A New approach for Design Technique of Multi-Rate Output Feedback Control using PID Controller

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Abstract: ***A new approach for designing a multirate output feedback control system using PID Controller, where the update interval of the control input is an integer multiple of the sampling interval of the plant output. The proposed method is designed by extending a conventional multirate output feedback control using the latest information of a plant. Hence, the proposed method can improve the control performance compared with conventional methods without the latest information using PID Controller. Furthermore, the obtained control structure can be simpler than conventional methods.***

Keywords: **SFG**-State Feedback Gain, MROF-Multirate Output Feedback Control, PID-Proportional Integral Derivative

1. Introduction

This method of sampling of control system, in which update interval of the control input is an integer multiple of the sampling interval of the plant output. In this method of feedback, we use latest information about the plant. We extended the multirate output feedback control method by adding a past plant output to obtain a robust feedback controller against round off error. In this method, the closed-loop stability of the multirate output feedback control remains, and control performance can be improved because the feedback gain can be redesigned by selecting additional design parameters. Although this we also proposes an extending method of the multirate output feedback control, the proposed method is designed using the present output. Hence, the problem of time delay is resolved because the latest information of a plant is employed. Furthermore, the structure of the proposed method can be described simpler as compared with the conventional method. In multirate system, only the system outputs and past control inputs are used to compute the control input.

$$u(k) = y(k)+u(k-1) \tag{1}$$

where *u(k)* is input sequence *,y(k)* is output sequence and u(k-1) is the past input sample. Hence multirate output feedback control for uncertain discrete-time systems is proposed. In this technique, the system output is sampled at a rate faster than the control input. Means output is sampled at faster rate and control input is updated at slower rate. Consequently, the control algorithm is based on output feedback and at the same-time, is applicable to all controllable and observable systems. This feature makes the proposed control algorithm more practical in comparison to state feedback based methods. Thus, it has the advantages of both state feedback and output feedback control philosophies.

2. Multirate Output Feedback Control

For the simplicity of description, this paper deals with the case of $n = N = 2$. Hence, the multirate sampled output vector y_k is expressed as:

$$y_k = \begin{bmatrix} y\big((k-1)\tau\big) \\ y((k-1)\tau+\Delta) \end{bmatrix} \tag{2}$$

Then, using a feedback gain F_x, the control input $F_x x(k\tau)$ is realized as:

$$u(k\tau) = F_y y_k + F_u u((k-1)\tau) \tag{3}$$

where

$$F_y = F_x L_y = [F_{y1}\ F_{y2}] \tag{4}$$

$$F_u = F_x L_u \tag{5}$$

$$Ly = \Phi_\tau\ {C_0}^{-1} \tag{6}$$

$$Lu = \Gamma_\tau - \Phi_\tau {C_0}^{-1} D_0 \tag{7}$$

In the case of $F_{y1} \cong -F_{y2}$, if sampling interval is small, $F_y y_k$ is almost 0 due to numerical round off error.

This is because that

$$y((k-1)\tau) \cong y((k-1)\tau+\Delta).$$

We apply same procedure using present and past plant output and getting result. Using a reaching law approach proposed for the quasi sliding mode control of discrete-time LTI systems, a control law is designed. The reaching law approach given by satisfying the condition:

$$s(k+1) - s(k) = -q\tau s(k) - \varepsilon\,\mathrm{sgn}(s(k)) \tag{8}$$

where $1 - q\tau > 0$, $\varepsilon > 0$ and, $s(k)$ is the sliding surface given as:

$$s(k) = c^T x(k) \quad (9)$$

The state feedback control law that satisfies the reaching condition is derived as:

$$u(k) = -(c^T\Gamma_\tau)^{-1} \times ((c^T\Phi_\tau c^T + q\tau c^T)x(k) + \varepsilon\tau \mathrm{sgn}(c^T x(k))) \quad (10)$$

