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Existence Results for Quasilinear Delay Integrodifferential Equations with Nonlocal Conditions Via Measures of Noncompactness

Francis Paul Samuel[1], Tumaini RukikoLisso[2]

[1]Department of Mathematics and Physics, University of Eastern Africa, Baraton, Eldoret 2500 - 30100 Kenya

[2]Department of Mathematics and Physics, University of Eastern Africa, Baraton, Eldoret 2500 - 30100 Kenya

Abstract: ***We study the existence of mild solutions for quasilinear delay integro differential equations with nonlocal Cauchy problem in Banach spaces. The results are established by using Hausdorff's measure of non-compactness.***

Keywords: Mild solution, nonlocal conditions; Hausdorff's measure of noncompactness

1. Introduction

The notion of a measure of non-compactness turns out to be a very important and useful tool in many branches of mathematical analysis. The notion of a measure of weak compactness was introduced by De Blasi [9] and was subsequently used in numerous branches of functional analysis and the theory of differential and integral equations. El-Sayed [11] proves the existence theorem of monotonic solutions for a nonlinear functional integral equation of convolution type by Hasusdorff measure of non-compactness.

Fan et al [12] discussed semi linear differential equations with nonlocal condition using measure of non-compactness. Balachandran and Ilamaran [2] studied an existence theorem for functional integral equations with deviating arguments by using the measure of weak noncompactness.

The study of abstract nonlocal initial value problems was initiated by Byszewski [6, 7]. Balachandran and Paul Samuel [3] give the existence and uniqueness of mild and classical solutions for quasi linear delay integro differential equations when f, g and k satisfy Lipschitz-type conditions. The problem of existence of solutions of quasi linear evolution equations in Banach space has been studied by several authors [3, 8, 10, 14]. Pazy [14] considered the following quasi linear equation with local condition of the form

$$u'(t)+A(t,u)u(t)=0, 0<t<T$$
$$u(0)=u_0,$$

and discussed the mild and classical solutions by using the fixed point argument. In this paper, we shall consider the problem of the existence of mild solutions of quasi linear delay integro differential equations with non local condition of the form

$$u'(t)+A(t,u)u(t) = f(t,u(\alpha(t))) + \int_0^t g(t,s,u(\beta(s)))ds,\ t\epsilon[0,T], \quad (1)$$

$$u(0)+h(u)=u_0, \quad (2)$$

Where $A: [0,T]\times X \to X$ are continuous functions in Banach space X, $u_0 \epsilon X, f:[0,T]\times X\to X, g:\Delta\times X\to X, h:C([0,T];X)\to X$ and α, β are given functions, Here $\Delta=\{t,s,0\le s\le t\le T\}$. The results obtained in this paper generalize the results of[2, 3, 9, 10].

2. Preliminaries

Let X be a Banach space with norm $||\cdot||$. Let $C([0,T];X)$ be the space of X-valued continuous functions on $[0,T]$withthe norm$||u||=sup\{||u(t)||, t\epsilon[0,T]\}$for u$\epsilon$ $C([0,T];X)$, and denoted $L(0,T;X)$ by the space of X-valuedBochner integrable functions on $[0,T]$ with the norm$||u||_L=\int_0^T||u(t)||dt$. The Hausdorff's measure of non compactness χY is defined byχ(B) $=inf\{r>0, B$ can be covered by finite number of balls with radii $r\}$for bounded set B in a Banach space Y .

Lemma 2.1 [4]. Let Y be a real Banach space and $B,E\subseteq Y$be bounded, with the following properties:

1) B is pre compact if and only if $\chi_X(B)=0$.
2) $\chi_Y(B)=\chi_Y(\bar{B})=\chi_Y(conv\ B)$, where$B$and $conv\ B$means the closure and convex hull of B respectively.
3) χ_Y(B)$\le\chi_Y$(E),where $B\subseteq E$.
4) χ_Y (B + E)) $\le\chi_Y$(B) + χ_Y (E), where $B+E=\{x+y: x\epsilon B, y\epsilon E\}$
5) $\chi_Y(B\cup E)\le\max\{\chi_Y(B),\chi_Y(E)\}$.
6) $\chi_Y(\lambda B)\le|\lambda|\chi_Y(B) for\ any\ \lambda\in R$
7) If the map $F:D(F)\subseteq Y\to Z$ is Lipchitz continuous with constant k the $\chi_Y(FB)\le k\chi_Y(B)$ for any bounded subset $B\subseteq D(F)$, where Z is Banach space.
8) $\chi_Y(B)=\inf\{d_Y(B,E); E\subseteq Y$is finite valued, where $d_Y(B,E)$ means the nonsymmetric (or symmetric) Hausdorff distance between B and E in Y.
9) If $\{W_n\}_{n=1}^{+\infty}$ is a decreasing sequence of bounded closed nonempty subset of Y and $\lim_{n\to\infty}\chi_Y(W_n)=0$, then $\bigcap_{n=1}^{+\infty}W_n$ is nonempty and compact in Y.

Lemma 2.2 (Darbo-Sadovskii [4]). If W⊆ Y is bounded closed and convex, the continuous map $F : W \to W$ is a $\chi_Y$contraction, then the map F has at least one fixed point in W. In this we denote by χ the Hausdorff's measure of non-compactness of X and denote _c by the Hausdorff's measure of non compactness of $C([a,T];X)$. To discuss the existence, we need the following Lemmas in this paper.

Lemma 2.3 [4]. If $W \subseteq C([0,T];X)$ is bounded, Then χ (W(t)) ≤ χ_c(W) for all $t \epsilon [0,T]$, where $W(t) = \{u(t);\ u \epsilon W\} \subseteq X$. Furthermore if W is equi-continuous on $[a,T]$, then $\chi(W(t))$ is continuous on $[a,T]$ and $\chi_c(W) = sup\{\chi(W(t)), t \epsilon [a,T]\}$.

Lemma 2.4 [13]. If $\{u_n\}_{n=1}^{\infty} \subset L^1(a,T;X)$ is uniformly integrable, then the function $\chi(\{u_n\}_{n=1}^{\infty})$ is measurable and

$\chi(\{\int_0^t u_n(s)ds\}) \leq 2\int_0^t \chi\{u_n\}_{n=1}^{\infty} ds)$(3)

Lemma 2.5 [4]. If $W \subseteq C([0,T];X)$ is bounded and equi-continuous, then then $\chi(W(t))$ is continuous and

$\chi\left(\{\int_0^t W(s)ds\}\right) \leq \int_0^t \chi\, W(s)ds$ (4)

for all $t \epsilon [0,T]$, where $\int_0^t W(s)ds$)= $\{\int_0^t u(s)ds; u \epsilon W\}$.

The C_0 - semigroup $U_u(t,s)$ is said to be equi continuous for $t > 0$ for all bounded set B in X. The following lemma is obvious.

Lemma 2.6. If the evolution family $\{U_u(t,s)\}_{0\leq s\leq t\leq T}$ is equi-continuous and $\eta \in L(0,T;R^+)$, then the set $\{\int_0^t U_u(t,s)u(s)ds\}, ||u(s)|| \leq \eta(s)$ for i.e $s \in [0,T]$ is equi-continuous for $t \in [0,T]$.

From [8], we know that for any fixed∈ $C([0,T];X)$, there exist a unique continuous function $U_u : [0,T]\ \mathrm{x}[0,T] \to B(X)$ defined on $[0,T]\mathrm{x}[0,T]$ such that

$U_u(t,s) = \int_s^t A_u U_u(\mathrm{w},s)dw$ (5)

where $B(X)$ denote the Banach space of bounded linearoperators from X to X with the norm $||F|| = sup\{||Fu||: ||u|| = 1\}$, and I stands for the identity operator on $X, A_u(t) = A(t,u(t))$. We have $U_u(t,t) = I$, $U_u(t,s)U_u(s,r) = U_u(t,r)$, where $(t,s,r) \in [0,T]$ x $[0,T]\mathrm{x}[0,T], \frac{\partial U_u(t,s)}{\partial t} = A_u(t)U_u(t,s)$ for almost all ∈ $[0,T]$.

