Proper Limit Rule

Kanay Barik[1], Asha Rani Paul[2]

[1]M.Sc., B.Ed. , Research Scholar, Kolhaan University, Jamshedpur, Jharkhand (India)

[2]M.Sc Scholar, Kolhaan University, Jamshedpur, Jharkhand (India)

Abstract: ***We can check the correctness of formula by using Dimensional Analysis. In physics, there are many types of energies, forces etc, which are dimensionally same but we cannot equate them. For example:- Mass energy relation cannot be equated with non relativistic kinetic and potential energy of a body, electrostatic force of repulsion between two protons cannot be equated with Gravitational force of attraction between them, but they are dimensionally same. Whether two equations of same dimension can be equated or not, can be check by proper limit rule.***

Keywords: Equation, integration, differentiation, limits, dimensions.

1. Introduction

1.1 Proper limit rule

Whether two equation of same dimension can be equated or not can be check by Proper Limit Rule. Two equations of same dimension can only be equated if the variables of left hand side depend upon the variables of right hand side, directly or indirectly. Since integration is the reverse process of differentiation. After integration and differentiation, if the left hand side of equated equation will equal to right hand side of equated equation, then only we can equate the relation of same dimension.

Steps

Step 1- At first we have to assume that, two equation of same dimension can be equated.
Step 2- Then we have to differentiate the left and right hand side of the equated equation.
Step 3- Then we have to integrate the left and right hand side of the equated equation within proper limit.
Step 4- After differentiation and integration, if the left hand side of equated equation will equal to right hand side, then only we can equate the relation of same dimension otherwise not.

2. Examples of Proper limit rule

Example 1. Whether kinetic energy can be equated with Electrostatic potential energy?
Let us check using proper limit rule
Step1. Let us consider $\frac{1}{2}\text{mv}^2 = \text{qV}$
Step2. Differentiating the variables
$\frac{1}{2}\text{md}(\text{v}^2) = \text{qdV}$
mvdv = qdV
Step3.Integrating within proper limit
$\text{m}\int_0^{\text{v}} \text{vdv} = \text{q}\int_0^{\text{V}} \text{dV}$
When V = 0, v = 0
Or $\frac{1}{2}\text{mv}^2 = \text{qV}$
So they can be equated.

Example 2. Whether hν can be equated with $\frac{1}{2}\text{mv}^2$?
Let us check using proper limit rule,
Step1. Let us assumed $\text{h}\nu = \frac{1}{2}\text{mv}^2$
Step2. Differentiating the variables
$\text{hd}\nu = \frac{1}{2}\text{md}(\text{v}^2)$
hdν = mvdv
Step3.Integrating within proper limit
$\text{h}\int_0^{\nu} \text{d}\nu = \text{m}\int_0^{\text{v}} \text{vdv}$
$\text{h}\nu = \frac{1}{2}\text{mv}^2$
So our assumption was right.

Example 3. Whether Electrostatic force of repulsion between two protons can be equated with Gravitational force of attraction between them?
Step1. Let us consider $\frac{kq^2}{R^2} = \frac{Gm^2}{R^2}$
Or $kq^2 = \text{G}m^2$
Where q is charge on proton and m is the mass of photon.
Step2. Differentiating the variables
$\text{k}dq^2 = \text{Gd}m^2$
k(2qdq) = G(2mdm)
kqdq = Gmdm
Since charge and mass are independent of each other, so they cannot be equated.

Example 4. Whether E = msT can be equated with E = mgh?
Let us check using proper limit rule
Step1. Let us consider msT = mgh
Step2. Differentiating the variables
msdT = mgdh
Since there is no relation between Temperature and height, so we cannot equate them.

Example 5. Whether kinetic energy can be equated with potential energy?
Let us check using proper limit rule.
Step1. Let us assumed that kinetic energy can be equated with potential energy
$\frac{1}{2}\text{mv}^2 = \text{mgh}$
Step2. Differentiating the variables,
$\frac{1}{2}\text{md}(\text{v}^2) = \text{mgdh}$
mvdv = mgdh
Step3. Integrating within proper limit
$\text{m}\int_0^{\text{v}} \text{vdv} = \text{mg}\int_{\text{h}}^{0} \text{dh}$

$\frac{1}{2}mv^2 = -mgh$

So they are inversely proportional to each other.

Example 6. Whether Heat energy can be equated with Einstein's mass energy relation?

