Approximation by Stancu Type Generalization of Beta Operators

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Abstract: In this paper, we are dealing with Stancu Beta operators $V_n^{\alpha,\beta}$ defined by (1.5). We establish direct and local approximation properties of these operators.

Keywords: Beta operators, Modulus of continuity, Rate of convergence, Direct and local approximation

1. Introduction

Gupta and Ahmad [9] introduced the Durrmeyer variant of the discrete beta operators to approximate Lebesgue integrable functions on the interval $[0,\infty)$. The beta operators from $C[0,\infty)$ into $C[0,\infty)$, the class of all bounded and continuous functions on $[0,\infty)$, are defined as

(1.1) $$(V_n f)(x)=\frac{1}{n}\sum_{v=0}^{\infty} b_{n,v}(x) f\left(\frac{v}{n+1}\right),$$

where

(1.2) $$b_{n,v}(x)=\frac{1}{B(v+1,n)}\frac{x^v}{(1+x)^{n+v+1}},\ x\in[o,\infty)$$

and $B(v+1,n)$ denotes the Beta function given by $\Gamma(v+1).\Gamma(n)/\Gamma(n+v+1)$.

In [4] D. D. Stancu introduced the following genrealization of Bernstein polynomials

(1.3) $$S_n^{\alpha}(f,x)=\sum_{k=0}^{\infty} f\left(\frac{k}{n}\right)P_{n,\alpha}^k(x),\ 0\le x\le 1,$$

where $P_{n,\alpha}^k(x)=\binom{n}{m}\dfrac{\prod_{s=0}^{k-1}(x+\alpha s)\prod_{s=0}^{n-k-1}(1-\infty+\alpha s)}{\prod_{s=0}^{n-1}(1+\alpha s)}$. We get the classical Bernstein polynomials by putting α = 0 in (1.3). Starting with two parameters α, β satisfying the conditions $0\le\alpha\le\beta$ in 1983, the other generalization of Stancu operators was given in [3] and studied the linear positive operators $S_n^{\alpha,\beta}:C[0,1]\to C[0,1]$ defined for any $f\in C[0,1]$ as follows:

(1.4) $$S_n^{\alpha,\beta}(f,x)=\sum_{k=0}^{\infty} P_{n,k}(x) f\left(\frac{k+\alpha}{n+\beta}\right),\ 0\le x\le 1,$$

where $P_{n,k}(x)=\binom{n}{k}x^k(1-x)^{n-k}$ are the fundamental Bernstein polynomials (cf. [8]). For $\alpha=\beta=0$, the polynomials in (1.4) are Bernstein polynomials. Atakut [2] gave Stancu type generalisation of Baskakov operators as follows:

$$L_n^{\alpha,\beta}(f,x)=\sum_{k=0}^{\infty}\frac{(-x)^k}{k!}\varphi_n^{(k)}(x) f\left(\frac{k+\alpha}{n+\beta}\right),$$

where $\varphi_n(x)=(1+x)^{-n}$ and established some approximation properties of these operators. She also shown the convergence of the derivative $\dfrac{dr}{dx^r}L_n^{\alpha,\beta}(f,x)$ to $f^r(x), r=1,2,....$ as $n\to\infty$ provided $f^r(x)$ exists.

Recently, Ibrahim [1] introduced Stancu-Chlodowsky polynomials and investigated convergence and approximation properties of these operators. Motivated by such type operators we introduce the operators as follow:

(1.5) $$V_n^{\alpha,\beta}(f,x)\equiv\left(V_n^{\alpha,\beta}f\right)(x)=\frac{1}{n}\sum_{k=0}^{\infty} b_{n,k}(x) f\left(\frac{k+\alpha}{n+1+\beta}\right),$$

where $b_{n,k}(x)$ is given in (1.2). $\left(V_n^{\alpha,\beta} f\right)(x)$ is called Beta Stancu operators. For $\alpha = 0 = \beta$, we get 1.1.

In the present paper, we study the rate of convergence and approximation properties of these operators by using modulus of continuity and K-functional of Peetre.