For a mild solution of (1) - (2) we mean a function $u \in C([0,T];X)$ and $u_0 \in X$ satisfying the integral equation

$u(t) =$
$U_u(t,0)\,u_0 - U_u(t,0)h(u) + \int_0^t U_u(t,s)\big[f(s,u(\alpha(s))) +$
$+ \int_0^s g(s,\tau,u(\beta(\tau)))d\tau\big]ds$,(6)

3. The Existence of Results for Compact$_h$

In this section, we give some existence results when h is compact and f satisfy the conditions with respect to Hasudorff's measure of non-compactness and its applications in differential equations in Banach spaces. We give some existence results of the nonlocal problem (1)- (2). We assume the following assumptions:

(H1) The evolution family $\{U_u(t,s)\}_{0\leq s\leq t\leq T}$ generated by $A(t,u)$ is equi-continuous, and $||U_u(t,s)|| \leq M_0$ for almost $t,s \in [0,T]$

(H2) $f : [0,T] \times X \to X$ satisfies the Caratheodory type conditions and there exist $m_1 \in L[0,T]$ and $b_1 \geq 0$ such that

$$|f(t,u)| \leq m_1(t)b_1||u||, t \text{ a.e in } [0,T], u \in R^+$$

(H3) (a) $h : C([0,T];X) \to X$ is continuous and compact.
(b) There exist $N > 0$ such that $||h(u)|| \leq N_0$ for all $u \in C([0,T];X)$.
(H4) (a) $g : [0,T] \times [0,T] \times X \to X$ satisfies the Caratheodory-type conditions. (i.e.) $g(.,.,u)$ is measurable for all $u \in X$ and $g(t,s,.)$ is continuous for a.e $t,s \in [a,T]$, there exist $m_2 \in L[0,T]$ and $b_2 \geq 0$ such that $\int_0^t |g(t,s,u)|\,ds \leq m_2(t,s)b_2||u||$, for t a.e in $[0,T], u \in R^+$

(b) There exist $\eta \in L(0,T;R^+), \zeta \in L(0,T;R^+)$, such that $\zeta(g(t,s,D) \leq \eta(t)\zeta(s)\chi(D)$ for a.e $t,s \in [0,T]$, and for any bounded subset $D \subset C([0,T],X)$. Here we let $\eta(t) \leq K_1$ and $\zeta(t) \leq K_2$.
(H5) $\alpha,\beta : [0,T] \to [0,T]$ is non decreasing and there exist positive constants δ_1 and δ_2 such that $\alpha'(t) \geq \delta_1$ and $\beta'(t) \geq \delta_2$ respectively for $t \in [0,T]$.

Theorem: 3.1 Under the assumptions $(H1) - (H2)$ are satisfied, then the nonlocal initial value problem. Let $u_0 \in Y$ and the family $A(t,b)$ of linear operators for $t \in I\ [0,T]$ and $(1) - (2)$ has at least one mild solution.

Proof.
Let $\Omega(t)$ be a solution of the scalar equation

$$\Omega(t) = M_0N_0 + M_0(m_1(t)\,b_1/\delta_1 + m_2(t,s)\,b_2/\delta_2 \int_0^t \Omega(s)ds \quad (7)$$

for $t \in [0,T]$.

Consider the map $F : C([0,T];X) \to C([0,T];X)$ defined by

$$(Fu)(t) = U_u(t,0)h(u) + \int_0^t U_u(t,s)\left[f\left(s,u(\alpha(s))\right) + \int_0^s g(s,\tau,u(\beta(\tau)))d\tau\right]ds \quad (8)$$

for all $u \in C([0,T;X)$. We can show that F is continuous by the usual techniques.
Let us take

$$W_0 = \{u \in C([0,T];X), \|u(t)\| \leq \Omega(t), \text{for all } t \in [0,T]\}.$$

Then $W_0 \subseteq C([0,T];X)$ is bounded and convex. We define $W_1 = \overline{conv}\,K(W_0)$, where $conv$ means the closure of the $convex$ hull in $C([0,T];X)$. As $U_u(t,s)$ is equi-continuous, g is compact and $W_0 \subseteq C([0,T];X)$ is bounded, due to Lemma 2.7 and the assumption(H4)(b), $W_1 \subseteq C([0,T];X)$ is bounded closed convex non empty and equi-continuous on $[0,T]$.