Substitution of equations gives a multirate output feedback quasi sliding mode control law:

$$u(k) = \widetilde{F_y}\,\widetilde{y_k} + \widetilde{F_u}u((k-1)\tau) - (c^T\Gamma_\tau)^{-1}\varepsilon\tau \mathrm{sgn}(c^T(\widetilde{L_y}\,\widetilde{y_k} + \widetilde{L_u}\,u((k-1)\tau))) \quad (11)$$

where Fx is set to

$$-(c^T\Gamma_\tau)^{-1} \times ((c^T\Phi_\tau - c^T + q\tau c^T)$$

3. Extension of MROF using past plant output

The coefficients Ly and Lu determined by the above equations have no redesign parameters. Hence, extended the multirate sampled outputs y_k, and design parameters are newly introduced. Using new design parameters, the coefficients can be redesigned independent of the value of $x(k\tau)$.

Using y_k and $y((k-1)\tau - \Delta)$, y_k is defined as:

$$\overline{y_k} = \begin{bmatrix} y((k-1)\tau - \Delta) \\ y_k \end{bmatrix} \quad (12)$$

$$= \begin{bmatrix} y((k-1)\tau - \Delta) \\ y((k-1)\tau) \\ y((k-1)\tau + \Delta) \end{bmatrix} \quad (13)$$

Then,

$$\overline{y_{k+1}} = \begin{bmatrix} y(k\tau - \Delta) \\ y(k\tau) \\ y(k\tau + \Delta) \end{bmatrix}$$

$$= \begin{bmatrix} Cx(k\tau - \Delta) \\ Cx(k\tau) \\ Cx(k\tau + \Delta) \end{bmatrix} \quad (14)$$

$x(k\tau - \Delta)$ is obtained as:

$$x(k\tau - \Delta) = \Phi^{-1}x(k\tau) - \Phi^{-1}\Gamma\, u(k\tau - \Delta)$$

$$\overline{y_{k+1}} = \begin{bmatrix} C\Phi^{-1}x(k\tau) - C\Phi^{-1}\Gamma\, u(k\tau - \Delta) \\ Cx(k\tau) \\ C\Phi x(k\tau) + C\Gamma u(k\tau) \end{bmatrix}$$

$$\overline{y_{k+1}} = \overline{C_0}x(k\tau) + \bar{\Gamma}u_k \quad (15)$$

Where,

$$\overline{C_0} = \begin{bmatrix} C\Phi^{-1} \\ C \\ C\Phi \end{bmatrix} \quad (16) \qquad \bar{\Gamma} = \begin{bmatrix} -C\Phi^{-1}\Gamma & 0 \\ 0 & 0 \\ 0 & C\Gamma \end{bmatrix} \quad (17)$$

$$u_k = \begin{bmatrix} u((k-1)\tau) \\ u(k\tau) \end{bmatrix} \quad (18)$$

$\overline{C_0}^*$ is defined as satisfying, $\overline{C_0}^*\,\overline{C_0} = I_{2\times 2}$ and then,

$$x(k\tau) = \overline{C_0}^*\,(\overline{y_{k+1}} - \bar{\Gamma}u_k)$$

$$x((k+1)\tau) = .\overline{L_y}\,\overline{y_{k+1}} + .\,\overline{L_u}u_k \quad (19)$$

where

$$\overline{L_y} = \Phi_\tau\overline{C_0}^* \quad (20)$$

$$\overline{L_u} = \bar{\Gamma}_\tau - \Phi_\tau\overline{C_0}^*\bar{\Gamma} \quad (21)$$

$$\bar{\Gamma}_\tau = [0\ \Gamma_\tau] \quad (22)$$

Hence, the present state variable is obtained as:

$$x(k\tau) = \overline{L_y}.y_k + \overline{L_u}.u_{k-1} \quad (23)$$

Then, the control input $u(k\tau) = F_x x(k\tau)$ is expressed as:

$$u(k\tau) = \overline{F_y}\,\overline{y_k} + \overline{F_u}u_{k-1} \quad (24)$$

where,

$$\overline{F_y} = F_x\overline{L_y} \quad (25)$$

$$\overline{F_u} = F_x\overline{L_u} \quad (26)$$

Using design parameters introduced newly, $\overline{F_y} = F_x\overline{L_y}$ is designed independent of the value of $x(k\tau)$ to obtain a robust controller against round off error.