2. Preliminaries

In this section we require the following results:

Lemma 1. *For the functions* t^m, $m = 0, 1, 2$ *we have*

$$V_n^{\alpha,\beta}(1,x) = 1,\ V_n^{\alpha,\beta}(t,x) = \frac{n+1}{n+1+\beta}x + \frac{\alpha}{n+1+\beta}$$

$$V_n^{\alpha,\beta}(t^2,x) = \frac{(n+1)(n+2)}{(n+1+\beta)^2}x^2 + \frac{(n+1)(1+2\alpha)}{(n+1+\beta)^2}x + \frac{\alpha^2}{(n+1+\beta)^2}.$$

Proof. The operators $V_n^{\alpha,\beta}$ are well defined on functions 1, t, t^2 and

$\sum_{v=0}^{\infty} b_{n,k}(x) = n$. Then for every n Î ¥ and $x \in [0,\infty)$, we obtain

$$V_n^{\alpha,\beta}(1,x) = \frac{1}{n}\sum_{v=0}^{\infty} b_{n,k}(x) = 1.$$

Similarly,

$$V_n^{\alpha,\beta}(t,x) = \frac{1}{n}\sum_{k=0}^{\infty} b_{n,k}(x)\left(\frac{k+\alpha}{n+1+\beta}\right)$$

$$= \frac{1}{n}\sum_{k=0}^{\infty} b_{n,k}(x)\frac{k}{n+1+\beta} + \frac{\alpha}{n+1+\beta}$$

$$= \frac{1}{n(n+1+\beta)}\sum_{k=1}^{\infty} \frac{(n+k)!}{(k-1)!(n-1)}\frac{x^k}{(1+x)^{n+k+1}} + \frac{\alpha}{n+1+\beta}$$

$$= \frac{1}{(n+1+\beta)}\sum_{k=0}^{\infty} \frac{(n+k+1)!}{k!n!}\frac{x^{k+1}}{(1+x)^{n+k+2}} + \frac{\alpha}{n+1+\beta}$$

$$= \frac{x}{(n+1+\beta)}\sum_{k=0}^{\infty} b_{n+1,k}(x) + \frac{\alpha}{n+1+\beta}$$

$$= \frac{(n+1)x}{(n+1+\beta)} + \frac{\alpha}{n+1+\beta}.$$

Finally,

$$V_n^{\alpha,\beta}(t^2,x) = \frac{1}{n}\sum_{k=0}^{\infty} b_{n,k}(x)\left(\frac{k+\alpha}{n+1+\beta}\right)^2$$

$$= \frac{1}{n(n+1+\beta)^2}\sum_{k=0}^{\infty} \frac{(n+k)!}{k!(n-1)!}\left(k^2 + 2k\alpha + \alpha^2\right)\frac{x^k}{(1+x)^{n+k+1}}$$

$$= \frac{1}{(n+1+\beta)^2}\left[\frac{1}{n}\sum_{k=1}^{\infty} \frac{(n+k)!}{k!(n-1)!}k^2 + 2\alpha\frac{1}{n}\sum_{k=1}^{\infty}\frac{(n+k)!}{k!(n-1)!}k\right.$$

$$\left. + \alpha^2\frac{1}{n}\sum_{k=0}^{\infty}\frac{(n+k)!}{k!(n-1)!}\right]\frac{x^k}{(1+x)^{n+k+1}}$$

$$=\frac{1}{(n+1+\beta)^2}\sum_{k=0}^{\infty}\frac{(n+k+1)!}{k!(n-1)!}(k+1)\frac{x^k+1}{(1+x)^{n+k+2}}+\frac{x}{(n+1+\beta)^2}\sum_{k=0}^{\infty}b_{n+1,k}(x)+\frac{\alpha^2}{(n+1+\beta)^2}$$

$$=\frac{1}{(n+1+\beta)^2}\sum_{k=0}^{\infty}\frac{(n+k+2)!}{k!(n-1)!}\frac{x^{k+2}}{(1+x)^{n+k+3}}+\frac{(1+2\alpha)}{(n+1+\beta)^2}x\sum_{k=0}^{\infty}b_{n+1,k}(x)+\frac{\alpha^2}{(n+1+\beta)^2}$$

$$=\frac{(n+1)x^2}{(n+1+\beta)^2}\sum_{k=0}^{\infty}b_{n+2,k}(x)+\frac{(n+1)(1+2\alpha)x}{(n+1+\beta)^2}+\frac{\alpha^2}{(n+1+\beta)^2}$$

$$=\frac{(n+1)(n+2)x^2}{(n+1+\beta)^2}+\frac{(n+1)(1+2\alpha)x}{(n+1+\beta)^2}+\frac{\alpha^2}{(n+1+\beta)^2}$$

