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Derivation of Einstein's Energy Equation from Maxwell's Electric Wave Equation

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Abstract: ***The derivation of the equivalence of mass-energy from the principles of special relativity theory (SRT) by Einstein is arguably the most famous equation in physics. Since this derivation was published, it has been the subject of continuing controversy. In this paper, Einstein's Mass-Energy Equation will derive from Maxwell's electric wave equation. In addition to that, the evaluation of photon mass can be found. The electric field wave equation for polar material from utility of Maxwell's equations, beside the relations that expressed macroscopic polarization of the medium, permittivity and permeability of free space were derived.***

Keywords: Einstein Equation, Maxwell's Wave Equation, Photon mass, polarization, Einstein's equation

1. Introduction

The special relativity theory (SRT) [1] has removed the barrier between matter and energy, but it has created a new barrier that cannot be transcended. This barrier separates what is known as non-relativistic from relativistic physics. Einstein was the first to derive mass-energy equivalence from the principles of SRT [2]. Since this derivation was published, it has been the subject of continuing controversy. Therefore, the relativistic mass in this case is a purely kinematical effect. Hence, according to Einstein the relativistic mass is not a physical effect but rather the result of the effect of relative motion on observation. Einstein then derives his equation mathematically and under special conditions, which requires the above speculation. Therefore, the relativistic mass based on Lorentz transformation of moving frame of reference can appear mysterious and are put in doubt [3, 4].

However, Maxwell unified electromagnetic theory, assembled the laws of Ampere, Faraday and Gauss into a set of four equations called Maxwell electromagnetic equations. He also added another term to Ampere law to include a time changing displacement current density [1-5]. Maxwell's equations are Physical laws which unify electric and magnetic phenomena [1-3]. The importance of these equations relies in their wide-spread applications in our day life and modern technology [2]. Despite the fact that Maxwell's equations are more than hundred years old. They still are subject to changes in content or notion. The complete sets of equations of Maxwell are known in electrodynamics since 1865. These equations have been defined for 20 field variables [5, 6]. Later Oliver Heaviside and William Gibbs have transformed these equations into used notion with vectors. At that time many scientists-one of them has been Maxwell himself was convinced, that the correct notion for electrodynamics must be possible with quaternions. Conceptually, Maxwell's equations describe how electric charges and electric currents act as sources for the electric and magnetic fields. Further, it describes how a time varying electric field generates a time varying magnetic field and vice versa. Maxwell's equations have two major variants. The set of Maxwell's equations use total charge and total current including the difficult to calculate atomic level charges and currents in materials. The four equations of Maxwell, the first two equations are scalar equations, while the last two are vector equations. Gauss' law for electricity, more commonly simply referred to as Gauss' law. Gauss' law for magnetism is remarkably similar to Gauss' law for electricity, but means something rather different [7-9].

2. Maxwell's Electric Wave Equation

The M.Es used to describe the behavior of electromagnetic waves, which are [10-12];

$$\nabla . D=\rho,\ \nabla . B=0,\ \nabla\times E=-\frac{\partial B}{\partial t},$$
$$\nabla\times H=J+\frac{\partial D}{\partial t} \tag{1}$$

Where, D, B, E, H and J are density of electric flux, density of magnetic flux, electric field, magnetic field and current density, respectively. Satisfy the following relations [13];

$$B=\mu_0 H, J=\sigma E, D=\varepsilon_0 E+P \tag{2}$$

Where P, ε_0 and μ_0 are macroscopic polarization of the medium, the permittivity of free space and the permeability of free space, respectively. Applying the curl operator to both sides of 3rd equation in (1) one obtains;

$$\nabla\times(\nabla\times E)=-\nabla\times\frac{\partial B}{\partial t}=-\frac{\partial}{\partial t}(\nabla\times B) \tag{3}$$

Using the identity **[14, 15]**;

$$\nabla\times(\nabla\times E)=\nabla(\nabla . E)-\nabla^2 E \tag{4}$$

Equation (3) gives;

$$\nabla(\nabla.E)-\nabla^2 E=-\frac{\partial}{\partial t}(\nabla\times B) \quad (5)$$

From (2) since;

$$B=\mu_0 H \quad (6)$$

Then (5) becomes;

$$\nabla(\nabla.E)-\nabla^2 E=-\frac{\partial}{\partial t}(\nabla\times\mu_0 H) \quad (7)$$

From equation (7) since;

$$\nabla\times H=J+\frac{\partial D}{\partial t} \quad (8)$$

From (1) gets;

$$\nabla(\nabla.E)-\nabla^2 E=-\frac{\partial}{\partial t}\left(\mu_0 J+\mu_0\frac{\partial D}{\partial t}\right) \quad (9)$$

But;

$$D=\varepsilon_0 E+P \quad (10)$$

Therefore;

$$\nabla(\nabla.E)-\nabla^2 E=-\mu_0\frac{\partial J}{\partial t}-\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}-\mu_0\frac{\partial^2 P}{\partial t^2} \quad (11)$$

Where,

$$J=\sigma E \quad (12)$$

Then:

$$\nabla(\nabla.E)-\nabla^2 E+\mu_0\frac{\partial J}{\partial t}+\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}=-\mu_0\frac{\partial^2 P}{\partial t^2} \quad (13)$$

The polarization, p, thus acts as a source term in the equation for radiation field [16].