Let us check using proper limit rule

Step1. Let us consider $msT = mc^2$

Step2. Differentiating the variables

$$smdT = c^2dm$$
$$smdT = c^2dm$$

Step3.Integrating within proper limit

$sm\int \; dT = c^2 \int \; dm$

Since there is no relation between relativistic mass and Temperature, so we cannot equate them.

Example 7. Whether mc^2 can be equated with $\frac{1}{2}mv^2$?

Let us check within proper limit rule

Step1. Let us consider $mc^2 = \frac{1}{2}mv^2$

Step2. Differentiating the variables

In the above relation, in left hand side mass is variable and speed of light is constant but in the right hand side mass is constant and velocity is variable. So we cannot equate them.

Example 8. Whether $E = h\nu$ can be equated with $E = mc^2$ for photon?

Let us check using proper limit rule.

For Photon

Step1. Let us assumed that, $h\nu = mc^2$

Step2. Now we have to differentiate the variables

$hd\nu = c^2dm$

Step3. Then we have to integrate the above equation within proper limit

$h\int_0^{\nu} d\nu = c^2 \int_0^{m} dm$

Since photon does not exist at rest but in motion its frequency is ν and mass becomes m.

$h(\nu - 0) = c^2 (m - 0)$

$h\nu = mc^2$

So our assumption was right. So $h\nu$ can be equated with mc^2 in case of photon.

Example 9. Whether $E = h\nu$ can be equated with $E = mc^2$ for electron?

Let us check using proper limit rule

For Electron

Step1. Let us assumed that, $h\nu = mc^2$

Step2. Now we have to differentiate the variables

$hd\nu = c^2dm$

Step3. Then we have to integrate the above equation within proper limit

$h\int_0^{\nu} d\nu = c^2 \int_{m_0}^{m} dm$

When the velocity of particle is zero, then according to de-Broglie's hypothesis, wavelength will be infinite; frequency is zero and mass is m_0. It should be noted that for equivalence we can write the frequency of electron at rest as ν_0, this does not means that electron wave at rest vibrate with frequency ν_0, if this is true then at rest it must has wavelength, which is not possible because wave is only associated with motion. When the velocity of particle is v, and then its frequency may be ν and mass becomes m.

$h(\nu - 0) = c^2 (m - m_0)$

$h\nu = \Delta mc^2$

So for electron, we cannot equate $h\nu = mc^2$, as the rest mass of electron is not zero. Further we cannot use this equation to proof de-Broglie hypothesis.

Example 10. Whether $\lambda = 2d\sin\theta$ can be equated

$\lambda = \frac{h}{mv} = \frac{h}{(2mE_k)^{\frac{1}{2}}}$

Let us check using Proper limit rule

Step1. Let us assumed that $2d\sin\theta = \frac{h}{(2mE_k)^{\frac{1}{2}}}$

$\frac{h^2}{2mE_k} = 4d^2 \sin^2\theta$

Or $E_k = \frac{h^2}{8md^2\sin^2\theta}$

Since m, h &d are constant. So from above relation we can say that K.E is inversely proportional to $\sin^2\theta$. But there is no relation between θ and K.E of electron, K.E depends on scattering angle φ relative to incident beam which is 50^0 in a particular case of Davisson- Germer Experiment when the kinetic energy was 54eV. Again the energy of electron increases when it enters a crystal by an amount equal to work function of the surface. Hence the electron speeds will increases inside the crystal and its wavelength becomes shorter. Another complication arises from interference between waves diffracted by different families of Bragg planes.

So we cannot equate $2d\sin\theta$ with $\frac{h}{mv}$. In particular case they may be same, but not always.

3. Conclusion

Two equations of same dimension cannot be equated always.

4. Scope

This rule can guide us to equate the equations of same dimensions.

Reference

[1] Louis de-Broglie-Revolution in modern physics
[2] Niels Bohr- Atomic physics and Human Knowledge
[3] Ashoke Mukhopadhay- Physics and philosophy
[4] David Bohm- The undivided universe
[5] H.C Verma- The concept of physics
[6] J.N Kapoor – Mathematical Modeling
[7] George Greenstein -The Quantum challenge
[8] Arthur Beiser -Modern Physics, p- 116.

Author Profile

Kanay Barik has done M.Sc., B.Ed. He is a Research Scholar at Kolhan University,–Kendadih, PS- Jadugora, Via – Ghatsila, Singhbhum (Jharkhand), PIN-832303

Asha Rani Paul is doing M.Sc. at Kolhan University, Kendadih, PS- Jadugora, Via – Ghatsila, Singhbhum (Jharkhand), PIN-832303