For any $u \in F(W_0)$, we know

$$||u(t)|| \leq M_0 N_0 + M_0[\int_0^t \left\|f\left(s,u(\alpha(s))\right)\right\| ds + \int_0^t \int_0^s \left\|g\left(s,\tau,u(\beta(\tau))\right)\right\| d\tau ds]$$

$$\leq M_0 N_0 + M_0[\int_0^t m_1(s) b_1 ||u(\alpha(s)|| ds + \int_0^s m_2(s,\tau) b_2 ||u(\beta(\tau)|| d\tau]$$

$$\leq M_0 N_0 + M_0[m_1(t) b_1 \int_0^t ||u(\alpha(s)|| \alpha'(s)/\delta_1 ds + m_2(s,\tau) b_2 \int_0^s ||u(\beta(\tau)|| \beta'(\tau)/\delta_2 d\tau]$$

$$\leq M_0 N_0 + M_0[m_1(t) b_1/\delta_1 \int_0^t ||u((s)|| ds + m_2(s,\tau) b_2/\delta_2 \int_0^s ||u(\tau)|| d\tau]$$

$$\leq M_0 N_0 + M_0[m_1(t) b_1/\delta_1] + m_2(t,s) b_2/\delta_2 \int_0^t \Omega(s) ds,$$

$=\Omega(t)$, for $t, s, \tau \in [0,T]$.

It follows that $W_1 \subset W_0$. We define $W_{n+1} = \overline{conv}\, F(W_n)$, for $n = 1,2,3,\cdots$ From above we know that $\{W_n\}_{n=1}^{\infty}$ is a decreasing sequence of bounded, closed, convex, equi-continuous on $[0,T]$ and non empty subsets in $C([0,T];X)$.

Now for $n \geq 1$ and $t \in [0,T]$, $W_n(t)$ and $F(W_n(t))$ are bounded subsets of X, hence, for any $\in > 0$, there is a sequence $\{u_k\}_{k=1}^{\infty} \subseteq W_n$ such that (see, e.g.[5],pp125).

$$\chi(W_{n+1})) = \chi(FW_n(t))$$

$$\leq \chi(\int_0^t U_u(t,s)[f(s,\{u_k \alpha(s))\}_{k=1}^{\infty})ds + \int_0^s g(s,\tau,\{u_k \beta(\tau))\}_{k=1}^{\infty})d\tau]ds$$

$$\leq 2M_0 \int_0^t [\chi\left(f(s,\{u_k \alpha(s))\}_{k=1}^{\infty}\right)ds] + 4 M_0 \int_0^t \int_0^s [\chi(g(s,\tau,\{u_k \beta(\tau))\}_{k=1}^{\infty})d\tau)ds]$$

$$\leq 2M_0 K_1 \int_0^t \chi(\{u_k \alpha(s)\}_{k=1}^{\infty})ds] + 4 M_0 K_1 K_2 \int_0^t \int_0^s [\chi(\{u_k \beta(\tau))\}_{k=1}^{\infty})d\tau)ds]$$

$$\leq 2M_0 K_1/\delta_1 \int_0^t \chi(\{u_k(s)\}_{k=1}^{\infty})ds] + 4 M_0 K_1 K_2/\delta_2 \int_0^t \chi(\{u_k(s)\}_{k=1}^{\infty})ds + \epsilon$$

$$\leq (2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2}\right]) \int_0^t \chi(\{u_k(s)\}_{k=1}^{\infty})ds + \epsilon$$

$$\leq (2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2}\right]) \int_0^t \chi(W_n(s))ds + \epsilon$$

Since $\epsilon > 0$ is arbitrary, it follows that from the above ine quality that

$$\chi(W_{n+1}(t))) \leq (2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2}\right]) \int_0^t \chi(W_n(s))ds \quad (9)$$

for all $t \in [0,T]$. Because W_n is decreasing for n, we have

$$\sigma(t) = \lim_{n\to\infty} \chi(W_n(t))$$

for all $t \in [0,T]$. From (9), we have

$$\sigma(t) \leq (2M_0 K_1 \left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2}\right]) \int_0^t (\sigma(s))ds$$

for $t \in [0,T]$, which implies that $\sigma(t) = 0$ for all $t \in [0,T]$. By Lemma 2.3, we know that $\lim_{n\to\infty} \chi(W_n(t)) = 0$.

