Investigation of PAPR Reduction in MIMO-OFDM System using PTS and SLM Techniques

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Abstract: ***Communication is one of the important aspects of life. With the advancement in age and its growing demands, there has been rapid growth in the field of communication. In recent years, the communication industry has started focusing on fourth generation (4G) mobile communication systems. It is expected that 4G will provide a comprehensive and secure IP solution where voice, data, and multimedia can be offered to users at "anytime, anywhere" with higher data rates than previous generations. For better spectrum efficiency and obtains as high as 100Mbps wireless transmission rate, 4G requires more advanced communication techniques to be employed. Multiple input multiple output (MIMO) and orthogonal frequency division multiplexing (OFDM) have, therefore, been adopted due to their superior performance. OFDM is an attractive modulation technique for transmitting large amount of digital data over radio waves. However, the principal drawback of OFDM is that the peak transmitted power can be substantially larger than the average power; abbreviated as PAPR. It is preferred to have a minimum PAPR, as it will allow a higher average power to be transmitted for a fixed peak power; and thus, improving the overall signal to noise ratio at the receiver. Therefore, this paper investigates of significant reduction in PAPR has been achieved using selected mapping (SLM) and partial transmit sequence (PTS) in MIMO-OFDM systems. From the analysis, it is inferred that PTS method provides a better PAPR reduction performance compared to SLM method.***

Keywords: Peak to Average Power Ratio, PTS and SLM; OFDM conventional; Multi-Carrier Systems.

1. Introduction

Communication is one of the important aspects of life. With the advancement in age and its growing demands, there has been rapid growth in the field of communication. In recent years, the communication industry has started focusing on fourth generation (4G) mobile communication systems. It is expected that 4G will provide a comprehensive and secure IP solution where voice, data, and multimedia can be offered to users at "anytime, anywhere" with higher data rates than previous generations [1]. For better spectrum efficiency and obtain as high as 100Mbps wireless transmission rate, 4G requires more advanced communication techniques to be employed. Multiple input multiple output (MIMO) and orthogonal frequency division multiplexing (OFDM) have, therefore, been adopted due to their superior performance. OFDM is an attractive modulation technique for transmitting large amount of digital data over radio waves. OFDM is preferred as it offers a high tolerance to multipath signals and is spectrally efficient; which makes it suitable for future wireless communication systems. However, it has few disadvantages; with the key one is that the peak of the OFDM signal can be up to N times the average power (where N is the number of carriers). These large peaks increase the amount of inter-modulation distortion resulting in an increase in the error rate. This ratio of peak power of signal to average power is called peak-to-average power ratio; abbreviated as PAPR. It is preferred to have a minimum PAPR, as it will allow a higher average power to be transmitted for a fixed peak power; and thus, improving the overall signal to noise ratio at the receiver. A few examples of the effect of high PAPR is shown in Figures 1 & 2. There are a number of techniques dealing with the problem of PAPR. Some of these include: constellation shaping, nonlinear companding transforms [2], tone reservation [3] and tone injection (TI) [4], clipping and filtering [5], signal scrambling techniques [6] and precoding based techniques. Among these, two signal scrambling techniques, namely, selected mapping (SLM) and partial transmit sequence (PTS) are investigated in this paper.

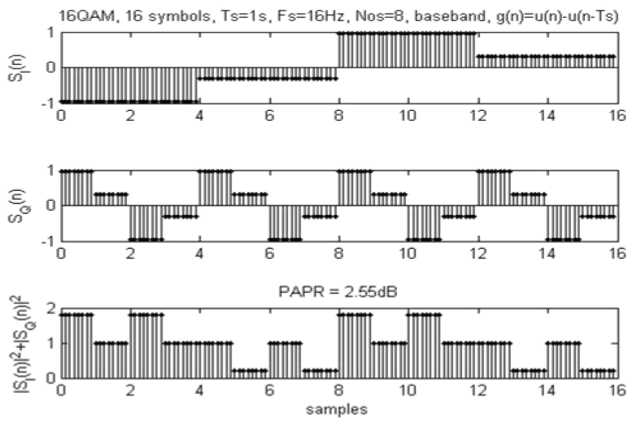

Figure 1: An example of high PAPR in a typical digital QAM modulated OFDM signal.