Remark 1. By simple computation we have

$$V_n^{\alpha,\beta}(t-x,x)=\frac{\alpha-\beta x}{n+1+\beta}$$

$$V_n^{\alpha,\beta}\left((t-x)^2,x\right)=\frac{(n+1+\beta^2)}{(n+1+\beta)^2}x^2+\frac{(n+1-2\alpha\beta)}{(n+1+\beta)^2}x+\frac{\alpha^2}{(n+1+\beta)^2}.$$

Lemma 2. *For* n Î ¥ , *we have*

$$V_n^{\alpha,\beta}\left((t-x)^2,x\right)\le\frac{(n+1+\beta^2)x(1+x)+\alpha^2}{(n+1+\beta)^2}.$$

Proof. For $0\le\alpha\le\beta$ and from the Remark 1, we have

$$V_n^{\alpha,\beta}\left((t-x)^2,x\right)=\frac{(n+1+\beta^2)}{(n+1+\beta)^2}x^2+\frac{(n+1-2\alpha\beta)}{(n+1+\beta)^2}x+\frac{\alpha^2}{(n+1+\beta)^2}$$

$$\le\frac{(n+1+\beta^2)}{(n+1+\beta)^2}x^2+\frac{(n+1-2\beta^2)}{(n+1+\beta)^2}x+\frac{\alpha^2}{(n+1+\beta)^2}$$

$$\le\frac{(n+1+\beta^2)x(1+x)+\alpha^2}{(n+1+\beta)^2}.$$

3. Main Results

In this section we establish direct and local approximation theorems in connection with the operators $V_n^{\alpha,\beta}$. Let $C[0,\infty)$ be the space of all real valued, bounded and uniformly continuous function on $[0,\infty)$ endowed with the norm $\|f\| = \sup\{|f(x)| : x \in [0,\infty)\}$.

Theorem 1. *For any* $f \in C[0,\infty)$, *one has for n sufficiently large. Then, for every* $x \in [0,\infty)$ *we have*

$$\left|V_n^{\alpha,\beta}(f,x)-f(x)\right|\le 2\omega(f,\delta),$$

where $\delta=\sqrt{\dfrac{(n+1+\beta^2)x(1+x)+\alpha^2}{(n+1+\beta)^2}}$ *and* $\omega(f,.)$ *is the usual modulus of continuity of f.*

Proof. Using the relation $\sum_{v=0}^{\infty}b_{n,v}(x)=n$, we have

$$V_n^{\alpha,\beta}(f,x)-f(x)=\frac{1}{n}\sum_{k=0}^{\infty}b_{n,k}(x)\left[f\left(\frac{k+\alpha}{n+1+\beta}\right)-f(x)\right]$$

and so

$$V_n^{\alpha,\beta}(f,x)-f(x)\le\frac{1}{n}\sum_{k=0}^{\infty}b_{n,k}(x)\left[f\left(\frac{k+\alpha}{n+1+\beta}\right)-f(x)\right]$$

taking $y=\dfrac{k+\alpha}{n+1+\beta}$ and $|y-x|\le\lambda\delta$, we have

$$|f(y)-f(x)|\le\omega(f,\lambda\delta)\le(1+\lambda)\omega(f,\delta)$$

Thus, we have

$$\left|f\left(\frac{k+\alpha}{n+1+\beta}\right)-f(x)\right|\le\left(1+\frac{\left|\frac{k+\alpha}{n+1+\beta}-x\right|}{\delta}\right)\omega(f,\delta)$$

$$\left|V_n^{\alpha,\beta}(f,x)-f(x)\right|\le\left(1+\frac{1}{n}\sum_{k=0}^{\infty}b_{n,k}(x)\frac{\left|\frac{k+\alpha}{n+1+\beta}-x\right|}{\delta}\right)\omega(f,\delta)$$

applying Cauchy Schwarz inequality, we have

$$\left|V_n^{\alpha,\beta}(f,x)-f(x)\right|\le\left(1+\frac{1}{\delta}\left\{\frac{1}{n}\sum_{k=0}^{\infty}b_{n,k}(x)\left(\frac{k+\alpha}{n+1+\beta}-x\right)^2\right\}^{1/2}\right)\omega(f,\delta)$$

$$\le\omega(f,\delta)\left(1+\frac{1}{\delta}\left\{V_n^{\alpha,\beta}\left((t-x)^2,x\right)\right\}^{1/2}\right).$$