Since;

$$D=\varepsilon_0 E,\nabla.D=\rho,\rho=0 \quad (14)$$

Therefore;

$$\varepsilon\nabla.E=\rho=0,\nabla.E=0 \quad (15)$$

Therefore equation (13) becomes;

$$-\nabla^2 E+\mu_0\frac{\partial J}{\partial t}+\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}=-\mu_0\frac{\partial^2 P}{\partial t^2} \quad (16)$$

This represents the wave equation for electric field.

3. Derivation of Einstein's Energy Equation from Maxwell's Electric Wave Equation

To derive Einstein's energy equation, the equation (16) of wave for electric field will used. For polar materials, having electric dipole moment, p, for, N, atoms having displacement, x, is given by [17];

$$P=-eNx \quad (17)$$

Then;

$$\frac{\partial^2 P}{\partial t^2}=-eN\ddot{x} \quad (18)$$

When an electron is affected by electric force, eE, and resistive force $\gamma\dot{x}$ beside a magnetic force $Be\dot{x}$, its motion equation becomes;

$$m_e\ddot{x}=eE_0e^{i\omega t}-\gamma\dot{x}-Be\dot{x} \quad (19)$$

To solve this equation let;

$$x=x_0e^{i\omega t} \quad (20)$$

Then;

$$\dot{x}=i\omega x_0e^{i\omega t},\ddot{x}=-\omega^2x_0e^{i\omega t} \quad (21)$$

Where ω is the angular frequency of vibrating electron.

Thus from (17), (18), (19), (20) and (21) obtain;

$$\ddot{x}=\frac{1}{m_e}\left[eE_0e^{i\omega t}-i\gamma\omega E_0(\frac{x_0}{E_0})e^{i\omega t}-iBe\omega E_0(\frac{x_0}{E_0})e^{i\omega t}\right] \quad (22)$$

From which;

$$\ddot{x}=\frac{1}{m_e}\left[e-i\gamma\omega(\frac{x_0}{E_0})-iBe\omega(\frac{x_0}{E_0})\right]E \quad (23)$$

With the aid of equations (18) and (23) equation (16) becomes;

$$-\nabla^2E+\mu_0\sigma\frac{\partial E}{\partial t}+\varepsilon_0\mu_0\frac{\partial^2E}{\partial t^2}=-\frac{\mu_0 eN}{m_e}\left[e-\frac{i\gamma\omega x_0}{E_0}-\frac{iBe\omega x_0}{E_0}\right]E \quad (24)$$

The solution of (24) can be in the form;

$$E=E_0e^{i(\omega_0 t-kx)} \quad (25)$$

From which;

$$\frac{\partial^2E}{\partial x^2}=-K^2E=\nabla^2E \quad (26)$$

and

$$\frac{\partial^2 E}{\partial t^2} = -\omega_0{}^2 E \qquad (27)$$

Where ω_0 is the angular frequency of electromagnetic waves?

Substituting (26) and (27) in (24) one gets;

$$K^2 E - i\omega\mu_0\sigma E - \varepsilon_0\mu_0\omega_0^2 E = -\frac{\mu_0 eN}{m_e}\left[e - \frac{i\gamma\omega x_0}{E_0} - \frac{iBe\omega x_0}{E_0}\right]E \qquad (28)$$

From which;

$$K^2 - i\omega\mu_0\sigma - \varepsilon_0\mu_0\omega_0^2 = -\frac{\mu_0 e^2 N}{m_e} + \frac{i\gamma\omega x_0\mu_0 eN}{E_0} + \frac{iBe^2\omega x_0\mu_0 N}{E_0} \qquad (29)$$

Equating the real and the imaginary parts of (29) yields;

$$K^2 - \varepsilon_0\mu_0\omega_0^2 = -\frac{\mu_0 e^2 N}{m_e} \qquad (30)$$

From which;

$$K^2 = \varepsilon_0\mu_0\omega_0^2 - \frac{\mu_0 e^2 N}{m_e} = \frac{\omega_0^2}{C^2} - \frac{\mu_0 e^2 N}{m_e} \qquad (31)$$