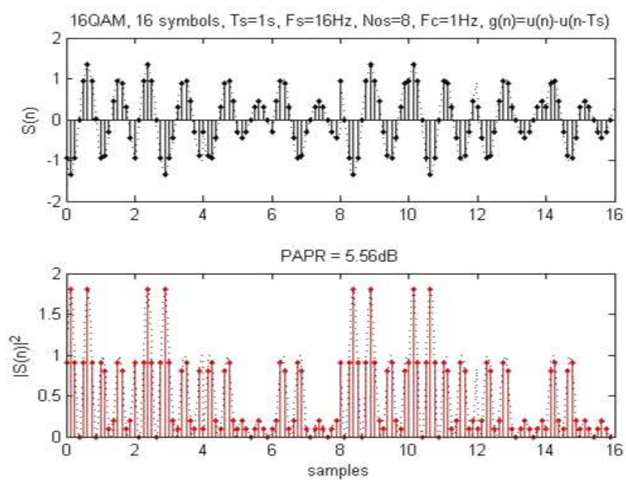

Figure 2: An example of high PAPR in a typical analog QAM modulated OFDM signal.

2. System Model

2.1 OFDM System Model

OFDM is a multicarrier system, with the functional block diagram shown in Figure 3 below.

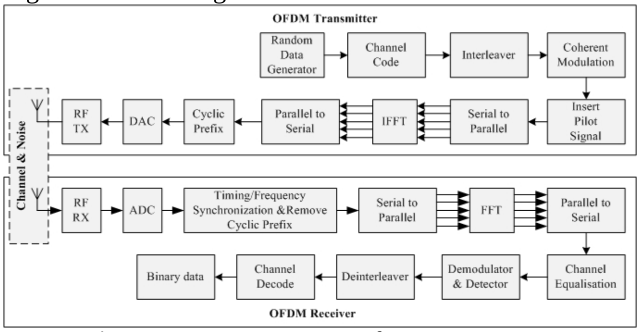

Figure 3: Basic structure of OFDM system.

In Figure 3, at the transmitting end, the input binary serial data stream is first processed by channel encoder, constellation mapping and serial to parallel (S/P) conversion. A single signal is divided into N parallel routes after N-point inverse fast Fourier transform (IFFT). Each orthogonal sub-carrier is modulated by one of the N data routes independently. By definition, the N processed points constitute one OFDM symbol. In Figure 3, these baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N. This complex vector of size N can be mathematically expressed as

$$\mathrm{X} = [\mathrm{X}_0, \mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3 \ldots \mathrm{X}_{N-1}] \quad (1)$$

X is then passed through the IFFT block to give

$$x = WX \quad (2)$$

Where, W is the $N \times N$ IFFT matrix. Thus, the complex baseband OFDM signal with N sub carriers can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} dX_k e^{\frac{j2\pi kn}{N}} \quad n = 0, 1, \ldots, N-1. \quad (3)$$

After parallel-to-serial conversion, a cyclic prefix (CP) with a length of N_g samples is appended before the IFFT output to form the time-domain OFDM symbol, $s = [s_0, \ldots, s_{N+Ng-1}]$, where, $s_i = x_{(i-N_g)_N}$. The useful part of OFDM symbol does not include the N_g prefix samples and has duration of T_u seconds. The samples (s) are then amplified, with the amplifier characteristics is given by function F. The output of amplifier produces a set of samples given by:

$$y = [y_0, y_1, \ldots, y_{N+Ng-1}] \quad (4)$$

At the receiver front end, the received signal is applied to a matched filter and then sampled at a rate $T_s = T_u/N$. The received signals are then fed into an analog to digital (A/D) converter, sample output and take timing estimation to find initial position of OFDM symbol. After dropping the CP samples (N_g), the received sequence z, assuming an additive white Gaussian noise (AWGN) channel, can be expressed as

$$z = F(Wd) + \eta \quad (5)$$

Where, the noise vector η consists of N independent and normally distributed complex random variables with zero mean and variance $\sigma_n^2 = E\{|\eta_n|^2\}$. Subsequently, the sequence z is fed to the fast Fourier transform (FFT), which produces the frequency-domain sequence r as

$$r = W^H z \quad (6)$$

Where, k_{th} element of r is given by

$$r_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} z_n e^{\frac{-j2\pi kn}{N}} \quad k=0, 1, 2, \ldots, N-1 \quad (7)$$