In view of Lemma 2, by choosing $\delta=\sqrt{\dfrac{(n+1+\beta^2)x(1+x)+\alpha^2}{(n+1+\beta)^2}}$,

$$\left|V_n^{\alpha,\beta}(f,x)-f(x)\right|\le2\omega\left(f,\sqrt{\frac{(n+1+\beta^2)x(1+x)+\alpha^2}{(n+1+\beta)^2}}\right).$$

Hence, the required result.

Let $B_{x^2}[0,\infty)$ = {f: for every $x\in[0,\infty)$, $|f(x)|\le M_f(1+x^2)$, M_f being a constant depending of f}. By $C_{x^2}[0,\infty)$, we denote the subspace of all continuous functions belonging to $B_{x^2}[0,\infty)$. Also, $C^*_{x^2}[0,\infty)$ is subspace of all functions $f\in C_{x^2}[0,\infty)$ for which $\lim_{x\to\infty}\dfrac{f(x)}{1+x^2}$ is finite. The norm on $C^*_{x^2}[0,\infty)$ is $\|f\|_{x^2}=\sup_{x\in[0,\infty)}\dfrac{|f(x)|}{1+x^2}$.

For any positive number a, by

$$\omega_a(f,\delta)=\sup_{\substack{|t-x|\le\delta\\ x,t\in[0,a]}}|f(t)-f(x)|$$

we denote the usual modulus of continuity of f on the closed interval $[0, a]$. We know that for a function $f\in C_{x^2}[0,\infty)$, modulus of continuity $\omega_\alpha(f,\delta)$ tends to zero as $\delta\to0$.

Theorem 2. *Let* $f\in C_{x^2}[0,\infty)$ *and* ω_{a+1} *be its modulus of continuity of finite interval* $[0, a+1]\subset[0,\infty)$ *where* $a>0$. *Then for every n*

$$\left\|V_n^{\alpha,\beta}(f)-f\right\|_{C[0,a]}\le K\,6\left(\frac{(n+1+2\beta^2)a(1+a)+\alpha^2}{(n+1+\beta)^2}\right)$$

$$+2\omega_{a+1}\left(f,\sqrt{\frac{\left(n+1+2\beta^2\right)a\left(1+a\right)+\alpha^2}{\left(n+1+\beta\right)^2}}\right),$$

where $K=6M_f\left(1+a^2\right)$.

Proof. For $x\in[0,a]$ and $t>a+1$. Since $t-x>1$, we have

$$\left|f(t)-f(x)\right|\le M_f\left(2+x^2+t^2\right)$$
$$\le M_f\left(2+3x^2+2(t-x)^2\right)$$
$$\le 3M_f\left(1+x^2+(t-x)^2\right)$$
$$\le 6M_f\left(1+x^2\right)(t-x)^2$$

(3.1) $\le 6M_f\left(1+a^2\right)(t-x)^2$.

For $x\in[0,a]$ and $t\le a+1$, we have

(3.2) $\left|f(t)-f(x)\right|\le\omega_{a+1}\left(f,|t-x|\right)\le\left(1+\frac{|t-x|}{\delta}\right)\omega_{a+1}(f,\delta)$

with $\delta>0$.
From (3.1) and (3.2), we can write

(3.3) $\left|f(t)-f(x)\right|\le 6M_f\left(1+a^2\right)(t-x)^2+\left(1+\frac{|t-x|}{\delta}\right)\omega_{a+1}(f,\delta)$

For $x\in[0,a]$ and $t\ge 0$

$$\left|V_n^{\alpha,\beta}(f,x)-f(x)\right|\le V_n^{\alpha,\beta}\left(\left|f(t)-f(x)\right|,x\right)$$
$$\le 6M_f\left(1+a^2\right)V_n^{\alpha,\beta}\left((t-x)^2,x\right)+\omega_{a+1}(f,\delta)\left(1+\frac{1}{\delta}\left\{V_n^{\alpha,\beta}\left((t-x)^2,x\right)\right\}^{1/2}\right)$$