Using Lemma 2.1 we know that $W = \cap_{n=1}^{\infty} W_n$ is convex compact and nonempty in $C([0,T];X)$ and $F(W) \subset W$. By the famous Schauder's fixed point theorem, there exist at leastone mild solution u of the initial value problem $(1)-(2)$), where $u \in W$ is a fixed point of the continuous map F.

Remark 3.2. If the functions f and g are compactor Lipchitz continuous (see e.g. [6, 7, 15], then $(H4)$ is automatically satisfied. In some of the early related results in references and above results, it is supposed that the map h isuniformly bounded. We indicate here that this condition can be released. In fact, if h is compact, then it must be bounded on bounded set. Here we give an existence result under growth condition of f and g, when h is not uniformly bounded. Precisely, we replace the assumptions $(H4)$ by

$(H6)$ There exist a functions $p \in L(0,T;R^+)$ and $q \in L(0,T;R^+)$. The Constants $b_1, b_2 > 0$ such that

$$||f(t,u)|| \leq p(t) b_1 ||u||,$$

$$\int_0^t ||g(t,s,u)|| ds \leq q(t,s) b_2 ||u||$$

for a.e $t \in [0,T]$ and all $u \in C([0,T];X)$.

Theorem: 3.2 Suppose that the assumptions $(H1)-(H6)$ are satisfied, then the equation $(1)-(2)$ has atleast one mild solution if

$$\lim_{r\to\infty} \sup \frac{M_0}{r}\left(\varphi(r) + rT\left[\frac{pb_1}{\delta_1} + \frac{qb_2}{\delta_2}\right]\right) < 1. \quad (10)$$

Where $\varphi(r) = \sup\{||h(u)||, ||u|| \leq r\}$.

Proof. The inequality (10) implies that there exist a constant $r > 0$ such that

$$M_0\left(\varphi(r) + rT\left[\frac{pb_1}{\delta_1} + \frac{qb_2}{\delta_2}\right]\right) < r.$$

Just as in the proof of Theorem 3.1, let $W_0 = \{u \in C([0,T];X), ||u(t)|| \leq r\}$ and $W_1 = \overline{conv} F W_0$. Then for any $u \in W_1$, we have

$$||u(t)|| \leq ||U_u(t,0)h(u)|| + \int_0^t U_u(t,s)\,[f\left(s,u(\alpha(s))\right) + \int_0^s g(s,\tau,u(\beta(\tau)))d\tau]ds,$$

$$\leq M_0\varphi(r) + M_0\left[\int_0^t p(s)b_1||u(\alpha(s))||ds + \int_0^s q(s,\tau)b_2||u(\beta(\tau))||d\tau\right]$$

$$\leq M_0\varphi(r) + M_0\left(\left[\frac{p(t)b_1rT}{\delta_1} + \frac{q(s,\tau)b_2rT}{\delta_2}\right]\right)$$

$$||u(t)|| \leq M_0\left(\varphi(r) + rT\left[\frac{pb_1}{\delta_1} + \frac{qb_2}{\delta_2}\right]\right) < r$$

for $t \in [0,T]$. It means that $W_1 \subset W_0$. So, can complete the proof similarly to Theorem 3.1.

4. The Existence of Results for Lipschitz$_h$

In the previous section, we obtained the existence results when h is compact but without the compactness of $U_u(t,s)_{0\leq s\leq t\leq T}$or f and g. In this section, we discuss the equation (1) $-$ (2) when h is Lipschitz and$f$ and g are not Lipschitz. Again we assume that $(H7)$ his a Lipschitz continuous in X, there exist a constant $L_0 > 0$ such that

$$\|h(u) - h(v)\| \leq L_0\|u - v\|, u, v \in C([0,T];X).$$

Theorem: 4.1 Suppose that the assumptions $(H1)$, $(H2)$, $(H4)$, $H(6)$ are satisfied, then the equation$(1) - (2)$ has at least one mild solution provided that

$$\left(M_0L_0 + TM_0K_1\left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2}\right]\right) < 1. \quad (11)$$