Finally, the estimated symbols vector $\hat{d}$ can be obtained from r by passing the same through the channel decoder, which eventually recover the original data. The real and imaginary parts of complex factor corresponding to in-phase components and quadrature components of OFDM symbols, respectively. It is to be noted that in ideal cases, the demodulation is performed based on the assumption of perfect symbol timing, carrier frequency, and phase synchronization. This is usually not practically possible to achieve; therefore, the demodulated signal will not be the exact replica of input signal; resulting in bit error rate (BER). The term BER can be mathematically expressed as the difference of the received demodulated data and the input data.

2.1.1 PAPR analysis

Mathematically, large peaks in OFDM system can be expressed in terms of Peak-to-Average Power Ratio (also abbreviated PAPR) as:

$$PAPR = \frac{P_{peak}}{P_{average}} = 10log_{10} \frac{\max[|x_n|^2]}{\mathrm{E}[|x_n|^2]} \quad (8)$$

Where P_{peak} represents peak output power, $P_{average}$ means average output power, E[.] denotes the expected value and x_n represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols X_k. However in practice, it is preferred to take the probability of PAPR exceeding a threshold as the measurement index to represent the distribution of PAPR. This is described as Complementary Cumulative Distribution Function (CCDF), and is mathematically expressed as:

$$P(PAPR > z) = 1 - (1 - \exp(-z))^N \quad (9)$$

2.1.2 Key PAPR reduction techniques

There are many different techniques that have been proposed to provide a solution to the problem of high PAPR of OFDM system. These PAPR reduction solutions can be divided into three categories: signal distortion, signal scrambling techniques and coding techniques. Among these techniques, this paper is focused on the comparison of two signal scrambling techniques, Selective Mapping (SLM) and Partial Transmit Sequences (PTS). These are discussed below.

2.1.3 SLM for reducing PAPR

The block diagram of SLM technique for reducing PAPR is shown in Figure 4.

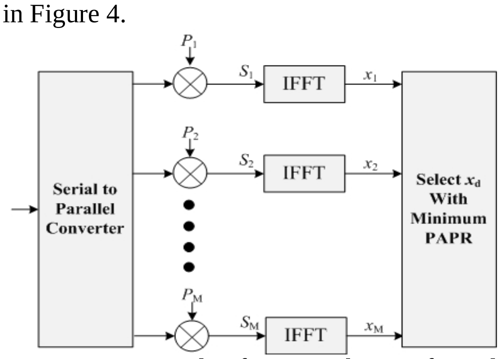

Figure 4: Basic principle of SLM technique for reducing PAPR.

In this approach, it is assumed that M OFDM symbols carry the same information and that these are statistically independent of each other. In this case, the probability of PAPR greater than z is equal to the product of each independent signal and can be written as

$$P(PAPR_{low} > z) = ((1-\exp(-z))^N)^M \qquad (10)$$

In SLM method, shown in Figure 10, firstly M statistically independent sequences which represent the same information are generated, and next, the resulting M statistically independent data blocks $S_m = [S_{m,0}, S_{m,1}, \dots, S_{m,N-1}]^T$, m=1,2,…,M are then forwarded into IFFT operation simultaneously. Finally, at the receiving end, OFDM symbols $x_m = [x_1, x_2, \dots, x_N]^T$ in discrete time-domain are acquired, and then the PAPR of these M vectors are calculated separately. The sequences x_d with the smallest PAPR will be elected for final serial transmission.