Hence, by Schwartz's inequality and Lemma 2, for every $x\in[0,a]$

$$\left|V_n^{\alpha,\beta}(f,x)-f(x)\right|\le 6M_f\left(1+a^2\right)V_n^{\alpha,\beta}\left((t-x)^2,x\right)$$
$$+\ \omega_{a+1}(f,\delta)\left(1+\frac{1}{\delta}\left\{V_n^{\alpha,\beta}\left((t-x)^2,x\right)\right\}^{1/2}\right)$$
$$\le 6M_f\left(1+a^2\right)\frac{\left(n+1+2\beta^2\right)x(1+x)+\alpha^2}{(n+1+\beta)^2}$$
$$+\ \omega_{a+1}(f,\delta)\left(1+\frac{1}{\delta}\left(\frac{\left(n+1+2\beta^2\right)x(1+x)+\alpha^2}{(n+1+\beta)^2}\right)^{1/2}\right)$$

taking $\delta=\sqrt{\frac{\left(n+1+2\beta^2\right)x(1+x)+\alpha^2}{(n+1+\beta)^2}}$

$$\left\|V_n^{\alpha,\beta}f-f\right\|_{C[0,a]}\le 6M_f\left(1+a^2\right)\left(\frac{\left(n+1+2\beta^2\right)a(1+a)+\alpha^2}{(n+1+\beta)^2}\right)$$
$$+\ 2\omega_{a+1}\left(f,\sqrt{\frac{\left(n+1+2\beta^2\right)a(1+a)+\alpha^2}{(n+1+\beta)^2}}\right).$$

which completes the proof.

Let the space $C[0,\infty)$ be endowed with the norm $\|f\|=\sup\{|f(x)|:x\in[0,\infty)\}$. Further let us consider the following Peetre's K-fucntional:

$$K(f,\delta)=\inf_{g\in C^2[0,\infty)}\left\{\|f-g\|+\delta\|g''\|\right\},$$

(cf. [6]).
It is clear that if $f\in C[0,\infty)$, $\delta>0$, then we have $\lim_{\delta\to 0}K(f,\delta)=0$. Some further results on Peetre's K-fucntional may be found in [10].

Theorem 3. *Let* $f \in C[0,\infty)$. *Then, for every* $x \in [0,\infty)$, *we have*

$$\left|V_n^{\alpha,\beta}(f,x) - f(x)\right| \le 2K(f,\delta) + \omega\left(f, \frac{|\alpha - \beta x|}{n+1+\beta}\right),$$

where $K(f,\delta)$ *is Peetre's K functional defined above and*

$$\delta = \frac{(n+1+2\beta^2)x^2(n+1-4\alpha\beta)x + \alpha^2 + \alpha}{(n+1+\beta)^2}.$$

Proof. We introduce the auxiliary operators defined by

$$V_n^{*\alpha,\beta}(f,x) = V_n^{\alpha,\beta}(f,x) - f\left(\frac{(n+1)x+\alpha}{n+1+\beta}\right) + f(x),$$

$x \in [0,\infty)$. These operators are linear and preserves the linear functions i.e. $V_n^{*\alpha,\beta}(t-x,x) = 0$

Let $g \in C^2[0,\infty)$. From Taylor's expansion of g

$$g(t) = g(x) + g'(x)(t-x) + \int_x^t (t-u)g''(u)\,du, \quad t \in [0,\infty)$$

we have

$$V_n^{*\alpha,\beta}(g,x) - g(x) = V_n^{*\alpha,\beta}\left(\int_x^t (t-u)g''(u)\,du, x\right)$$

$$\left|V_n^{*\alpha,\beta}(g,x) - g(x)\right| \le \left|V_n^{\alpha,\beta}\left(\int_x^t (t-u)g''(u)\,du, x\right)\right| + \left|\int_x^{\frac{(n+1)x+\alpha}{n+1+\beta}} \left(\frac{(n+1)x+\alpha}{n+1+\beta} - u\right)g''(u)\,du\right|$$

$$\le V_n^{\alpha,\beta}\left(\left|\int_x^t (t-u)g''(u)\,du, x\right|\right) + \int_x^{\frac{(n+1)x+\alpha}{n+1+\beta}} \left|\frac{(n+1)x+\alpha}{n+1+\beta} - u\right|\left|g''(u)\right|du$$

