u(s, r) = U_u(t, r)$, where $(t, s, r) \in [0, T] \times [0, T] \times [0, T]$, $\frac{\partial U_u(t, s)}{\partial t} = A_u(t)U_u(t, s)$ for almost all $\in [0, T]$. For a mild solution of (1) - (2) we mean a function $u \in C([0, T]; X)$ and $u_0 \in X$ satisfying the integral equation

$$u(t) = U_u(t, 0)u_0 - U_u(t, 0)h(u) + \int_0^t U_u(t, s)[f(s, u(\alpha(s))) + \int_0^s g(s, \tau, u(\beta(\tau)))d\tau]ds, \quad (6)$$

3. The Existence of Results for Compact,

In this section, we give some existence results when h is compact and f satisfy the conditions with respect to Hausdorff's measure of non-compactness and its applications

in differential equations in Banach spaces. We give some existence results of the nonlocal problem (1)- (2). We assume the following assumptions:

(H1) The evolution family $\{U_u(t, s)\}_{0 \leq s \leq t \leq T}$ generated by $A(t, u)$ is equi-continuous, and $\|U_u(t, s)\| \leq M_0$ for almost $t, s \in [0, T]$

(H2) $f : [0, T] \times X \rightarrow X$ satisfies the Caratheodory type conditions and there exist $m_1 \in L[0, T]$ and $b_1 \geq 0$ such that

$$|f(t, u)| \leq m_1(t)b_1\|u\|, \quad t \text{ a.e in } [0, T], u \in R^+$$

(H3) (a) $h : C([0, T]; X) \rightarrow X$ is continuous and compact.

(b) There exist $N > 0$ such that $\|h(u)\| \leq N_0$ for all $u \in C([0, T]; X)$.

(H4) (a) $g : [0, T] \times [0, T] \times X \rightarrow X$ satisfies the Caratheodory-type conditions. (i.e.) $g(\cdot, \cdot, u)$ is measurable for all $u \in X$ and $g(t, s, \cdot)$ is continuous for a.e $t, s \in [a, T]$, there exist $m_2 \in L[0, T]$ and $b_2 \geq 0$ such that $\int_0^t |g(t, s, u)| ds \leq m_2(t)b_2\|u\|$, for a.e in $[0, T], u \in R^+$

(b) There exist $\eta \in L(0, T; R^+), \zeta \in L(0, T; R^+)$, such that $\zeta(g(t, s, D)) \leq \eta(t)\zeta(s)\chi(D)$ for a.e $t, s \in [0, T]$, and for any bounded subset $D \subset C([0, T], X)$. Here we let $\eta(t) \leq K_1$ and $\zeta(t) \leq K_2$.

(H5) $\alpha, \beta : [0, T] \rightarrow [0, T]$ is non decreasing and there exist positive constants δ_1 and δ_2 such that $\alpha'(t) \geq \delta_1$ and $\beta'(t) \geq \delta_2$ respectively for $t \in [0, T]$.

Theorem: 3.1 Under the assumptions (H1) – (H2) are satisfied, then the nonlocal initial value problem. Let $u_0 \in Y$ and the family $A(t, b)$ of linear operators for $t \in I [0, T]$ and (1) – (2) has at least one mild solution.

Proof.

Let $\Omega(t)$ be a solution of the scalar equation

$$\Omega(t) = M_0 N_0 + M_0(m_1(t)b_1/\delta_1 + m_2(t, s)b_2/\delta_2 \int_0^t \Omega(s)ds) \quad (7)$$

for $t \in [0, T]$.

Consider the map $F : C([0, T]; X) \rightarrow C([0, T]; X)$ defined by

$$(Fu)(t) = U_u(t, 0)h(u) + \int_0^t U_u(t, s)[f(s, u(\alpha(s))) + \int_0^s g(s, \tau, u(\beta(\tau)))d\tau]ds \quad (8)$$

for all $u \in C([0, T]; X)$. We can show that F is continuous by the usual techniques.