Design of $\overline{\boldsymbol{C_0}}^*$ **:** To design of $\overline{\boldsymbol{C_0}}^*$, $\overline{\boldsymbol{C_{00}}}^*$ and $\overline{\boldsymbol{C_{01}}}^*$ are defined as:

$$\overline{\boldsymbol{C_{00}}}^* = \boldsymbol{\Phi}\begin{bmatrix} \boldsymbol{C_0}^{-1} & 0 \\ & 0 \end{bmatrix} \quad (27)$$

$$\overline{\boldsymbol{C_{01}}}^* = \boldsymbol{\Phi}\begin{bmatrix} -\phi_1 & -\phi_2 & 1 \\ -\phi_1 & -\phi_2 & 1 \end{bmatrix} \quad (28)$$

where ϕ_1 and ϕ_2 are the coefficients of the characteristic equation of Φ given as:

$\lambda^2 - \phi_2\lambda - \phi_1 = 0$ (29)

Using (28) and (29), $\overline{C_0}^*$ is designed as:

$$\overline{C_0}^* = \overline{C_{00}}^* + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\overline{\boldsymbol{C_{01}}}^* \quad (30)$$

Where λ_1 and λ_2 are design parameters.
Using (28),

$$\overline{C_0}^*\overline{C_0} = I \quad (31)$$

Based on Cayley-Hamilton theorem

$$\Phi^2 - \phi_2\Phi - \phi_1 I = 0 \quad (32)$$

and using (28),

$$\overline{C_{01}}^{\star}\overline{C_0}=0_{2\times 2} \qquad (33)$$

As a Result, $\overline{C_0}^{\star}\overline{C_0} = I$ can be achieved independent of the selection of λ_1 and λ_2. Then,

$\bar{F_y}$ = [$\overline{F_{y1}}\,\overline{F_{y2}}\,\overline{F_{y3}}$] can be designed using λ_1 and λ_2 independent of the value of x ($k\tau$). Hence, the effect of round off error can be improved.

However, the latest information of a plant is not employed in the conventional methods, and the extended method needs the calculation of the inverse of ϕ.

4. Extension of MR OF us ing pr esent pla nt output

The proposed method extends the conventional multirate output feedback control [3] using the latest plant output $y(k\tau)$ instead of a past plant output $y((k-1)\tau-\Delta)$. Consequently, the effect of round off error can be improved the same as the conventional method [4] and further, the calculation of Φ^{-1} is not required.

Using y_k and $y(k\tau)$, $\widetilde{y_k}$ is defined as:

$$\widetilde{y_k} = \begin{bmatrix} y_k \\ y(k\tau) \end{bmatrix}$$

$$= \begin{bmatrix} y\big((k-1)\tau\big) \\ y\big((k-1)\tau+\Delta\big) \\ y(k\tau) \end{bmatrix} \qquad (34)$$

Then,

$$\widetilde{y_{k+1}} = \begin{bmatrix} y(k\tau) \\ y\big((k-1)\tau-\Delta\big) \\ y((k+1)\tau) \end{bmatrix} \qquad (35)$$

$$= \begin{bmatrix} Cx(k\tau) \\ C\Phi x(k\tau) - C\Gamma u(k\tau) \\ C\Phi^2 x(k\tau) - C(\Phi+I)\Gamma u(k\tau) \end{bmatrix} \qquad (36)$$

$$= \widetilde{C_0}\, x(k\tau) + \tilde{\Gamma}\, u(k\tau) \qquad (37)$$

Where,

$$\widetilde{C_0} = \begin{bmatrix} C \\ C\Phi \\ C\Phi^2 \end{bmatrix},$$

$$\tilde{\Gamma} = \begin{bmatrix} 0 \\ C\Gamma \\ C(\Phi+I)\Gamma \end{bmatrix} \qquad (38)$$