$$\le V_n^{\alpha,\beta}\left(\left|\int_x^t (t-x)g''(u)\,du, x\right|\right) + \int_x^{\frac{(n+1)x+\alpha}{n+1+\beta}} \left|\frac{\alpha-\beta x}{n+1+\beta}\right|\left|g''(u)\right|du$$

$$\le \left[V_n^{\alpha,\beta}\left((t-x)^2, x\right) + \left(\frac{\alpha-\beta x}{n+1+\beta}\right)^2\right]\|g''\|$$

$$\le \left[\frac{(n+1+2\beta^2)x(1+x) + \alpha^2 + (\alpha-\beta x)^2}{(n+1+\beta)^2}\right]\|g''\|$$

$$\le \left[\frac{(n+1+4\beta^2)x(1+x) + 2\alpha^2}{(n+1+\beta)^2}\right]\|g''\|$$

$$\left|V_n^{\alpha,\beta}(f,x) - f(x)\right| \le \left|V_n^{*\alpha,\beta}(f-g,x) - (f-g)(x)\right| + \left|V_n^{*\alpha,\beta}(g,x) - g(x)\right|$$

$$+\left|f\left(\frac{(n+1)x+\alpha}{n+1+\beta}\right) - f(x)\right|$$

$$\le 2\|f-g\| + \left|V_n^{*\alpha,\beta}(g,x) - g(x)\right| + \left|f\,\frac{(n+1)x+\alpha}{n+1+\beta} - f(x)\right|$$

$$\le 2\|f-g\| + \left[\frac{(n+1+4\beta^2)x(1+x) + 2\alpha^2}{(n+1+\beta)^2}\right]\|g''\|$$

$$+\left|f\left(\frac{(n+1)x+\alpha}{n+1+\beta}\right) - f(x)\right|$$

$$\leq 2\|f-g\|+\left[\frac{\left(n+1+4\beta^{2}\right)x(1+x)+2\alpha^{2}}{(n+1+\beta)^{2}}\right]\|g''\|+\omega\left(f,\frac{|\alpha-\beta x|}{n+1+\beta}\right),$$

where $\omega(f,.)$ is the usual modulus of continuity of *f*.

Taking infimum over all $g \in C^2[0,\infty)$, we have

$$\left|V_n^{\alpha,\beta}(f,x)-f(x)\right| \leq 2K\left(f,\frac{\left(n+1+4\beta^{2}\right)x(1+x)+2\alpha^{2}}{(n+1+\beta)^{2}}\right)+\omega\left(f,\frac{|\alpha-\beta x|}{n+1+\beta}\right).$$

This completes the proof of the theorem.

References

[1] B. Ibrahim, Approximation by Stancu-Chlodowsky polynomials, Comput. Math. with Appl. 59, 274-282 (2010).
[2] C. Atakut, On the approximation of functions together with derivatives by certain linear positive operators, Commun. Fac. Sci. Univ. Ank. Ser. Phys.-Tech. Stat. 46 (1-2) 57-65 (1997).
[3] D. D. Stancu, Approximation of functions by means of a new generalized Bernstein operator, Calcolo 20, 211-229 (1983).
[4] D. D. Stancu, Approximation of functions by a new class of linear polynomial operators, Rev. Roumaine Math. Pures Appl. 13, 1173-1194 (1968).
[5] Ingrid Oancea, A Bernstein Stancu type operator which preserves e2, An.S_t. Univ. Ovidius Constant_a, 17 (1), 145-152 (2009).
[6] J. Peetre, A theory of interpolation of norned spaces, Notes Mat. 39, 1-86 (1968).
[7] J. P. King, Positive linear operators which preserve x2, Acta Math. Hunger. 99 (3), 203-208 (2003).
[8] S. N. Bernstein, D_emonstration du th_eor_eme de Weierstrass fond_ee sur le calcul de probabilit_es, Commun. Soc. Math. Kharkow, 13 (2), 1-2 (1912-1913).
[9] V. Gupta, and A. Ahmad, Simultaneous approximation by modi_ed Beta operators, Instanbul Uni. Fen. Fak. Mat. Der. 54, 11-22 (1995).
[10] Z. Ditzian and V. Totik, Moduli of Smoothness, Springer, Berlin, 1987.

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