The key point of selected mapping method lies in how to generate multiple OFDM signals when the information is same. For this purpose, firstly different pseudo-random sequences $P_m = [P_{m,0}, P_{m,1}, \dots, P_{m,N-1}]^T$, m=1,2,…,M, are defined; where $P_{m,n} = e^{j\varphi_{m,n}}$ and stands for the rotation factor. $P_{m,n}$ is also known as the weighting factor and $\varphi_{m,n}$ is uniformly distributed in [0 2π]. The N different sub-carriers are modulated with these vectors respectively so as to generate candidate OFDM signals. This process can also be seen as performing dot product operation on a data block X_n with rotation factor P_m.

In practice, all the elements of phase sequence P_1 are set to 1 so as to make this branch sequence the original signal. The symbols in branch m can be expressed as

$$S_m = [X_0 P_{m,0}, X_1 P_{m,1,\dots,} X_{N-1} P_{m,N-1}]^T, m = 1,2,\dots,M \qquad (10)$$

These M OFDM frames are then transformed from frequency domain to time domain by performing IFFT calculation. The entire process can be mathematically expressed as

$$x_m(t) = \frac{1}{\sqrt{N}} \sum_0^{N-1} X_n\, P_{m,n}.\, e^{j2\pi n\Delta f t}, 0 \le y \le NT, m = 1,2,\dots,M \qquad (11)$$

Finally, the one which possess the smallest PAPR value is selected for transmission. Its mathematical expression is given as

$$x_d = argmin_{1\le m\le M}(PAPR(x_m)) \qquad (12)$$

Where, argmin (.) represents the argument with minimum value. At the receiver, in order to correctly demodulate the received signal, it is necessary to know which sequence is linked to the smallest PAPR among M different candidates after performing the dot product. Thus, the receiver is required to learn information about selected phase vector sequence and ensure that the vector sequence is received correctly. An intuitive approach is to select the whole sequence of branch number m as side information transmitted to the receiving end. However in practice, it can be realized by sending the route number of the vector sequence instead. This is only possible when the receiving end is able to restore the random phase sequence by means of look-up table or any other method.

2.1.4 Partial Transmit Sequence for reducing PAPR

The functional block diagram of partial transmit sequence based PAPR reduction technique is shown in Figure 5. It is based on dividing the original OFDM sequence into several sub-sequences; and for each sub-sequence, multiplying by different weights until an optimum value is achieved.

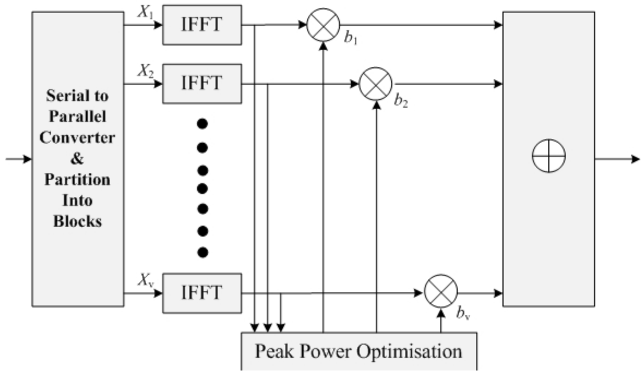

Figure 5: Block diagram of PTS technique for reducing PAPR.

In Figure 5, the data information in frequency domain X is separated into V non-overlapping sub-blocks and each sub-block vectors has the same size N. Thus, each sub-block contains N/V non-zero elements; with the rest part is zero. Mathematically, these sub-blocks can be expressed as

$$\hat{X} = \sum_{v=1}^{V} b_v X_v \qquad (13)$$

where, $b_v = e^{j\varphi_v}(\varphi_v \in [0,2\pi])\,\{v = 1,2,\dots,V\}$ is a weighting factor used for phase rotation. The signal in time domain is obtained by applying IFFT operation on X_v.