Proof. Consider the map $F_1, F_2 : C([0,B];X) \rightarrow C([0,B];X)$defined by$F_1 + F_2 = F$, where

$$F_1(u)(t) = U_u(t,0)h(u).$$

$$F_2(u)(t) = \int_0^t U_u(t,s)\,[f(s,u(\alpha(s))) + \int_0^s g(s,\tau,u(\beta(\tau)))d\tau]ds,$$

for $u \in C([0,B];X)$. As defined in the proof of Theorem3.1. We define$W_0 = \{u \in C([0,B];X : ||u(t)|| \leq \Omega(t)$ for all $t \in [0,T]$ and let $W = \overline{conv}FW_0$. Thenfrom the proof of Theorem 3.1 we know that W is abounded closed convex and equi continuous subset of$C([0,T];X)$ and $FW \subset W$. We shall prove that F isχ_c-contraction on W. Then Darbo-Sadovskii's fixed point theorem can be used to get a fixed point of Fin$W$, which is a mild solution of (1) $-$ (2). First, for every bounded subset $B \subset W$, from the $(H7)$ and Lemma 2.1, we have

$$\chi_c(F_1B) = \chi_c(U_B(t,0)h(B))$$

$$\leq M_0\chi_c(h(B))$$

$$\leq M_0L_0\chi_c(B) \quad (12)$$

Next, for every bounded subset $B \subset W$, for $t \in [0,T]$ and every $\epsilon > 0$, there is a sequence $\{u_k\}_{k=1}^{\infty} \subset B$,such that

$$\chi(F_2B(t)) \leq 2\chi\{F_2u_k(t)\}_{k=1}^{\infty} + \epsilon.$$

Note that B and F_2B are equi-continuous, we can get from Lemma 2.1, Lemma 2.4, Lemma 2.5 and $(H4)$ that

$$\chi(F_2B(t)) \leq \chi\left(\int_0^t U_u(t,s)\left[f(s,\{u_k(\alpha(s))\}_{k=1}^{\infty}) + \int_0^s g(s,\tau,\{u_k(\beta(\tau))\}_{k=1}^{\infty})d\tau\right]ds\right)$$

$$\leq 2M_0\int_0^t \left[\chi\left(f\left(s,\{u_k(\alpha(s))\}_{k=1}^{\infty}\right)\right)\right]ds + 4M_0\int_0^t\int_0^s [\chi(g(s,\tau,\{u_k(\beta(\tau))\}_{k=1}^{\infty}))]d\tau ds$$

$$\leq \left(2M_0K_1\left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2}\right]\right)\chi_c(B)T + \epsilon$$

for all $t \in [0,T]$.

Since $\epsilon > 0$ is arbitrary, we have

$$\chi(F_2B(t)) \leq T\left(2M_0K_1\left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2}\right]\right)\chi_c(B) \quad (13)$$

For any bounded $B \subset W$.

Now, for any subset $B \subset W$, due to Lemma 2.1,(12) and (13) we have

$$\chi_c(FB) \leq \chi_c(F_1B) + \chi_c(F_2B)$$

$$\chi_c(FB) \leq \left[M_0L_0 + T\left(2M_0K_1\left[\frac{1}{\delta_1} + \frac{2K_2}{\delta_2}\right]\right)\chi_c(B)\right].$$

We know that F is aχ_c-contraction on W. By Lemma2.2, there is a fixed point $u$of F in W, which is asolution of (1) $-$ (2). This completes the proof.

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Author Profile

Paul Samuel Francis holds M.Sc., M. Phil., Ph. D., degrees in Mathematics from Bharathiar University, Coimbatore, Tamil Nadu, India. He has served as a Lecturer in Mathematics at the Spicer Memorial College, Pune, Maharashtra State, India, and also worked as an Assistant Professor in Mathematics at the Karunya University, Coimbatore, Tamil Nadu, India. He is currently the Chairperson in the Department of Mathematics and Physics at the University of Eastern Africa, Baraton, Eldoret, Kenya.

Tumaini Rukiko Lisso holds M.Sc. degree in Mathematics from the University of Dar es Salaam, Tanzania. He is also a holder of DEA in Mathematics Education from the University of Rene Descartes, Paris V in France. He has served as a Mathematics tutor at the Dar es Salaam Institute of Technology in Tanzania and as Lecturer of Mathematics at the University of Eastern Africa, Baraton, Eldoret, Kenya. He served as Chairperson of the Department of Mathematics and Physics from 2006 to 2013.He is currently the Acting Director of Affiliations, Linkages, and Extension Programmes at the University of Eastern Africa, Baraton, Eldoret, Kenya.