Let us take

$$W_0 = \{u \in C([0, T]; X), \|u(t)\| \leq \Omega(t), \text{ for all } t \in [0, T]\}.$$

Then $W_0 \subseteq C([0, T]; X)$ is bounded and convex. We define $W_1 = \overline{\text{conv}} K(W_0)$, where conv means the closure of the convex hull in $C([0, T]; X)$. As $U_u(t, s)$ is equi-continuous, g is compact and $W_0 \subseteq C([0, T]; X)$ is bounded, due to Lemma 2.7 and the assumption (H4)(b), $W_1 \subseteq C([0, T]; X)$ is bounded closed convex non empty and equi-continuous on $[0, T]$.

For any $u \in F(W_0)$, we know

$$\begin{aligned} \|u(t)\| &\leq M_0 N_0 \\ &\quad + M_0 \left[\int_0^t \|f(s, u(\alpha(s)))\| ds \right. \\ &\quad \left. + \int_0^t \int_0^s \|g(s, \tau, u(\beta(\tau)))\| d\tau ds \right] \\ &\leq M_0 N_0 + M_0 \left[\int_0^t m_1(s) b_1 \|u(\alpha(s))\| ds \right. \\ &\quad \left. + \int_0^s m_2(s, \tau) b_2 \|u(\beta(\tau))\| d\tau \right] \\ &\leq M_0 N_0 + M_0 [m_1(t) b_1 \int_0^t \|u(\alpha(s))\| \alpha'(s) / \delta_1 ds \\ &\quad + m_2(s, \tau) b_2 \int_0^s \|u(\beta(\tau))\| \beta'(\tau) / \delta_2 d\tau] \\ &\leq M_0 N_0 + M_0 [m_1(t) b_1 / \delta_1 \int_0^t \|u(s)\| ds \\ &\quad + m_2(s, \tau) b_2 / \delta_2 \int_0^s \|u(\tau)\| d\tau] \\ &\leq M_0 N_0 + M_0 [m_1(t) b_1 / \delta_1 + m_2(t, s) b_2 / \delta_2 \int_0^t \Omega(s) ds, \\ &= \Omega(t), \text{ for } t, s, \tau \in [0, T]. \end{aligned}$$

It follows that $W_1 \subset W_0$. We define $W_{n+1} = \overline{c \circ n \circ v} F(W_n)$, for $n = 1, 2, 3, \dots$. From above we know that $\{W_n\}_{n=1}^\infty$ is a decreasing sequence of bounded, closed, convex, equicontinuous on $[0, T]$ and non empty subsets in $C([0, T]; X)$.

Now for $n \geq 1$ and $t \in [0, T]$, $W_n(t)$ and $F(W_n(t))$ are bounded subsets of X , hence, for any $\epsilon > 0$, there is a sequence $\{u_k\}_{k=1}^\infty \subseteq W_n$ such that (see, e.g. [5], pp125).

$$\begin{aligned} \chi(W_{n+1}) &= \chi(FW_n(t)) \\ &\leq \chi \left(\int_0^t U_u(t, s) [f(s, \{u_k \alpha(s)\}_{k=1}^\infty) ds \right. \\ &\quad \left. + \int_0^s g(s, \tau, \{u_k \beta(\tau)\}_{k=1}^\infty) d\tau ds \right] \\ &\leq 2M_0 \int_0^t [\chi(f(s, \{u_k \alpha(s)\}_{k=1}^\infty) ds] \\ &\quad + 4M_0 \int_0^t \int_0^s [\chi(g(s, \tau, \{u_k \beta(\tau)\}_{k=1}^\infty) d\tau ds] \\ &\leq 2M_0 K_1 \int_0^t \chi(\{u_k \alpha(s)\}_{k=1}^\infty) ds \\ &\quad + 4M_0 K_1 K_2 \int_0^t \int_0^s [\chi(\{u_k \beta(\tau)\}_{k=1}^\infty) d\tau ds] \\ &\leq 2M_0 K_1 / \delta_1 \int_0^t \chi(\{u_k(s)\}_{k=1}^\infty) ds \\ &\quad + 4M_0 K_1 K_2 / \delta_2 \int_0^t \chi(\{u_k(s)\}_{k=1}^\infty) ds + \epsilon \\ &\leq (2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2} \right]) \int_0^t \chi(\{u_k(s)\}_{k=1}^\infty) ds + \epsilon \end{aligned}$$

$$\leq (2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2} \right]) \int_0^t \chi(W_n(s)) ds + \epsilon$$