In the next step, a suitable factor combination; $b = [b_1, b_2, \dots, b_v]$ is made to find the optimum value suitable for reducing PAPR. Mathematically, it is given by:

$$b = [b_1, b_2, \dots, b_v] \qquad (14)$$
$$= argmin_{(b_1,b_2,\dots,b_v)}\left(max_{1\le n\le N}\left|\sum_{v=1}^{V} b_v.\, x_v\right|^2\right)$$

Where, argmin (*) is the condition to find the best value of ***b*** in order to optimize the PAPR performance. However on its downside, it increases the complexity as extra V-1 times IFFT operations needs to be performed. In addition, it will also require the PAPR value to be calculated at each step of the optimization technique. This usually involves a large number of trials before an optimum value can be obtained. In addition, to enable the receiver to identify different phases; it is also required to send phase factor (***b***) to receiver as sideband information. In this case, it is usual practice to set the first sub-block b_1 to unity for simplicity. So, the redundancy bits account for $(V-1)log_2W$ in which V represents the number of sub-block and W indicates possible variations of the phase. This increases the computational complexity of PTS. For example, if φ_v contains W possible values; then theoretically, b will have W^v different combinations. Therefore, a total of $V.W^v$ IFFTs will be introduced. By increasing V and W, the computational cost of PTS technique will increase. For instance, if phase factor b_v is defined for four possible values; then it means $b_v \in [\pm 1, \pm j]$ and for each OFDM symbol 2·(V-1) bits are transmitted as side information. Therefore in practical applications, computation complexity can be reduced by limiting the value range of phase factor $b = [b_{1,} b_{2,\dots,} b_v]$ to a proper level. At the same time, it can also be changed by different sub-block partition schemes.

3. Results & Discussion

3.1 Simulation of SLM scheme.

From the above made discussion, it can be interpreted that the ability of PAPR reduction using SLM is affected by the route number M and subcarrier number N. Therefore, the same is simulated in Matlab with different values of M and N. These are discussed below in two cases. The key parameters used for performing these simulations are mentioned in Table 1.

Table 1: Key parameters used for the simulations of SLM scheme

Parameters	*Value/Description*
Number of random data bits	10000
Modulation	QPSK
Over-sampling factor (Case I)	8
Route Number (M) (Case I)	2,4,8 & 16
Route Number (M) (Case II)	8
Number of Subcarriers (N) (Case II)	32,64,128 & 256

Case I: Comparison of PAPR reduction performance with different values of M while number of sub-carriers (N) is fixed at 128.

In this case, it is to be noted that the rotation factor is defined as $P_{m,n} \in [\pm1, \pm j]$. From Figure 6, it can be observed that the SLM method displays a significant level of PAPR reduction compared to the original OFDM signal. It can also be noted that increasing M leads to the improvement of PAPR reduction performance. However, it is difficult to achieve a linear growth of PAPR reduction performance with further increase in the value of M (like M>=8). Therefore in practical applications, it is preferred to take M=8, so as to avoid introducing too much computational complexity.

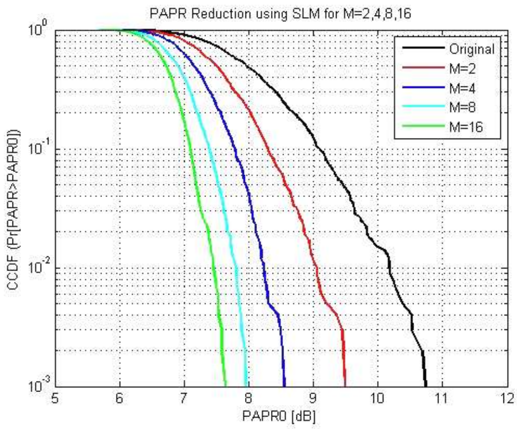

Figure 6: Performance of SLM PAPR reduction scheme with different values of *M*.

Case II: Comparison of PAPR reduction performance with different number of subcarriers (N) values; while, M is fixed at 8. It can be seen from Figure 7 that SLM technique is particularly suitable for OFDM with large number of sub-carriers.