Therefore;

$$K^2 = K_0^2 - \frac{\mu_0 e^2 N}{m_e} \qquad (32)$$

The expression for energy and momentum of a harmonic oscillator is given by [18];

$$E = \hbar\omega_0, P = \hbar K \qquad (33)$$

When equation (32) is multiplied by $\hbar^2 C^2$ one gets;

$$\hbar^2 K^2 C^2 = \hbar\omega_0^2 - \frac{\mu_0 e^2 N}{m_e}\hbar^2 C^2 \qquad (34)$$

With aid of (33) equation (34) becomes;

$$P^2 C^2 = E_0^2 - \frac{\mu_0 e^2 N}{m_e}\hbar^2 C^2 \qquad (35)$$

Therefore;

$$E_0{}^2 = P^2 C^2 + \frac{\mu_0 e^2 N}{m_e}\hbar^2 C^2 \qquad (36)$$

The functional form of the second term on the right hand side of equation (36) can be found, by assuming that in the absence of electrons or electrons or electromagnetic sources $N = 0$;

$$E_0{}^2 = P_0^2 C^2 = m_0^2 C^4 \qquad (37)$$

Where the photon momentum is given by;

$$P_0 = m_0 C^2 \qquad (38)$$

Thus;

$$E_0{}^2 = P^2 C^2 + P_0^2 C^2 = P^2 C^2 + m_0^2 C^4 \qquad (39)$$

This resembles Einstein's energy equation;

$$E_0{}^2 = P^2 C^2 + m_0^2 C^4 \qquad (40)$$

M.Es can also be obtained by adding to equation (19) the force of binding energy through the term $k_0 x$;

$$m_e\ddot{x} = eE - k_0 x - m_e\gamma\dot{x} \qquad (41)$$

Here the magnetic field is assumed to absent. In this case the electron equation of motion becomes;

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{e}{m_e}E \qquad (42)$$

Where;

$$\omega_0 = \sqrt{\frac{k}{m_0}}, k = m_0\omega_0^2 \qquad (43)$$

In view of equations (17), (18), (19) and (25);

$$P = -eNx, \quad x = x_0 e^{i\omega t}, \quad E = E_0 e^{i(\omega t - Kx)} \qquad (44)$$

Here the frequency of electron vibration and electromagnetic waves is ω, when electromagnetic field is applied. For free vibration without electromagnetic wave frequency is ω. From (44);

$$\frac{\partial P}{\partial t} = -eN\dot{x}, \quad \frac{\partial^2 P}{\partial t^2} = -eN\ddot{x} \qquad (45)$$

$$\dot{x} = -i\omega x_0 e^{i\omega t},\ \ddot{x} = -\omega^2 x \tag{46}$$

$$\frac{\partial E}{\partial t} = iKE,\ \frac{\partial^2 E}{\partial t^2} = -K^2 E \tag{47}$$

$$\frac{\partial^2 E}{\partial t^2} = \nabla^2 E,\ \nabla^2 E = -K^2 E \tag{48}$$

$$\text{and } \frac{\partial E}{\partial t} = i\omega E,\ \frac{\partial^2 E}{\partial t^2} = -\omega^2 E \tag{49}$$

From (42) and (43) one gets;

$$\ddot{x} = \frac{e}{m_e} E - \frac{k}{m_e} x - \gamma \dot{x} \tag{50}$$

Substitution of (50) in (45) gives;

$$\frac{\partial^2 P}{\partial t^2} = -eN\ddot{x} = -eN\left[\frac{e}{m_e} E - \frac{k}{m_e} x - \gamma \dot{x}\right] \tag{51}$$

From equations (51) and (16) gets;

$$-\nabla^2 E + \mu_0 \sigma \frac{\partial E}{\partial t} + \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = -\mu_0 eN\left[\frac{e}{m_e} E - \frac{k}{m_e} x - \gamma \dot{x}\right] \tag{52}$$

From equations (48) and (49) in equation (52) yields;

$$K^2 E + i\omega\mu_0 \sigma E - \omega^2 \varepsilon_0 \mu_0 E = -\frac{\mu_0 eN}{m_e} E + \frac{kx_0 \mu_0 eN}{m_e E_0} E + \frac{i\omega\gamma x_0}{E_0} E \tag{53}$$

Since;

$$x = x_0 e^{i\omega t} = \frac{x_0}{E_0} E_0 e^{i\omega t} = \frac{x_0}{E_0} E \tag{54}$$

It follows;

$$K^2 E - i\omega\mu_0 \sigma E - \varepsilon_0 \mu_0 \omega^2 E = -\frac{\mu_0 e^2 N}{m_e} E + \frac{kx_0 \mu_0 eN}{m_e E_0} E + \frac{i\omega\gamma x_0}{E_0} E \tag{55}$$