$\widetilde{\boldsymbol{C_0}}^{\star}$ is defined as satisfying $\widetilde{\boldsymbol{C_0}}^{\star}\widetilde{C_0} = I_{2\times 2}$, and then,

$$x(k\tau) = \widetilde{\boldsymbol{C_0}}^{\star}\,(\widetilde{y_{k+1}} - \tilde{\Gamma}u(k\tau)) \qquad (39)$$

Using (39) is rewritten as:

$$x((k+1)\tau) = \widetilde{L_y}\,\widetilde{y_{k+1}} + \widetilde{L_u}\,u(k\tau) \qquad (40)$$

where

$$\widetilde{L_y} = \Phi_\tau \widetilde{C_0}^{\star} \qquad (41)$$

$$\widetilde{L_u} = \Gamma_\tau - \Phi_\tau \widetilde{C_0}^{\star}\tilde{\Gamma} \qquad (42)$$

Hence, the present state variable is given as:

$$x(k\tau) = \widetilde{L_y}\,\widetilde{y_k} + \widetilde{L_u}\,u((k-1)\tau) \qquad (43)$$

Then, the control input $u(k\tau) = \widetilde{F_x}x(k\tau)$ is expressed as:

$$u(k\tau) = \widetilde{F_y}\,\widetilde{y_k} + \widetilde{F_u}\,u((k-1)\tau) \qquad (44)$$

where,

$$\widetilde{F_y} = \widetilde{F_x}\widetilde{L_y} \qquad (45)$$

$$\widetilde{F_u} = \widetilde{F_x}\widetilde{L_u} \qquad (46)$$

New design parameters are newly introduced the same as the conventional method [4],

and the effect of round off error is improved independent of the value of $x(k\tau)$ by designing $\widetilde{F_y}$.

$\widetilde{C_0}^{\star}$ is designed as:

$$\widetilde{C_0}^{\star} = \widetilde{C_{00}}^{\star} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \widetilde{C_{01}}^{\star} \qquad (47)$$

$$\widetilde{C_{00}}^{\star} = \begin{bmatrix} C_0^{-1} & 0 \\ & 0 \end{bmatrix} \qquad (48)$$

$$\widetilde{C_{01}}^{\star} = \begin{bmatrix} -\phi_1 & -\phi_2 & 1 \\ -\phi_1 & -\phi_2 & 1 \end{bmatrix} \qquad (49)$$

5. PID Controller

The PID Controller is the most popular feedback controller used within the process industries. It is a robust easily understood algorithm that can provide excellent control performance despite the varied dynamic characteristics of process plant. Owing to the fact that they have only three terms to adjust their input-output behaviour, their implementation is very simple and can be performed even without the use of sophisticated microcontrollers and/or microcomputers. Although simple in structure, their field of applicability is quite versatile and this is the primary reason behind their widespread use in the industry. Adjusting the PID gains to achieve a good response could be problematic, especially for an inexperienced user. As a result, most commercial PID controllers have functions to tune the 3 parameters automatically. This is normally called “autotunig” feature. There are some variants of autotunig methods. Here mention one of them, the relay feedback, which is closely related to a manual tuning scheme known as Ziegler-Nichols Frequency Domain (ZNFD) method.