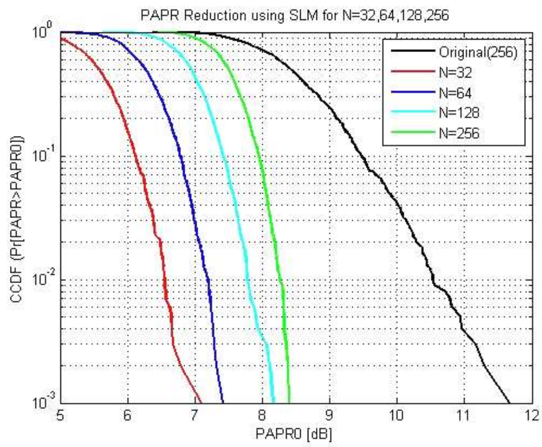

Figure 7: Performance of SLM PAPR reduction technique for different values of *N*.

Thus from the above made discussion, SLM approach can significantly reduce the PAPR of OFDM signals. The increase of the number of OFDM signal frames M will raise the complexity dramatically; but, with benefit of small improvement of PAPR reduction performance. SLM technique can be adapted to any length of FFT frame; which means it can be used for different OFDM systems with different number of carriers. It is particularly suitable for the OFDM system with a large number of sub-carriers (more than 128). In terms of complexity, every time when SLM technique is applied, it requires calculating the M group IFFTs at the transmitter compared to only one on ordinary OFDM system, and its M of N points IFFTs operation needs $n_{mul} = M.\frac{N}{2}log_2N$ complex multiplication and $n_{add} = M.\frac{N}{2}log_2N$ addition, separately. These problems usually pose high difficulties on real OFDM implementation; therefore, it is required to reduce the computational complexity. Therefore in practical applications, to compromise with the computing complexity and improve the performance, M<=8 is usually taken.

3.1.2 Simulation of PTS scheme.

From the above made discussion, the performance of PTS approach depends upon the number of sub-blocks V and the number of possible phase value W. The impact of these two parameters on PTS scheme is discussed below.

Case I: Effect of number of sub-blocks (V) on PAPR reduction using PTS scheme.

In this case, QPSK modulation is applied for number of subcarriers (N=256) and V = *0, 2, 3 & 4*. The results obtained are shown in Figure 8.

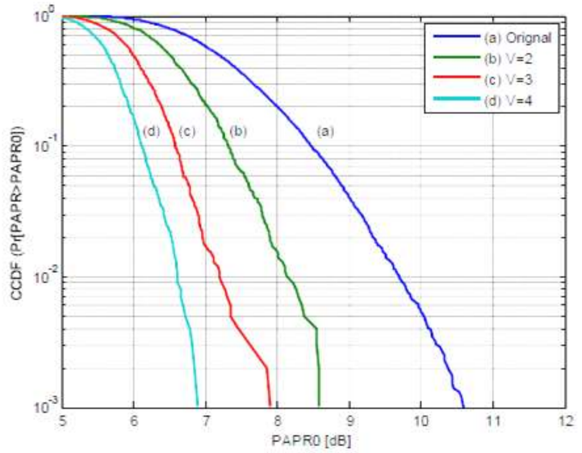

Figure 8: Performance of PTS PAPR reduction scheme for different values of *V*.

It can be seen from Figure 8 that PTS technique significantly reduces the PAPR of OFDM system. It can also be observed that with increasing the value of *V*, the PAPR is more reduced. However for higher values of *V*, the CCDF curve's fall off is less with increase in the value of *V*. This means large sub-block numbers *V* will result in small improvement of PAPR reduction performance and increases the hardware complexity. Therefore in practice, it is advisable to choose a suitable value of *V* to achieve a tradeoff in the use of PTS.

Case II. Effect of different value range W on the reduction of PAPR using PTS scheme.

The simulation results for this case are shown in Figure 9.