Since $\epsilon > 0$ is arbitrary, it follows that from the above inequality that

$$\chi(W_{n+1}(t)) \leq (2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2} \right]) \int_0^t \chi(W_n(s)) ds \quad (9)$$

for all $t \in [0, T]$. Because W_n is decreasing for n , we have

$$\sigma(t) = \lim_{n \rightarrow \infty} \chi(W_n(t))$$

for all $t \in [0, T]$. From (9), we have

$$\sigma(t) \leq (2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2} \right]) \int_0^t (\sigma(s)) ds$$

for $t \in [0, T]$, which implies that $\sigma(t) = 0$ for all $t \in [0, T]$. By Lemma 2.3, we know that $\lim_{n \rightarrow \infty} \chi(W_n(t)) = 0$.

Using Lemma 2.1 we know that $W = \bigcap_{n=1}^\infty W_n$ is convex compact and nonempty in $C([0, T]; X)$ and $F(W) \subset W$. By the famous Schauder's fixed point theorem, there exist at least one mild solution u of the initial value problem (1) – (2), where $u \in W$ is a fixed point of the continuous map F .

Remark 3.2. If the functions f and g are compactor Lipschitz continuous (see e.g. [6, 7, 15]), then (H4) is automatically satisfied. In some of the early related results in references and above results, it is supposed that the map h is uniformly bounded. We indicate here that this condition can be released. In fact, if h is compact, then it must be bounded on bounded set. Here we give an existence result under growth condition of f and g , when h is not uniformly bounded. Precisely, we replace the assumptions (H4) by

(H6) There exist functions $p \in L(0, T; R^+)$ and $q \in L(0, T; R^+)$. The Constants $b_1, b_2 > 0$ such that

$$\begin{aligned} \|f(t, u)\| &\leq p(t) b_1 \|u\|, \\ \int_0^t \|g(t, s, u)\| ds &\leq q(t, s) b_2 \|u\| \end{aligned}$$

for a.e. $t \in [0, T]$ and all $u \in C([0, T]; X)$.

Theorem: 3.2 Suppose that the assumptions (H1) – (H6) are satisfied, then the equation (1) – (2) has at least one mild solution if

$$\limsup_{r \rightarrow \infty} \frac{M_0}{r} \left(\varphi(r) + rT \left[\frac{pb_1}{\delta_1} + \frac{qb_2}{\delta_2} \right] \right) < 1. \quad (10)$$

Where $\varphi(r) = \sup\{\|h(u)\|, \|u\| \leq r\}$.

Proof. The inequality (10) implies that there exist a constant $r > 0$ such that

$$M_0 \left(\varphi(r) + rT \left[\frac{pb_1}{\delta_1} + \frac{qb_2}{\delta_2} \right] \right) < r.$$

Just as in the proof of Theorem 3.1, let $W_0 = \{u \in C([0, T]; X), \|u(t)\| \leq r\}$ and $W_1 = \overline{c \circ n \circ v} F W_0$. Then for any $u \in W_1$, we have

$$\begin{aligned} \|u(t)\| &\leq \|U_u(t, 0) h(u)\| \\ &\quad + \int_0^t U_u(t, s) [f(s, u(\alpha(s))) \\ &\quad + \int_0^s g(s, \tau, u(\beta(\tau))) d\tau] ds, \end{aligned}$$

$$\begin{aligned} &\leq M_0\varphi(r) + M_0 \left[\int_0^t p(s)b_1 \|u(\alpha(s))\| ds \right. \\ &\quad \left. + \int_0^s q(s,\tau)b_2 \|u(\beta(\tau))\| d\tau \right] \\ &\leq M_0\varphi(r) + M_0 \left(\left[\frac{p(t)b_1 r T}{\delta_1} + \frac{q(s,\tau)b_2 r T}{\delta_2} \right] \right) \\ \|u(t)\| &\leq M_0 \left(\varphi(r) + r T \left[\frac{p b_1}{\delta_1} + \frac{q b_2}{\delta_2} \right] \right) < r \end{aligned}$$

for $t \in [0, T]$. It means that $W_1 \subset W_0$. So, can complete the proof similarly to Theorem 3.1.

