A Study on Generalized Fuzzy Game Value for Three Players

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Abstract: ***Game theory is an integral part of decision making problems. In this paper, we analyze some strategies, players under consideration, the corresponding pay off's may be fall within the range rather than accurate values. The uncertainty in determinant matrix games, we consider three players inter valued game matrices also discussed. It may be extended further determinant games for more players.***

Keywords: decision making, strategy, pay off, crisp game value, saddle point

1. Introduction

Game theory is a decision theory applicable to competitive situations. It is usually used when two or more individuals or organisations with conflicting objectives try to make decisions. In such situations, decision made by one decision maker affects the decision made by one or more of the remaining decision makers. The fundamental problem of game theory is that the player makes decisions with crisp data. In a real life world, most games always takes place in uncertain environments. Because of uncertainty in real world applications, pay off's of a game may not be a fixed number. This situation gives the introduction of fuzzy game. Matrix games have many useful applications, especially in decision making systems. However, in real world applications, due to certain forms of uncertainty outcomes of a matrix game may not be a fixed number even though the players do not change their strategies. By noticing the fact that the payoffs may only vary within a designated range for fixed strategies, we propose to use an interval valued matrix, whose entries are closed intervals to model such kind of uncertainty. In this paper we assume that the intervals in the game matrix G are closed and bounded intervals of real numbers and represent uniformly distributed possible payoffs.

2. Crisp Game Value of the Matrix

Let us consider the game with two players A & B. The players A and B have two strategies. For player A, minimum value in each row represents the least gain to him, if he chooses a particular strategy. He will then select the strategy that maximizes his minimum gain. For player B, the maximum value in each column represents the maximum loss to him, if he chooses his particular strategy. He will then select the strategy that minimizes his maximum loss. If there exists a g_{ij} in a classical m × n game matrix G such that g_{ij} is simultaneously the minimum value of the i^{th} row and the maximum value of the j^{th} column of G, then g_{ij} is called a Saddle value of the game. If a matrix game has saddle value it is said to be strictly determined. If the maximum value equals the minmax value, then the game is said to have a saddle point and the corresponding strategies are called optimum stratagies. The amount of pay off at saddle point is called the crisp value of the game. Saddle point is the minimum value of the i^{th} row and the maximum value of the j^{th} column of a game.

3. Fuzzy Matrix Games

The elements of the game are affected by various sources of fuzziness. The gain or payoff function is not always defined numerically or sharply. It is formulated semantically and, at the same time, fuzzily, in such terms as excellent, good, or sufficiently reliable, durable, resistant etc. The strategies employed by players are usually marked by different levels of significance and intensity. These and other conditions account for the need to include the theory of fuzzy sets in the solution concept of the theory of games. Let G = $\{g_{ij}\}$ be an m×n interval valued matrix. The matrix G defines a zero sum interval matrix game provided whenever the row player uses his i^{th} strategy and the column player select his j^{th} strategy, then row player wins and the column player losses a common x€ g_{ij}. Let G be a m x n interval game matrix such that all intervals in the same row (or column) of G are crisply comparable. If there exists a g_{ij} € G such that g_{ij} is simultaneously crisply less than or equal to g_{ik}, for all k € {1, 2,...n} and crisply greater than or equal to g_{lj} for all l € {1,2,...m},then the interval g_{ij} is called a saddle interval of the game. An interval game is crisply determined if it has a saddle interval.

4. Comparing Intervals

In order to compare strategies and payoffs for an interval game matrix, we need to define a notion of interval inequality (both ≤ and ≥) that corresponds to an intuitive notion of a better possible outcome or payoff. Let x and y be two nonempty intervals. We will consider their relationship in the following different cases.

a) x ∩ y = 0 and x < y. In this case, every possible payoff value from y exceeds all of the possible payoffs from x. Therefore, we say that x < y and y < x crisply, which corresponds to the traditional definition of comparison used in interval computations.
b) x = y. We then define the crisp inequalities x ≤ y and y ≤ x, again paralleling common usage of existing interval inequality comparisons.

c) x ∩ y is not equal to 0 and x is not equal to y, we define x ≤ y crisply for this case as x offers no larger payoff than what is possible in y. We also define the crisp inequality y ≥ x. Both of these comparisons also mirror existing practice in interval computing. If x C y ,we can assume that y is nontrivial interval. Here we need use the width function w. As x is a proper subset y we know that w(y) – w(x) > 0 .