4. Results

Equating real parts on both sides of equation (55) then;

$$K^2 E - \varepsilon_0 \mu_0 \omega^2 E = -\frac{\mu_0 e^2 N}{m_e} E + \frac{kx_0 \mu_0 eN}{m_e E_0} E \tag{56}$$

Rearrange equation (56);

$$K^2 E = -\mu_0 eN\left(\frac{e}{m_e} E - \omega_0^2 \frac{x_0}{E_0} E\right) + \varepsilon_0 \mu_0 \omega^2 E \tag{57}$$

$$\text{Where, } \varepsilon_0 \mu_0 = \frac{1}{C^2}, \omega_0 = \frac{K}{m_e} \tag{58}$$

Therefore;

$$K^2 E = \left[-\mu_0 eN\left(\frac{e}{m_e} - \omega_0^2 \frac{x_0}{E_0}\right) + \frac{\omega^2}{C^2}\right] E \tag{59}$$

From which;

$$K^2 = -\mu_0 eN\left(\frac{e}{m_e} - \omega_0^2 \frac{x_0}{E_0}\right) + K_0^2 \tag{60}$$

Where;

$$\frac{\omega}{C} = K_0 \tag{61}$$

From equation (55) ,equating imaginary parts yields;

$$i\omega\mu_0 \sigma E = -i\omega\gamma \frac{x_0}{E_0} E \tag{62}$$

$$\text{Therefore: } \gamma = -\mu_0 \sigma E_0 \tag{63}$$

From equation (59) and (63) gets;

$$K^2 E = -\mu_0 eN\left(\frac{e}{m_e} - \frac{\omega_0^2 \mu_0 \sigma x_0}{\gamma}\right) E + \frac{\omega^2}{C^2} E \tag{64}$$

Multiplying equation (64) by $\hbar^2 C^2$ and dividing by E obtains;

$$\hbar^2 K^2 C^2 = -\mu_0 eN\hbar^2 C^2\left(\frac{e}{m_e} - \frac{\omega_0^2 \mu_0 \sigma x_0}{\gamma}\right) + \hbar^2 \omega^2 \tag{65}$$

From which;

$$P^2 C^2 = -\mu_0 eN\hbar^2 C^2\left(\frac{e}{m_e} - \frac{\omega_0^2 \mu_0 \sigma x_0}{\gamma}\right) + E^2 \tag{66}$$

Therefore;

$$P^2 C^2 = -\mu_0 eN\hbar^2 C^2\left(\frac{e}{m_e} - \frac{\omega_0^2 \mu_0 \sigma x_0}{\gamma}\right) + E^2$$

Rearranging which one gets;

$$E^2 = P^2C^2 + \mu_0 eN\hbar^2C^2\left(\frac{e}{m_e} - \frac{\omega_0^2\mu_0\sigma x_0}{\gamma}\right) \qquad (67)$$

This represents Einstein's Energy Equation.

Comparing equation (67) with Special Relativity energy equation [19];

$$E^2 = P^2C^2 + m_0^2C^4 \qquad (68)$$

One gets;

$$m_0^2C^4 = -\mu_0 eN\hbar^2C^2\left(\frac{e}{m_e} - \frac{\omega_0^2\mu_0\sigma x_0}{\gamma}\right) \qquad (69)$$

From which the photon mass m_0 is given by;

$$m_0 = \sqrt{\mu_0 eN\left(\frac{e}{m} + \frac{\omega_0^2\mu_0\sigma x_0}{\gamma}\right)}\frac{\hbar}{C} \qquad (70)$$

5. Discussion

In this work, the Maxwell's electric wave equation was derived using the well known Maxwell's equations, (see equation 1), beside the relations in (2). Meanwhile, the polarization P thus as source term in this equation. Then, by utilizing the relations (14) and (15) the final form obtained as in (16). To derive the Einstein's Energy Equation from Maxwell's electric wave equation, the polar material is considered. If the electron is affected by electric, resistive and magnetic force, equation (19) is gotten which represents the equation of its motion. Thus, by solving of (19) with (18) and (23) equation (16) yields (24) from the solution of this equation equations (26) and (27) is obtained by substituting this into (24) equation (29) obtained, from which (32) obtained. For energy and momentum of harmonic oscillator as in (33) the equation (36) was found. After many processes and mathematical operations the equation (67) obtained, comparing this with (68), one gets (69) from which (70) is given which represents the relation from which can get the value of the mass of photon.

6. Conclusion

The conclusion of this work is that; the wave equation of electric field for polar material form Maxwell's equations is deduced. Then, a new approach for relativistic energy equation has been attained and hence, from which one can calculate the value of photon mass after comparing it with the familiar relativistic energy equation.

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