4. The Existence of Results for Lipschitz_h

In the previous section, we obtained the existence results when h is compact but without the compactness of $U_u(t, s)_{0 \leq s \leq t \leq T}$ or f and g . In this section, we discuss the equation (1) – (2) when h is Lipschitz and f and g are not Lipschitz. Again we assume that $(H7)$ is a Lipschitz continuous in X , there exist a constant $L_0 > 0$ such that

$$\|h(u) - h(v)\| \leq L_0 \|u - v\|, u, v \in C([0, T]; X).$$

Theorem: 4.1 Suppose that the assumptions $(H1)$, $(H2)$, $(H4)$, $H(6)$ are satisfied, then the equation (1) – (2) has at least one mild solution provided that $\left(M_0 L_0 + T M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2} \right] \right) < 1$. (11)

Proof. Consider the map $F_1, F_2 : C([0, B]; X) \rightarrow C([0, B]; X)$ defined by $F_1 + F_2 = F$, where

$$\begin{aligned} F_1(u)(t) &= U_u(t, 0)h(u). \\ F_2(u)(t) &= \int_0^t U_u(t, s) [f(s, u(\alpha(s))) + \\ &\quad \int_0^s g(s, \tau, u(\beta(\tau))) d\tau] ds, \end{aligned}$$

for $u \in C([0, B]; X)$. As defined in the proof of Theorem 3.1. We define $W_0 = \{u \in C([0, B]; X) : \|u(t)\| \leq \Omega(t) \text{ for all } t \in [0, T] \text{ and let } W = \overline{\text{conv}} F W_0$. Then from the proof of Theorem 3.1 we know that W is a bounded closed convex and equi continuous subset of $C([0, T]; X)$ and $FW \subset W$. We shall prove that F is χ_c -contraction on W . Then Darbo-Sadovskii's fixed point theorem can be used to get a fixed point of F in W , which is a mild solution of (1) – (2). First, for every bounded subset $B \subset W$, from the $(H7)$ and Lemma 2.1, we have

$$\begin{aligned} \chi_c(F_1 B) &= \chi_c(U_B(t, 0)h(B)) \\ &\leq M_0 \chi_c(h(B)) \\ &\leq M_0 L_0 \chi_c(B) \quad (12) \end{aligned}$$

Next, for every bounded subset $B \subset W$, for $t \in [0, T]$ and every $\epsilon > 0$, there is a sequence $\{u_k\}_{k=1}^\infty \subset B$, such that $\chi(F_2 B(t)) \leq 2\chi\{F_2 u_k(t)\}_{k=1}^\infty + \epsilon$.

Note that B and $F_2 B$ are equi-continuous, we can get from Lemma 2.1, Lemma 2.4, Lemma 2.5 and $(H4)$ that

$$\begin{aligned} \chi(F_2 B(t)) &\leq \chi \left(\int_0^t U_u(t, s) \left[f(s, \{u_k(\alpha(s))\}_{k=1}^\infty) \right. \right. \\ &\quad \left. \left. + \int_0^s g(s, \tau, \{u_k(\beta(\tau))\}_{k=1}^\infty) d\tau \right] ds \right) \end{aligned}$$

$$\begin{aligned} &\leq 2M_0 \int_0^t \left[\chi \left(f \left(s, \{u_k(\alpha(s))\}_{k=1}^\infty \right) \right) \right] ds \\ &\quad + 4M_0 \int_0^t \int_0^s \left[\chi \left(g(s, \tau, \{u_k(\beta(\tau))\}_{k=1}^\infty) \right) \right] d\tau ds \\ &\leq \left(2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2} \right] \right) \chi_c(B) T + \epsilon \end{aligned}$$

for all $t \in [0, T]$.

Since $\epsilon > 0$ is arbitrary, we have

$$\chi(F_2 B(t)) \leq T \left(2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2} \right] \right) \chi_c(B) \quad (13)$$

For any bounded $B \subset W$.

Now, for any subset $B \subset W$, due to Lemma 2.1, (12) and (13) we have

$$\chi_c(FB) \leq \chi_c(F_1 B) + \chi_c(F_2 B)$$

$$\chi_c(FB) \leq \left[M_0 L_0 + T \left(2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2} \right] \right) \right] \chi_c(B).$$

We know that F is a χ_c -contraction on W . By Lemma 2.2, there is a fixed point u of F in W , which is a solution of (1) – (2). This completes the proof.

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