The matrix game is given as follows:

Table 1: Three Players Table

C

	B_1	B_2	B_3
A_1	(a_1, b_1)	(a_2, b_2)	(a_3, b_3)
A_2	(a_4, b_4)	(a_5, b_5)	(a_6, b_6)
A_3	(a_7, b_7)	(a_8, b_8)	(a_9, b_9)

The players A and B have strategies $[A_1, A_2]$ and $[B_1, B_2]$.In the first case, we assume the player C chooses the strategy A_1 and in the second case he chooses A_2 and in the third case the player chooses the strategy A_3.

5. Example

C

	B_1	B_2	B_3
A_1	[0,1]	[6,7]	[-2,0]
A_2	[5,6]	[2,7]	[1,3]
A_3	[-8,-5]	[-1,0]	[-4,-2]

5.1 Minimum Interval

If the player chooses the strategy A_1 , then

(a) [0,1]< [6,7] = 1 [0,1]< [-2,0] =0
[0,1]< [5,6] =1 [0,1]< [2,7] =1
[0,1]< [1,3] =1 [0,1]< [-8,-5] =0
[0,1]< [-1,0] =0 [0,1]< [-4,-2] =0
Min{1,0,1,1,1,0,0,0}=0

(b) [6,7]< [0,1] = 0 [6,7]< [-2,0] =0
[6,7]< [5,6] =0 [6,7]< [2,7] =0
[6,7]< [1,3] =0 [6,7]< [-8,-5] =0
[6,7]< [-1,0] =0 [6,7]< [-4,-2] =0
Min{0,0,0,0,0,0,0,0}=0

(c) [-2,0]< [0,1] = 1 [-2,0]< [6,7] =1
[-2,0]< [5,6] =1 [-2,0]< [2,7] =1
[-2,0]< [1,3] =1 [-2,0]< [-8,-5] =0
[-2,0]< [-1,0] =0.5 [-2,0]< [-4,-2] =0
Min{1,1,1,1,1,0,0.5,0}=0
Hence max {0,0,0} = 0

5.2 Maximum Interval

(a) [0,1]> [6,7] = 0 [0,1]> [-2,0] =1
[0,1]> [5,6] =0 [0,1]> [2,7] =0
[0,1]> [1,3] =0 [0,1]> [-8,-5] =1
[0,1]> [-1,0] =1 [0,1]> [-4,-2] =1
max{0,1,0,0,0,1,1,1}=1

(b) [6,7]> [0,1] = 1 [6,7]> [-2,0] =1
[6,7]> [5,6] =1 [6,7]> [2,7] =0.8
[6,7]> [1,3] =1 [6,7]> [-8,-5] =1
[6,7]> [-1,0] =1 [6,7]> [-4,-2] =1
max{1,1,1,0.8,1,1,1,1}=1

(c) [-2,0]> [0,1] = 0 [-2,0]> [6,7] =0
[-2,0]> [5,6] =0 [-2,0]> [2,7] =0
[-2,0]> [1,3] =0 [-2,0]> [-8,-5] =1
[-2,0]> [-1,0] =0 [-2,0]> [-4,-2] =1
max{0,0,0,0,0,1,0,1}=1
Hence min {1,1,1} = 1.

This corresponds to the interval [6, 7] and [2, 7].If the third player C chooses A_1, he wins. If he chooses other strategies, he loses the game.

6. Conclusion & Future Work

In this paper, we have discussed three person zero sum games under determinant values. The strategies for determinant matrix games fully analyzed under fuzzily matrix games. We are analyzing three and more players different strategies based on inter valued fuzzy game. The result of this paper can be extended to multiplayers under determinant interval matrix game method which may be helpful of handling the problem of uncertainty matrix games.

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