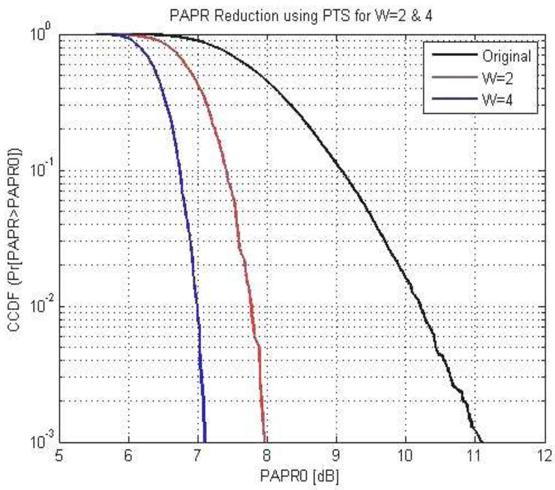

Figure 9: Performance of PTS PAPR reduction scheme for different values of *W*.

Figure 9 shows a varying PAPR reduction performance with different *W* (collection range of weighting factor) when using PTS reduction scheme. The parameters taken are: number of sub-carriers N = 128, QPSK modulation, oversampling factor (L=8) and the number of sub-blocks (V = 4). From Figure 4.6, it can be noticed that the CCDF curve has nearly 1dB improvement when W = 4, compared to W = 2, the 1% PAPR is about 7.5 dB. Therefore, it can be concluded that in a PTS OFDM system, the large value of W gives the better PAPR performance when the number of sub-block *V* is fixed.

3.3 Comparison of SLM and PTS Techniques

The comparison of SLM and PTS PAPR reduction techniques is shown in Figure. In PTS method, if the collection range of weighting factor is *W*; then for *V* sub-blocks, the system exhibits W^{V-1} types of auxiliary information sequence, and the number of redundant bits is $R_{ap} = (V-1)log_2 W$. For the same case in SLM method, if the length of sequence is *M*; then in SLM system, it requires redundant bits of $R_{ap} = log_2(M-1)$. Thus, PTS method requires a higher information redundancy compared to SLM technique under same circumstances.

Figure 10 shows the simulation results for reducing PAPR in an OFDM system using SLM and PTS methods. In PTS method, the number of sub-carriers is set to N = 128, modulation is QPSK and weighting factor is $b_v \in [\pm 1, \pm j]$; In SLM method, rotation factor is $P_{m,n} \in [\pm 1, \pm j]$.

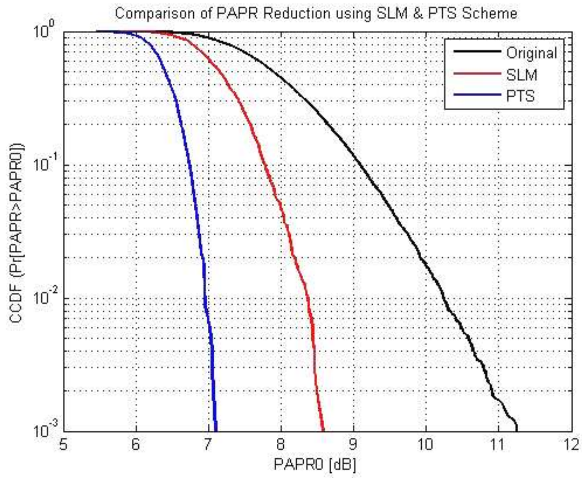

Figure 10: Comparison of PAPR reduction using PTS and SLM technique.

From the above mentioned analytical results, the IFFT calculations of these two methods is same when *V* = *M*. However for PTS method, it can provide more signal manifestations; thus, PTS method will provide a better performance on PAPR reduction. The same can be confirmed from simulation results shown in Figure 10. It can be observed from Figure 10 that with the same CCDF probability (1%), the PAPR value equals to 7dB when PTS is employed; while the PAPR rises up to 8.2dB when SLM is employed under the same circumstances.

4. Conclusion

This paper investigates the results obtained on applying a SLM & PTS scheme for reducing PAPR in OFDM systems. It has been observed from the results that PTS method provides a better PAPR reduction performance compared to SLM method. However, the transmitter and receiver complexity is very high. Thus in practical applications, a tradeoff needs to be made between good performance and auxiliary information. From the above made discussion, SLM technique is more suitable if system can tolerate more redundant information; otherwise, PTS technique is more acceptable when complexity becomes the first considering factor.

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