6. Numerical Example

Linear Model:
A controlled plant is given as:

$$\dot{x}(t) = \begin{bmatrix} -100.77 & 201.53 \\ -201.53 & 100.77 \end{bmatrix} x(t) + \begin{bmatrix} -100.77 \\ 100.77 \end{bmatrix} u(t)$$

$$y(t) = [1\ 0]\, x(t) \qquad (50)$$

τ and N are set to 0.006 and 2, respectively. The sliding surface and design parameters q and ε are set as:

$$c^T = [-0.738\ 1] \qquad (51)$$
$$q = 1 \qquad (52)$$
$$\varepsilon = 0.1 \qquad (53)$$

To improve the effect of quantization, the control law is extended by using $[\lambda_1\ \lambda_2] = [-0.5\ 1.0]$ in the conventional [2] and the proposed methods.

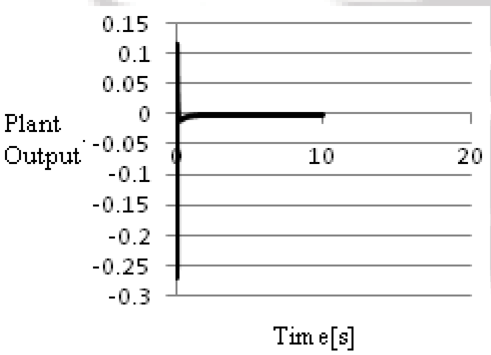

Figure 1: Proposed MROF

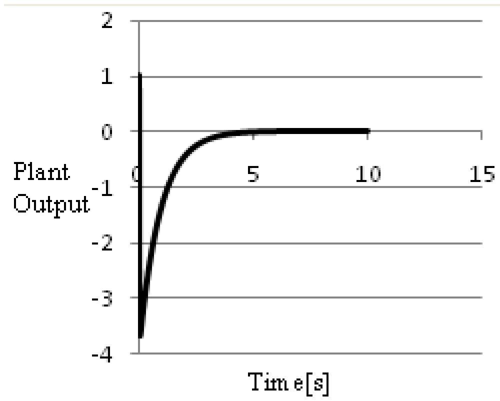

Figure 2: Conventional MROF

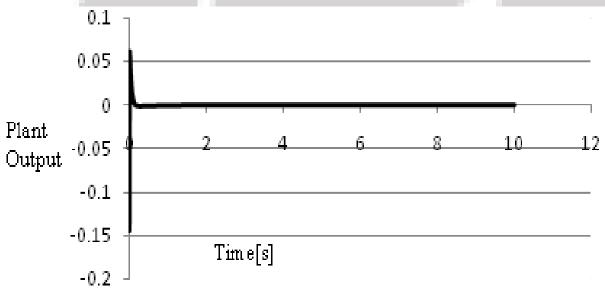

Figure 3: Proposed MROF Using PID Controller

7. Conclusion

After implementing sliding mode control law gain F_x=[0.2664 0.7319] and then using that control gain we calculated multirate output feedback gains of conventinal control law as

$F_y = [-1.7303\ 1.7292]$ and $F_u = 0.9995$.

and gains of extended control laws are

$\bar{F_y} = [-0.7302\ \ -0.4654\ 0.9990]$ and

$\bar{F_u} = \begin{bmatrix} 0.4880 & 0.0893 \\ 0.2441 & 1.1787 \end{bmatrix}$.

and gains of proposed control laws are

$\tilde{F_y} = [-0.3660\ \ -0.6339\ 1.3643\ ]$ and

$\tilde{F_u} = 0.6339$.

These gains are calculated with the help of MATLAB.

As a result, the calculation of the inverse of a state transition matrix is not needed, and the structure of the designed control system can be simply described as compared with the conventional method. Furthermore, the proposed method has potential to achieve higher performance more than the conventional method because the present plant output as the latest information of a plant is employed. By using conventional method system control input does not have latest information about plant dynamics but its extending approach as well as proposed approach has latest information about plant dynamics. Implementing this method on an linear plant, its performance be controlled or we can say improved and at the same time closed loop stability remains.

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